

The Transmission of Monetary Policy Operations through Redistributions and Durable Purchases*

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Abstract

A large literature has documented statistically significant effects of monetary policy on economic activity. The central explanation for how monetary policy transmits to the real economy relies critically on nominal rigidities, which form the basis of the New Keynesian (NK) framework. This paper studies a different transmission mechanism that operates even in the absence of nominal rigidities. We show that in an OLG setting, standard open market operations (OMO) conducted by Central Banks have important revaluation effects that alter the level and distribution of wealth in the economy and the incentives to work and save for retirement. Specifically, expansionary OMO lead households to front-load their purchases of durable goods and work and save more, thus generating a temporary boom in durables, followed by a bust. The mechanism can account for the empirical responses of key macroeconomic variables to monetary policy interventions. Moreover, the model implies that different monetary interventions (e.g., OMO versus helicopter drops) can have different qualitative effects on activity. The mechanism can thus complement the NK paradigm. We study extensions of the model incorporating labor market frictions and wage rigidities.

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1 Introduction

A central question in monetary economics is how monetary policy interventions transmit to the real economy. As emphasized by Woodford (2012) in his influential Jackson Hole symposium paper, in standard modern, general-equilibrium, frictionless asset pricing models, open market purchases of securities by Central Banks have no effect on the real economy. This result, which goes back to Wallace (1981)’s seminal article, is at odds with the widely held view that open market operations (OMO) by Central Banks affect interest rates—and at odds indeed with the very practice of Central Banks.

The irrelevance result is easiest to see in the context of a representative agent model, as explained by Woodford (2012);¹ however, Wallace (1981)’s widely cited result applies to a more general setting with heterogeneous agents. A key premise for Wallace’s irrelevance result, however, is that OMO by the Central Bank are accompanied by fiscal transfers that ensure no change in the income distribution following the monetary policy intervention. In other words, by construction, distributional effects of OMO are muted by fiscal transfers that neutralize distributional changes—and hence preclude any change in individuals’ decisions following the intervention.²

The goal of this paper is to study the effects of monetary policy interventions when, realistically, OMO are not accompanied by neutralizing fiscal transfers—nor is there a complete set of state-contingent securities that would ensure an unchanged income distribution following the policy intervention. The motivation is necessarily a practical one. When researchers estimate the causal effects of monetary policy interventions, they do not (cannot) abstract from or control for the distributional effects they cause—and there is no accompanying fiscal policy that undoes them. Hence, to understand the effects of those interventions on activity, researchers need to take into account the potential impact of the redistribution caused by the policy interven-

¹Suppose the central bank wishes to sell a risky asset (an asset with lower return in a bad state); one would think the private sector would be in principle only willing to buy it at a lower price. However, in the frictionless settings analyzed by Woodford (2012), even if the central bank keeps the risky asset, the risk does not disappear from the economy. The central bank’s earning on its portfolio are lower in the bad state and this means lower earnings distributed to the Treasury (and hence higher taxes to be collected from the private sector in the bad state). So the representative household’s after-tax income is equally exposed to risk, whether or not the household buys the asset. Thus asset prices are unaffected by the open-market operation.

²Wallace (1981) refers to this condition as “unchanged fiscal policy.” An unchanged fiscal policy in that context is one in which there is no change in government consumption and no change in the income or wealth distribution. To implement Wallace’s OMO without the redistributive effects, a Central Bank needs to ask the Treasury to change transfers and taxes in a particular way to keep the income distribution unchanged. An alternative way of obtaining this result would be to have a complete set of contingent securities that would undo any change in the income distribution.

tion. These redistributive effects can be significant. Indeed, Doepke and Schneider (2006a)'s empirical study points to an important revaluation of assets and redistributive effects from wealthy, old agents to younger generations and the government following monetary expansions. Similar results are found by Adam and Zhu (2014) for European countries and Canada. Relatedly, Coibion, Gorodnichenko, Kueng and Silvia (2012) find that monetary expansions reduce inequality, as measured by Gini coefficients, suggesting a redistribution away from wealthier individuals.

Under these premises, we analyse the effect of OMO on economic activity in a tractable dynamic, stochastic, general-equilibrium (DSGE) model. In the model, overlapping generations (OLG) of households use money, bonds, and durable goods to smooth consumption over their life cycle as well as the business cycle. In the baseline case, we assume that all prices are flexible and there are no labour market frictions—an assumption we later relax in the web Appendix. The inclusion of durable goods is motivated by the empirical finding that the response of economic activity to monetary policy shocks is almost entirely driven by the response of the durable-good sector, whereas the response of non-durable consumption is weak at best. The inclusion of government bonds is needed to capture the standard OMO, entailing sales and purchases of Treasury bonds, that is, changes in the composition of the Central Bank's balance sheet.

We show that in this setting, a monetary expansion carried out through OMO triggers a durable-driven boom in output, even under fully flexible prices and wages.³ Higher amounts of money in circulation lead to an increase in prices and a downward revaluation of nominal wealth. Thus, members of the public suffer a negative wealth effect, the magnitude of which depends on the size and composition of their asset portfolios. At the same time, the operation improves the financial position of the government via two channels. First, an increase in prices reduces the real value of government debt. Second, the operation increases the Central Bank's bonds holdings and consequently its stream of interest revenues, which are transferred to the Treasury.⁴ We assume that the Treasury follows a balanced budget policy and hence it uses the increased net income flows to finance a persistent reduction in (non-distortionary) taxes. While these tax cuts help to compensate households for their wealth losses, they do not prevent redistributive effects. In particular, old agents emerge as the biggest losers from the operation whereas future generations benefit the most. In between these extremes are agents who are in

³In a representative-agent model with durables, this nominal flexibility will immediately lead to money neutrality.

⁴In the data, these remittances amount to an average of two percent of government expenditures per year, with high variability over time.

the working phase of their lives when the shock hits. They suffer a negative revaluation of their retirement savings but do not receive full compensation from the Treasury once they retire. The outlook of a larger drop in consumption upon retirement creates incentives for young agents to work and save more, causing a decrease in the nominal and real interest rate. This drives up the demand for durable goods and increases output.⁵

To better understand the importance of agents' life cycle savings considerations, we also study a limit case of our baseline model with an infinitely-lived representative agent. In this limit case, monetary neutrality is obtained, as in Sidrauski (1967). This is because agents suffering a revaluation effect on their financial assets are compensated in equal amounts by current and future transfers from a fiscal authority rebating lump-sum transfers, thus precluding wealth effects and any change in behaviour. In the absence of nominal rigidities, real wages and relative prices are thus entirely determined by real factors. Nominal wage income and durable good prices therefore increase in tandem in the presence of inflation, and the increase in nominal wage income exactly offsets the desire to bring forward durable good purchases. This is true even though inflation does reduce the real value of financial wealth.⁶ Money neutrality in our model obtains under the same conditions in which Ricardian Equivalence holds (Barro 1974). By (realistically) precluding risk sharing of aggregate monetary policy shocks across generations, the model yields money non-neutrality even with flexible prices.⁷

We also show that in our model the real effects of open market operations can sharply differ from the effects of "helicopter drops," that is, tax cuts financed by an increase in the money supply. Although the effects of the two policies on nominal interest rates are very similar, an expansionary helicopter drop causes a bust in durables and a decline in output and hours. The

⁵In the Appendix we further extend the model to allow for search and matching frictions in the labour market and real wage rigidities. In this version of the model, the increase in employment due to the monetary expansion is caused by an increase in the demand for workers, rather than by the increased labour supply, as in our baseline model. The introduction of these rigidities—together with other frictions we abstract from—can help fine-tune the model to better match the data. For expositional clarity and to focus on the new mechanism, however, we keep the baseline model relatively simple.

⁶Recall the assumption that the government makes lump-sum transfers from seigniorage revenues to agents. Following Weil (1991)'s arguments, based on an endowment economy with helicopter drops, we show that also in an economy with production and durable goods, the reduction in wealth caused by OMO is exactly offset by future increases in government transfers, which renders money neutral. This intuition is perhaps not too easy to see in Weil (1991) because of a key mistake in the derivation of the formula for money holdings, going from equation 3.12 to 3.18, which blurs the interpretation.

⁷Allowing for fiscal transfers to exactly offset the heterogenous effects of monetary policy across different agents would restore the money neutrality in our model. Realistically, however, monetary policy shocks are not accompanied by targeted fiscal transfers aimed at undoing the monetary effects. Hence, to interpret the data and in particular the empirical evidence on the effect of monetary policy interventions, one cannot assume away the redistributive effects of monetary policy.

difference is driven by the distributional effects of the two policies. We therefore reach the conclusion that the precise instruments used to implement interest rate changes can become pivotal components of monetary policy once redistributive effects are taken into account.

The methodological strategy followed in this paper has much in common with the New Keynesian literature (e.g., Christiano, Eichenbaum, and Evans, 2005): we construct a dynamic equilibrium model and compare the responses to monetary policy shocks to those obtained from a structural Vector Auto-Regressive (VAR) model.⁸ We argue that our model can match many of the patterns in the data without relying on sticky prices and/or wages. We thus provide an alternative perspective on how monetary policy shocks transmit to the real economy, deliberately abstracting from sticky prices and wages. That said, there are no methodological barriers to incorporating nominal rigidities into our framework. In fact, doing so can address the criticism levied by Barsky, House, and Kimball (2003, 2007) against the New-Keynesian model. These authors integrate durable goods into an otherwise standard sticky-price framework and show that the model generates a counterfactual *decline* in durables following an expansionary monetary shock.⁹ By introducing a “retirement” phase, our model provides a mechanism that counteracts the channel highlighted by Barsky et al. (2007) and can thus help the NK model in mimicking the response of the economy.

The paper connects with a small but growing literature which seeks to study other channels for the transmission of monetary policy that can complement the standard channel based on nominal rigidities. Examples in this literature are Grossman and Weiss (1983), Rotemberg (1984), and Alvarez and Lippi (2012), who study the role of segmentation in financial markets and the redistributive effects caused by monetary policy.¹⁰ Lippi, Ragni, and Trachter (2013) provide a general characterization of optimal monetary policy in a setting with heterogeneous agents and incomplete markets. More quantitative analyses can be found in Doepke and Schneider (2006b), Meh, Ríos-Rull, and Terajima (2010), Algan, Allais, Challe and Ragot (2012) and

⁸Models in the New-Keynesian literature rarely allow for household heterogeneity due to computational challenges. An exception is Gorneman, Kuester and Nakajima (2012) who study monetary policy shocks in a model with sticky prices and uninsurable labor market risk. Other exceptions are McKay and Reis (2013) and Ravn and Sterk (2013).

⁹Specifically, when durable goods’ prices are relatively flexible, as appears to be the case in the data, these models predict that following a monetary expansion, non-durable purchases increase, while durable purchases, remarkably, decrease. In the case of fully flexible durable prices, the predicted contraction in the durable goods producing sector is so large that the monetary expansion has almost no effect on total aggregate output. See Klenow and Malin (2011) and references therein for a positive link between the durability of the good and the frequency of price adjustment.

¹⁰In our model, there is no financial segmentation: all agents can in principle participate in financial markets, though naturally some may endogenously choose not to hold any positions.

Gottlieb (2012). Like us, they numerically analyze the effects of monetary policy and/or inflation in a flexible price economy with aggregate dynamics and heterogeneous-agents. However, none of these papers models open market operations or consumer durables, both key elements of the transmission mechanism we highlight. Moreover, the heterogeneity in these models typically requires computationally heavy methods, which makes it difficult to incorporate shock processes that are realistic enough for a comparison to responses from a structural VAR.¹¹ By contrast, our model is solved quickly using standard linearization methods, allowing for a straightforward comparison to VARs as well as New-Keynesian DSGE models. To achieve this, we follow a simple stochastic ageing structure introduced in Gertler (1999), but work out a computational strategy that allows for standard preferences.¹²

The paper is organized as follows. Section 2 reviews the main empirical facts that motivate our model. Section 3 introduces the model. Section 4 performs various numerical exercises and discusses the findings in light of the empirical evidence. Section 5 offers concluding remarks.

2 Empirical Evidence

This Section first revisits the aggregate effects of monetary policy shocks on the macroeconomy, highlighting the role of durables. We do so using a structural VAR model. Following much of the literature, we identify monetary policy shocks using a recursive scheme. In addition, we consider a less restrictive set-identification strategy, following Uhlig (2005). In the web Appendix we study the data using Romer and Romer (2004)'s approach to identify exogenous monetary policy interventions. Finally, this section reviews the evidence on redistributive effects of monetary policy and inflation.

2.1 Monetary expansions and the response of durables

Policy and academic discussions on the economic effects of monetary policy interventions often rely on the relatively high sensitivity of the durables sector to interest rate changes. We corrob-

¹¹Doekpe and Schneider (2006) and Meh et al. (2010) model a one-time, unanticipated inflationary episode rather than recurring monetary policy shocks. Algan et al. (2012) and Gottlieb (2012) discretize the monetary shock process, giving rise to relatively stylized dynamics. Another substantial difference from Doekpe and Schneider (2006b) and Meh et al. (2010) is that both quantitative models counterfactually generate a contraction in activity following a monetary expansion, whereas our model generates a boom in activity driven by the durable good sector.

¹²Gertler's approach requires the utility function to be in a class of nonexpected utility preferences, excluding for example standard CRRA utility functions, whereas our model is instead compatible with the latter.

rate this premise by studying the U.S. evidence from 1966 until 2007 using a VAR approach. We find that monetary expansions lead to temporary booms in durables, with little or no increase in non-durables. This motivates the introduction of durables in our model, as the key variable driving the response of output.

The empirical analysis for measuring the effects of monetary policy shocks relies on a general linear dynamic model of the macroeconomy whose structure is given by the following system of equations (see for example, Olivei and Tenreyro, 2007):

$$Y_t = \sum_{s=1}^S \mathbf{A}_s Y_{t-s} + \sum_{s=1}^S \mathbf{B}_s P_{t-s} + \mathbf{G}^y v_t^y + \mathbf{G}^p v_t^p \quad (1)$$

$$P_t = \sum_{s=1}^S \mathbf{C}_s Y_{t-s} + \sum_{s=1}^S \mathbf{D}_s P_{t-s} + \mathbf{H}^y v_t^y + \mathbf{H}^p v_t^p \quad (2)$$

Uppercase letters are used to indicate vectors or matrices of variables or coefficients, lower-case letters denote vectors of structural disturbances, boldface letters denote time-invariant coefficients and S is the number of lags in the VAR. In particular, Y_t is a vector of non-policy macroeconomic variables (e.g., output, durable and non-durable purchases, aggregate and relative sectoral prices), and P_t summarizes the policy stance.

Equation (1) allows the non-policy variables Y_t to depend on lagged values of Y and P , on monetary policy shocks, denoted v_t^p , and on a vector of other macroeconomic disturbances, denoted v_t^y .¹³ Equation (2) states that the policy variable P_t depends on the same variables as Y_t , though of course with different coefficients.

In what follows we take the federal funds rate as the single measure of policy, and use innovations in the federal funds rate as a measure of monetary policy shocks, so P_t and v_t^y are scalars. Federal Reserve operating procedures have varied in the past 40 years, but several authors have argued that funds-rate targeting provides a good description of Federal Reserve policy over most of the period (see Bernanke and Blinder, 1992, and Bernanke and Mihov, 1998).

The time-invariant elements of \mathbf{B}_s , \mathbf{C}_s , \mathbf{D}_s and \mathbf{G}_s are estimated by ordinary least squares and do not depend on how structural shocks are identified. In the benchmark estimation, we use seasonally adjusted data from 1966:Q1 to 2008:Q1. The beginning of the estimation period

¹³Shocks are assumed to have zero mean and to be uncorrelated among each other and over time. Independence from contemporaneous economic conditions is considered part of the definition of an exogenous policy shock. The standard interpretation of v^p is a combination of various random factors that might affect policy decisions, including data errors and revisions, preferences of participants at the FOMC meetings, politics, etc. (See Bernanke and Mihov 1998).

is dictated by the behavior of monetary policy. After 1965 the federal funds rate started to exceed the discount rate, becoming the primary instrument of monetary policy. We stop in 2008:Q1 to avoid concerns with the potentially confounding effects from the financial crisis and the zero-lower bound.

The non-policy variables in the system include real GDP, the GDP deflator, durable-sector consumption purchases, non-durable consumption, and the relative price deflator of non-durables vis-à-vis durables.¹⁴ In our benchmark specification, all the variables in the vector Y are expressed in log levels. The policy variable, the federal funds rate, is expressed in levels. We formalize trends in the non-policy variables as deterministic, and allow for a linear trend in each of the equations of the VAR. We discuss next two different identification strategies.

2.1.1 Recursive identification of monetary policy shocks

To identify monetary policy shocks, we need to impose restrictions on \mathbf{G}^y , \mathbf{G}^p , \mathbf{H}^y and \mathbf{H}^p . Our first identification scheme embeds the restriction that policy shocks do not affect macro variables within the current period. That is, we assume that all elements in the vector \mathbf{G}^p are equal to zero. This assumption is sufficient to uniquely identify monetary policy shocks. Although debatable, this recursive identifying assumption is standard in VAR analyses.¹⁵ The effect of policy innovations on the non-policy variables is identified with the impulse-response function of Y to past realizations of v^p , with the federal funds rate placed last in the ordering.¹⁶ An estimated series for the policy shock can be obtained via a Choleski decomposition of the covariance matrix of the reduced-form residuals.

<<Figure 1 here>>

The estimated impulse-responses are depicted in Figure 1, together with 95 percent confidence bands around the estimated responses. We consider a monetary policy shock which corresponds to a 75-basis point decline in the funds rate on impact. For ease of comparison, the response of

¹⁴The source for all aggregates and their deflators is the Bureau of Economic Analysis, Quarterly National Income and Product Accounts. The sectoral deflators are chain-weighted indexes of the real deflators for the individual sub-categories, with the weight being the nominal shares of the sub-category on the sector's consumption. The source for the spot commodity price index is the Commodity Research Bureau.

¹⁵See, among others, Bernanke and Blinder (1992), Bernanke and Mihov (1998), Christiano et al. (1999) and (2005), Jean Boivin and Marc Giannoni (2006), and Julio Rotemberg and Michael Woodford (1997).

¹⁶The ordering of the variables in Y_t is irrelevant. Since identification of the dynamic effects of exogenous policy shocks on the macro variables Y only requires that policy shocks do not affect the given macro variables within the current period, it is not necessary to identify the entire structure of the model.

the three economic aggregates (GDP, durables, and non-durables) to the shock are graphed on the same scale across plots. The top left panel shows that the GDP response to the policy shock is persistent, peaking between 3 and 7 quarters after the shock and slowly decaying thereafter. The top right panel shows the response of durables, which, as it is apparent, is both fast and sizable. Durables increase on impact, reaching a level close to its maximum response 2 quarters after the shock. Moreover, the peak response for durables is more than three times as large as the GDP response. In contrast, the response of non-durables, depicted in the center left panel, is virtually insignificant, with its peak response being less than a fourth of that for durables. The price level (depicted in the center right panel) does not increase in the initial quarters following the shock, as it is standard in the literature. The plot shows that there is a slight “price puzzle,” although the change in prices is not statistically significant.¹⁷ It takes about 7 quarters after the shock for prices to start increasing. The bottom left panel shows that the response of the relative price of durables over nondurables is virtually insignificant. The response of the fed funds rate, shown in the bottom right panel, converges back to zero around the 8th quarter after the shock.

The differences in the responses of durables and non durables are substantial from an economic standpoint. Over the 2 years after the shock, the share of durable purchases in total GDP increases by 314 basis points, whereas the share of non-durables *decreases* by 94 basis points.

In the web Appendix we carry out an alternative exercise based on Romer and Romer’s narrative approach. The monetary policy shocks ε_t are quarterly averages of the monetary policy shocks identified by Romer and Romer (2004), extended through to 2008 by Coibion et al (2012). The results, illustrated in the web Appendix, are qualitatively similar (and quantitatively close) to those obtained using the VAR approach. The web Appendix also investigates the response of taxes following a monetary intervention, a response that becomes interesting in light of the model. With a monetary expansion, bond holdings held by the Central Bank increase—and so do its interest revenues. This leads to an increase in remittances from the Central Bank to the Treasury and thus to higher transfers (or lower taxes) to individuals, as appears to be the case in the data.

¹⁷We considered an extended VAR including a commodity price index, aimed at mitigating the “price puzzle” whereby a monetary tightening initially coincides with an increasing rather than a decreasing price level. Over our sample, however, we found only very small differences with the baseline VAR.

2.1.2 Set-identification of monetary policy shocks

Although standard in the literature, recursive identification of monetary policy shocks is controversial. In particular, the assumption that prices and quantities do not respond at all to a monetary policy shock within a quarter conflicts with most modern models of monetary policy, including standard sticky-price models. An additional limitation—which is especially important for our purpose—is that the assumed zero response of prices precludes an immediate redistribution among creditors and debtors of one-period nominal debt contracts.

To address these issues, we follow Uhlig (2005) and impose a weaker set of assumptions, merely restricting the sign of responses to monetary policy shocks and thus allowing all variables to move in the initial quarter the shock realizes. The cost we pay for weakening the identifying assumptions is that we no longer achieve point-identification of monetary policy shocks. Instead, we study a set of impulse responses. We impose the following set of restrictions on an expansionary monetary policy shock. First, the federal funds rate is assumed to remain negative during the three quarters after the initial quarter of the shock.¹⁸ This restriction avoids responses which feature only a very brief decline of the nominal interest rate followed by an interest rate hike; arguably, such patterns cannot be thought of as unambiguous monetary expansions. Second, we assume that the response of prices in the initial quarter of the shock is non-negative. This assumption is consistent with a wide range of monetary models, with and without nominal rigidities—and indeed with the model we postulate. Finally, we restrict the response of durables in the initial quarter to be non-negative. This restriction is perhaps the least innocuous one, but it is meant to exclude negative supply/demand shocks which trigger an endogenous reduction of the federal funds rate.¹⁹ We are reassured by the fact that in the estimated responses identified using Romer and Romer’s strategy—which imposes no restriction on the sign or magnitude of the durables response—durables increase significantly following a monetary expansion. More generally, note that our set of restrictions nests the responses obtained using the recursive identification scheme, as well as the responses based on Romer and Romer’s approach shown in the web Appendix. The restrictions are also consistent with our model, to be described in the next Section.

Figure 2 shows the set of responses identified using the sign-restrictions approach. For

¹⁸We also assume that the federal funds rate responds negatively in the initial quarter of the shocks, but this is merely a normalization.

¹⁹Conversely, one could be concerned that the identified policy shocks mainly pick up positive demand/supply shocks. But note that for this to be true it would have to be the case that the Fed responds to an expansionary macro shock by lowering the interest rate for a prolonged period of time.

comparability, all responses have been normalized to imply a decline in the federal funds rate of 75 basis points on impact. The light grey areas denote 90 percent bands of the responses which satisfy the sign restrictions, whereas the dark grey areas capture 70 percent bands. The green line is the average over the 0.1 percent responses which are closest to the median response of the price level in the initial period of the shock.

A striking feature of Figure 2 is the large range of price level responses, plotted in the middle panel in the right column. On impact, prices can respond up to 1.5 percent, sharply contrasting the zero-response imposed under the recursive identification scheme. Equally striking are the responses of durable purchases. These are in line with response under the recursive identification, but quantitatively the increase can be even much larger under the sign restrictions approach. The earlier key finding that durables respond much more to a monetary expansion than non-durables is preserved under the sign restrictions approach. In fact, non-durables respond negatively on impact in the majority of responses satisfying the sign restrictions.

2.2 Redistributive Effects of Monetary Policy

A main goal of our paper is to study the redistributive effects of monetary policy and their impact on aggregate variables in a quantitative model. A number of recent empirical papers substantiate our motivation. In particular, Doepke and Schneider (2006a) document significant wealth redistributions in the US economy following (unexpected) inflationary episodes. Their analysis is based on detailed data on assets and liabilities held by different segments of the population, from which they calculate the revaluation effects caused by inflation. The authors find that the main winners from a monetary expansion are the government and the young, whereas the losers tend to be rich, old households holding nominal bonds. Note that households as a whole are net creditors and the government is a net debtor in the US economy. Adam and Zhu (2014) document similar patterns for Euro area and Canada, and update the results for the United States. As for the US economy, in most euro-area countries, households are net creditors and the government is a net debtor.²⁰

Our model will embed these redistributive revaluation effects and will bring two additional considerations to the analysis. The first consideration is how these redistributive effects alter the various demographic groups' incentives to work, consume, and save in different types of assets, as well as how these changes affect the macroeconomy. The second consideration is

²⁰Looking at the disaggregated data for households, the age at which households become net creditors differs across countries.

how the Treasury redistributes the higher revenues stemming from an expansionary monetary policy intervention. These higher revenues consist of i) higher value of remittances received from the Central Bank as a result of the interest on bonds earned by the Central Bank; and ii) gains from the revaluation of government debt—assuming the government is a net debtor. The revaluation gains by the government can be large, as Doepke and Schneider (2006a)’s calculations illustrate. The remittances are also considerable, amounting to an average of two percent of total government revenues during our period of analysis, with significant volatility. In the baseline model, we assume that these remittances are rebated to the young (working agents), as in practice the taxation burden tends to fall on the working population. However, the framework can be adjusted to allow for different tax-transfer configurations.

An additional empirical paper motivating our analysis is Coibion et al. (2012), who find that unexpected monetary contractions as well as permanent decreases in the inflation target lead to an increase in inequality in earnings, expenditures, and consumption. Their results rely on the CEX survey, and thus exclude top income earners. The authors however argue that their estimates provide lower bounds for the increase in inequality following monetary policy contractions. This is because individuals in the top one-percent of the income distribution receive a third of their income from financial assets—a much larger share than any other segment of the population; hence, the income of the top one-percent likely rises even more than for most other households following a monetary contraction.

Consistent with these findings, in our model, monetary policy expansions cause a redistribution of income from old agents, who rely more heavily on wealth, to young agents and future tax payers. The consumption of goods by the young increases relative to that of old agents following a monetary expansion. These results are more directly examined by Wong (2014), who finds that total expenditures by the young increases relatively to those of older people following a monetary policy expansion, the latter identified through a recursive VAR assumption. In the web Appendix, using a different identification strategy, we study the responses by different demographic groups and furthermore explore the differences in the responses of durables and nondurables by the various groups. We find that indeed young households see an increase in expenditures relative to old households and that this response is almost entirely driven by the purchases of durable goods. These results lend support to the mechanism in our model, which generates a relative increase in durable consumption by young (working) agents vis-à-vis old (retired) agents following a monetary expansion.

3 Open market operations in a flex-price model

We study the dynamic effects of monetary policy shocks in a general equilibrium model which embeds overlapping generations and a parsimonious life cycle structure with two stages: working life and retirement. Transitions from working life to retirement and from retirement to death are stochastic but obey fixed probabilities, following Gertler (1999). (In the web Appendix we study a version of the model in which agents age deterministically and obtain results that are similar to the baseline model.) Financial markets are incomplete in the sense that there exists no insurance against risks associated with retirement and longevity. As a result, agents accumulate savings during their working lives, which they gradually deplete once retired. These savings can take the form of money, bonds, and durable consumption goods.

The money supply is controlled by a Central Bank, who implements monetary policy using open market operations, that is, by selling or buying bonds. Realistically, we assume that the Central Bank transfers its profits to the Treasury. The Treasury in turn balances its budget by setting lump-sum transfers to households. In this environment we study the dynamic effects of persistent monetary policy shocks. We contrast our benchmark model with an alternative economy in which the Central Bank uses “helicopter drops” of money rather than OMO to implement monetary policy.

We solve the model using a standard numerical method.²¹ This may seem challenging given the presence of heterogeneous households and incomplete markets. In particular, the presence of aggregate fluctuations implies that a time-varying wealth distribution is part of the state of the macroeconomy. To render the model tractable, we introduce a government transfer towards newborn agents which eliminates inequality among young agents.²² We show that aggregation then becomes straightforward and only the distribution of wealth between the group of young and old agents is relevant for aggregate outcomes. At the same time, our setup preserves the most basic life-cycle savings pattern: young agents save for old age and retired agents gradually consume their wealth.

Another advantage of our model with limited heterogeneity is that it straightforwardly nests a model with an infinitely-lived representative agent. One can show analytically that monetary policy shocks do not affect real activity under the representative agent assumption, provided that

²¹Specifically, we use first-order perturbation, exploiting its certainty-equivalence property. See the appendix for details.

²²Wealth inequality among retired agents, as well as between young and old, is preserved in our framework.

money and consumption enter the utility function separably.²³ This result is closely related to the fact that by construction redistributive effects are absent in an economy without heterogeneity. Also, the operating procedures of monetary policy (OMO versus helicopter drop) have the same effect on prices, a result that is broken down once we move beyond the representative agent assumption.

In the benchmark model discussed in this Section we do not incorporate any form of product or labor market friction. Hence, the monetary transmission in the model is very different to the transmission in New Keynesian models, which typically abstract from demographics and household heterogeneity in wealth. In the web Appendix we analyze the combined transmission of monetary policy shocks by introducing labor market frictions to the model.

3.1 Agents and demographics

We model a closed economy which consists of a continuum of households, a continuum of perfectly competitive firms and a government, which is comprised of a Treasury and a Central Bank. In every period a measure of new young agents is born. Young agents retire and turn into old agents with a time-invariant probability $\rho_o \in [0, 1)$ in each period. Upon retirement, agents face a time invariant death probability $\rho_x \in (0, 1]$ in each period, including the initial period of retirement. The population size and distribution over the age groups remains constant over time and the total population size is normalized to one. The fraction of young agents in the economy, denoted ν , can be solved for by exploiting the implication that the number of agents retiring equals the number of deaths in the population, i.e.

$$\rho_o \nu = \rho_x (1 - \nu + \rho_o \nu). \quad (3)$$

The age status of an agent is denoted by a superscript $\mathbf{s} \in \{\mathbf{n}, \mathbf{y}, \mathbf{o}\}$, with \mathbf{n} denoting a newborn young agent, \mathbf{y} a pre-existing young agent, and \mathbf{o} an old agent.

Households derive utility from non-durables, denoted $c \in \mathbb{R}^+$, a stock of durables, $d \in \mathbb{R}^+$, and real money balances, denoted $m \in \mathbb{R}^+$. They can also invest in nominal bonds, the real

²³This result by itself is not surprising, as (super)neutrality results for representative agent models with productive durables, have been known since the seminal work of Sidrauski (1967) and Fischer (1979). Sidrauski (1967) shows that when money enters the utility function separably, the rate of inflation does not affect real outcomes in the steady state. Fischer (1979) shows that under logarithmic utility this is also true along transition paths. Under alternative utility functions this is generally not true, but in quantitative exercises deviations from neutrality are often found to be quantitatively small, see for example Danthine, Donaldson and Smith (1987). In our benchmark model we will assume logarithmic utility and thus focus on a different source of non-neutrality.

value of which we label $b \in \mathbb{R}$. Bonds pay a net nominal interest rate $r \in \mathbb{R}^+$.

Young agents, including the newborns, supply labor to firms on a competitive labor market whereas old agents are not productive. Durables depreciate at a rate $\delta \in (0, 1)$ per period and are produced using the same technology as non-durables. Because of the latter, durables and non-durables have the same market price. All agents take laws of motion of prices, interest rates, government transfers and idiosyncratic life-cycle shocks as given. We describe the decision problems of the agents in turn.

3.2 Old agents

Agents maximize expected lifetime utility subject to their budgets, taking the law of motion of the aggregate state, denoted by Γ , as given. Letting primes denote next period's variables, we can express the decision problem for old agents ($\mathbf{s} = \mathbf{o}$) recursively and in real terms as:

$$\begin{aligned}
 V^{\mathbf{o}}(a, \Gamma) &= \max_{c, d, m, b} U(c, d, m) + \beta (1 - \rho_x) \mathbb{E}V^{\mathbf{o}}(a', \Gamma') \\
 \text{s.t.} & \\
 c + d + m + b &= a + \tau^{\mathbf{o}}, \\
 a' &\equiv (1 - \delta)d + \frac{m}{1 + \pi'} + \frac{(1 + r)b}{1 + \pi'}, \\
 c, d, m &\geq 0,
 \end{aligned} \tag{4}$$

where $V^{\mathbf{o}}(a, \Gamma)$ is the value function of an old agent which depends on the aggregate state and the real value of wealth, denoted by a , \mathbb{E} is the expectation operator conditional on information available in the current period, $\beta \in (0, 1)$ is the agent's subjective discount factor, and $\pi \in \mathbb{R}$ is the net rate of inflation. $U(c, d, m)$ is a utility function and we assume that $U_j(c, d, m) > 0$, $U_{jj}(c, d, m) < 0$ and $\lim_{j \rightarrow 0} U_j(c, d, m) = \infty$ for $j = c, d, m$. Finally, $\tau^{\mathbf{s}} \in \mathbb{R}$ is a transfer from the government to an agent with age status \mathbf{s} , so $\tau^{\mathbf{o}}$ is the transfer to any old agent.

The budget constraint implies that old agents have no source of income other than from wealth accumulated previously. Implicit in the recursive formulation of the agent's decision problem is a transversality condition $\lim_{t \rightarrow \infty} \mathbb{E}_t \beta^t (1 - \rho_x)^t U_{c,t} x_t = 0$, where $x = d, m, b$ and where $U_{c,t}$ denotes the marginal utility of non-durable consumption. Finally, we assume that agents derive no utility from bequests and that the wealth of the deceased agents is equally distributed among the currently young agents.

3.3 Young agents

Young agents supply labor in exchange for a real wage $w \in \mathbb{R}^+$ per hour worked. The optimization problem for newborn agents ($\mathbf{s} = \mathbf{n}$) and pre-existing young agents ($\mathbf{s} = \mathbf{y}$) can be written as:

$$V^{\mathbf{s}}(a, \Gamma) = \max_{c, d, m, b, h} U(c, d, m) - \zeta \frac{h^{1+\kappa}}{1+\kappa} + \beta(1 - \rho_o) \mathbb{E}V^{\mathbf{y}}(a', \Gamma') + \beta\rho_o(1 - \rho_x) \mathbb{E}V^{\circ}(a', \Gamma') \quad (5)$$

$\mathbf{s} = \mathbf{n}, \mathbf{y}$

s.t.

$$\begin{aligned} c + d + m + b &= a + wh + \tau^{bq} + \tau^{\mathbf{s}}, \\ a' &\equiv (1 - \delta)d + \frac{m}{1 + \pi'} + \frac{(1 + r)b}{1 + \pi'}, \\ c, d, m &\geq 0, \end{aligned}$$

where young agents too obey transversality conditions. The term $\zeta \frac{h^{1+\kappa}}{1+\kappa}$ captures the disutility obtained from hours worked, denoted h , with $\zeta > 0$ being a scaling's parameter and $\kappa > 0$ being the Frisch elasticity of labor supply. Bequests from deceased agents are denoted τ^{bq} ; as before, $\tau^{\mathbf{s}}$ is a lump-sum transfer from the government. When making their optimal decisions, young agents take into account that in the next period they may be retired, which occurs with probability $\rho_o(1 - \rho_x)$, or be deceased which happens with probability $\rho_o\rho_x$. We thus assume that upon retirement, young agents may be immediately hit by a death shock.

3.4 Firms

Goods are produced by a continuum of perfectly competitive and identical goods firms. These firms operate on a linear production function:

$$y_t = h_t. \quad (6)$$

Profit maximization implies that $w_t = 1$, that is, the real wage equals one.

3.5 Central bank

Although we do not model any frictions within the government, we make a conceptual distinction between a Central Bank conducting monetary policy and a Treasury conducting fiscal policy. We make this distinction for clarity and in order to relate the model to real-world practice.

The Central Bank controls the nominal money supply, $M_t \in \mathbb{R}^+$, by conducting open market operations. In particular, the Central Bank can sell or buy government bonds. We denote the nominal value of the bonds held by the Central Bank by $B_t^{\text{cb}} \in \mathbb{R}$. The use of these open market operations implies that in every given period the change in bonds held by the Central Bank equals the change in money in circulation, that is,

$$B_t^{\text{cb}} - B_{t-1}^{\text{cb}} = M_t - M_{t-1}. \quad (7)$$

The Central Bank transfers its accounting profit—typically called seigniorage- to the Treasury.²⁴ The real value of the seigniorage transfer, labeled $\tau_t^{\text{cb}} \in \mathbb{R}$, is given by:

$$\tau_t^{\text{cb}} = \frac{r_{t-1} b_{t-1}^{\text{cb}}}{1 + \pi_t}. \quad (8)$$

The above description is in line with how Central Banks conduct monetary policy, as well as with the typical arrangement between a Central Bank and the Treasury. By contrast, many models of monetary policy assume monetary policy is implemented using “helicopter drops,” that is expansions of the money supply that are not accompanied by a purchase of assets but instead by a fiscal transfer that is equal to the change in the money supply. Modern monetary models are often silent on how monetary policy is implemented and directly specify an interest rate rule. In our framework, however, the specific instruments used to implement monetary policy are critical, since the associated monetary-fiscal arrangements pin down redistributive effects and hence the impact of changes in monetary policy on the real economy.

When we implement the model quantitatively, we simulate exogenous shocks to monetary policy. We do so by specifying a stochastic process that affects the growth rate of the money supply M_t . The change in M_t is engineered using open market operations.

3.6 Treasury

The Treasury conducts fiscal policy. For simplicity, we abstract from government purchases of goods and assume that the Treasury follows a balanced budget policy. The government has an initial level of bonds B_{t-1}^{g} which gives rise to interest income (or expenditure if the government has debt) on top of the seigniorage transfer from the Central Bank. To balance its budget, the government makes lump-sum transfers to the households, which can be either positive or

²⁴We abstract from operational costs incurred by the central bank.

negative. The government's budget policy satisfies:

$$\nu \rho_o \tau_t^n + \nu (1 - \rho_o) \tau_t^y + (1 - \nu) \tau_t^o = \frac{r_{t-1} b_{t-1}^g}{1 + \pi_t} + \tau_t^{cb}. \quad (9)$$

Here, the left-hand side denotes the total transfer. In particular, $\nu \rho_o \tau_t^n$ is the total transfer to the newborns, $\nu (1 - \rho_o) \tau_t^y$ is the transfer to pre-existing young agents and b_t^g is the real value of government bonds. The right-hand side denotes total government income.

For tractability we also assume that the government provides newborn agents with an initial transfer that equalizes the wealth levels with the average after-tax wealth among pre-existing agents, that is,

$$\tau_t^n = a_t^y + \tau_t^y, \quad (10)$$

where $a_t^y \equiv \int_{i:s=y} a_{i,t} di$ is the *average* wealth among pre-existing young agents (before transfers). Since before-tax wealth is the only source of heterogeneity among young agents, all young agents make the same decisions and what arises is a representative young agent. This implication makes the model tractable. Note that although we eliminate heterogeneity among young agents by assumption, we do preserve heterogeneity between young and old agents, as well as heterogeneity among old agents.

Finally, we assume that only productive agents are affected by transfers/taxes, i.e. we set $\tau_t^o = 0$. This assumption is motivated by the reality that the majority of the tax burden falls on people in their working life, due to the progressivity of tax systems.²⁵

3.6.1 Market clearing and equilibrium

Aggregate non-durables and durables are given by:

$$c_t = \nu c_t^y + (1 - \nu) c_t^o, \quad (11)$$

$$d_t = \nu d_t^y + (1 - \nu) d_t^o, \quad (12)$$

²⁵We have solved a version of our model in which instead taxes are proportional to wealth levels, and obtained results similar to the ones obtained from our benchmark model. An alternative, behavioural assumption, suggested by David Laibson, would be to realistically assume that agents are not aware of these future transfers. We do not follow this avenue here, but we highlight that this would cause even bigger perceived wealth effects and intensify the aggregate responses we document.

where superscripts \mathbf{y} and \mathbf{o} denote the averages among young and old agents, defined analogously to the definition of $a_t^{\mathbf{y}}$.²⁶ Clearing in the markets for goods, money and bonds requires:

$$c_t + d_t = \nu h_t^{\mathbf{y}} + (1 - \delta) d_{t-1}, \quad (13)$$

$$m_t = \nu m_t^{\mathbf{y}} + (1 - \nu) m_t^{\mathbf{o}}, \quad (14)$$

$$0 = b_t^{\mathbf{s}} + b_t^{\mathbf{cb}} + \nu b_t^{\mathbf{y}} + (1 - \nu) b_t^{\mathbf{o}}. \quad (15)$$

Finally, the size of the bequest received per young agent is given by:

$$\tau_t^{bq} = \frac{\rho_x a_t^{\mathbf{o}} + \rho_o \rho_x a_t^{\mathbf{y}}}{\nu}. \quad (16)$$

We are now ready to define a recursive competitive equilibrium:

Definition. *A recursive competitive equilibrium is defined by policy rules for non-durable consumption, $c^{\mathbf{s}}(a, \Gamma)$, durable consumption, $d^{\mathbf{s}}(a, \Gamma)$, money holdings, $m^{\mathbf{s}}(a, \Gamma)$, bond holdings, $b^{\mathbf{s}}(a, \Gamma)$, labor supply, $h^{\mathbf{s}}(a, \Gamma)$, with $\mathbf{s} = \mathbf{n}, \mathbf{y}, \mathbf{o}, \mathbf{cb}, \mathbf{g}$, as well as laws of motion for inflation, the nominal interest rate and the real wage, such that households optimize their expected life-time utility subject to their constraints and the law of motion for the aggregate state, the Treasury and Central Banks follow their specified policies, and the markets for bonds, money, goods and labor clear in every period. The aggregate state Γ includes the value of the monetary policy shock, the distribution of wealth among agents, as well as the initial holdings of assets by households, the Treasury and the Central Bank.*

3.6.2 Three analytical results in a representative agent version

A special case of our model is obtained when we set the death probability to one, i.e. $\rho_x = 1$. In this case, agents immediately die upon retirement and old agents are effectively removed from the model. Given the absence of heterogeneity among young agents, the model becomes observationally equivalent to one with an infinitely-lived representative household with subjective discount factor $\tilde{\beta} = \beta(1 - \rho_o)$. In addition, we assume that non-durables, durables and real money balances enter the utility function separably. In particular, we assume the following logarithmic preferences: $U(c, d, m) = \ln c + \eta \ln d + \mu \ln m_t$, where $\eta, \mu > 0$ are preference parameters.

²⁶Due to the transfer to newborns $c_t^{\mathbf{y}} = c_t^{\mathbf{n}}$, $d_t^{\mathbf{y}} = d_t^{\mathbf{n}}$, $b_t^{\mathbf{y}} = b_t^{\mathbf{n}}$ and $m_t^{\mathbf{y}} = m_t^{\mathbf{n}}$.

This special case is useful to understand the role of household heterogeneity in the transmission of monetary policy, as several analytical results can be derived which contrast our numerical results to be presented in the next Section. The first result is:

Result 1. *Monetary policy is neutral with respect to real activity in the representative agent model.*

The arguments for the monetary neutrality follow Sidrauski (1967). The representative agents' first-order conditions for durables and labor supply, and the aggregate resource constraint are, respectively:

$$U_{c,t} = U_{d,t} + \tilde{\beta} (1 - \delta) \mathbb{E}_t U_{c,t+1}, \quad (17)$$

$$U_{c,t} = h_t^\kappa, \quad (18)$$

$$c_t + d_t = h_t + (1 - \delta) d_{t-1}, \quad (19)$$

where $U_{c,t} = \frac{1}{c_t}$, $U_{d,t} = \frac{\eta}{d_t}$ and for $t = 0, 1, \dots$. Given an initial level of durables and given that the utility function is separable in its arguments, these three equations pin down the equilibrium solution paths for c_t , d_t , and h_t in any period t without any reference to variables related to monetary policy. Given this solution it is straightforward to pin down output and the real interest rate as well.

Next, we consider how an unexpected monetary policy shock impacts on the price level in the representative agent world. We can derive the following result:

Result 2. *Monetary policy shocks impact on the prices solely through their effect on government transfers to the representative agent.*

This result can be seen from the government's consolidated (expected) present value budget constraint, which is derived in the web Appendix. It can be written as:

$$\mathbb{E}_t \sum_{s=t}^{\infty} D_s \left(\frac{r_s}{1+r_s} m_s - \tau_s^{\mathbf{g}} \right) = \frac{m_{t-1} - (1+r_{t-1})(b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}})}{1+\pi_t} \quad (20)$$

where $D_s \equiv \prod_{k=t}^{s-1} \frac{1+\pi_{k+1}}{1+r_k}$ is the agent's valuation of one unit of nominal wealth received in period $s > t$, $D_t \equiv 1$, and $\tau_t^{\mathbf{g}} \equiv \nu \rho_o \tau_t^{\mathbf{n}} + \nu (1 - \rho_o) \tau_t^{\mathbf{y}}$ is the total transfer to the households. The left-hand side represents the expected present value of the opportunity costs of holding money paid by the household, $\frac{r_s}{1+r_s} m_s$, which are income to the government, minus the transfers to

the households, τ_s^g . On the right hand side are the initial liabilities of the government. The web Appendix also demonstrates that from the neutrality of Result 1 it follows that D_s and $\frac{r_s}{1+r_s}m_s$, with $s \geq 1$, are unaffected by changes in monetary policy. Given that π_t is the only variable on the right-hand side that is not predetermined, it follows that its initial response to a monetary policy shock is fully pinned down by the change in the expected present value of the transfer. Thus, the impact of a monetary policy shock on the price level depends crucially on how seigniorage transfers to the households via the Treasury respond.

Intuition for Result 1, the irrelevance of monetary policy for real outcomes in the representative agent model, is obtained by considering the net wealth effects of changes in monetary policy (see Appendix). From the same equation above (Result 2) we can infer Result 3.

Result 3. *Changes in monetary policy do not create net wealth effects in the representative agent model.*

The flip-side of the government's budget constraint is the consolidated budget constraint of the households, excluding labor income. In particular, the initial liabilities of the government are equal to the value of bonds and money held by the public. The present value constraint shows that any such revaluation is *exactly* offset by a decline in the expected present value of transfers. Thus, a negative revaluation of the representative agent's nominal wealth following a surprise monetary expansion is exactly offset by a decline in transfers to be obtained from the government. Hence, there is no net wealth effect, an insight that is closely related to the seminal work of Barro (1978) and that was spelled out by Weil (1991) in the context of monetary model.

3.6.3 The transmission of OMOs with heterogeneous agents

The results derived for the special representative agent case help to understand the real effects of open market operations in the full model with heterogeneous agents. In this full version, agents are affected differently by monetary policy shocks for two reasons. The first is that portfolio size and compositions are heterogeneous across agents, affecting the extent to which they are affected by a surprise revaluation of nominal wealth. Second, agents are affected differently by a change in the path of transfers depending on their individual age status. Old agents can be expected to be disadvantaged by an expansionary monetary policy shock, since they suffer the negative revaluation of wealth but do not benefit from an increase in transfers. The same holds for a young agent who retires soon after a persistent and expansionary monetary shock. A monetary expansion also impacts directly on yet unborn generations, as the change in policy affects the

transfers they will receive from the government.

The redistributinal effects impact on agents' savings decisions. We can make this effect explicit by analyzing the young agents' first-order conditions for durables and bonds, respectively:

$$U_{c,t}^{\mathbf{y}} = U_{d,t}^{\mathbf{y}} + \beta(1 - \rho_o)(1 - \delta) \mathbb{E}_t U_{c,t+1}^{\mathbf{y}} + \beta \rho_o(1 - \rho_x)(1 - \delta) \mathbb{E}_t U_{c,t+1}^{\mathbf{yo}}, \quad (21)$$

$$U_{c,t}^{\mathbf{y}} = \beta(1 - \rho_o) \mathbb{E}_t \frac{(1 + r_t) U_{c,t+1}^{\mathbf{y}}}{1 + \pi_{t+1}} + \beta \rho_o(1 - \rho_x) \mathbb{E}_t \frac{(1 + r_t) U_{c,t+1}^{\mathbf{yo}}}{1 + \pi_{t+1}}, \quad (22)$$

where superscript **yo** denotes a newly retired agent. The right-hand sides of these equations capture the marginal utility benefits of saving through durables and bonds, respectively. Agents who retire face a reduction in their expected lifetime income and given the absence of insurance it holds in the stationary equilibrium that $U_c^{\mathbf{y}} < U_c^{\mathbf{yo}}$, i.e. the marginal utility of consumption (and wealth) increases upon retirement. Agents therefore have an incentive to save for retirement, mitigating the fall in consumption upon leaving the workforce.

Importantly, the redistributinal effect brought about by an expansionary monetary shock exacerbates the increase in the marginal utility of wealth upon retirement, i.e. $U^{\mathbf{yo}}$ increases further relative to $U^{\mathbf{y}}$. Young agents thus become more strongly incentivised to save for retirement during a monetary expansion. The above two first-order conditions make clear that this additional desire to save pushes down the real interest rate and encourages young agents to accumulate more durables. In our numerical simulations these effects result in an increase in aggregate durable expenditures in the equilibrium, pushing up aggregate output.

4 Quantitative simulations

In this Section we analyze the effects of open market operations in our model using numerical simulations. Before doing so we specify the details of household preferences and the monetary policy rule. We assume that the utility function is a CES basket of non-durables, durables and money, nested in a CRRA function:

$$U(c_{i,t}, d_{i,t}, m_{i,t}) = \frac{x_{i,t}^{1-\sigma} - 1}{1 - \sigma},$$

$$x_{i,t} \equiv \left[c_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta d_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu m_{i,t}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (23)$$

where $\epsilon, \sigma, \eta, \mu > 0$. Here, ϵ is the elasticity of substitution between non-durables, durables and money, σ is the coefficient of relative risk aversion, and η and μ are parameters giving utility

weights to durables and money, respectively. Computation of the dynamic equilibrium path seems complicated due to the high dimensionality of the aggregate state Γ_t . In the web Appendix we show that solving the model using a standard first-order perturbation (linearization) method is nonetheless straightforward under the above preference specification.²⁷

The Central Bank is assumed to set the money supply according to the following process:

$$\frac{M_t}{M_{t-1}} = 1 + z_t \quad (24)$$

where z_t is an exogenous shock process to the rate of nominal money growth, assumed to be of the following form:

$$z_t = \xi (\bar{m} - m_{t-1}) + \varepsilon_t, \quad \xi \in (0, 1), \quad (25)$$

where ε_t is an i.i.d. shock innovation and \bar{m} is the steady-state value of real money balances. A positive shock increases the money supply on impact. The above feedback rule implies that this increase is gradually reversed in subsequent periods when $\xi \in (0, 1)$.²⁸

4.1 Parameter values

Parameter values correspond to a model period of one quarter. The subjective discount factor, β , is set to 0.9732 which implies an annual real interest rate of 4 percent in the deterministic steady state. The steady state real interest rate is lower than the subjective discount rate, $1/\beta - 1$, due to the retirement savings motive arising in the presence of incomplete insurance markets. The durable preference parameter η is chosen to target a steady-state spending ratio of 20 percent on durables. To set the money preference parameter, we target a quarterly money velocity, defined as $\frac{y}{m}$, of 1.8. The intratemporal elasticity of substitution between non-durables, durables and money, ϵ , is set equal to one, as is the coefficient of relative risk aversion, σ . These two parameter settings imply that money and consumption enter the utility function additively in logs. Hence, our benchmark results are not driven by non-separability of money and consumption in the utility function. We set the Frisch elasticity of labor supply κ equal to one following many macro studies. The parameter scaling the disutility of labor, ζ , is set so as

²⁷In particular, we exploit the properties of first-order perturbation and show that the implied certainty equivalence with respect to the aggregate state allows us to express the decision rules of the old agents as linear functions of their wealth levels. This in turn implies that aggregation is straightforward and that only the distribution of wealth between *between* old agents and young agents is relevant for aggregate outcomes.

²⁸In equilibrium, both real and nominal money balances increase following the shock. Also, the rule implies that the net rate of inflation is zero in the steady state.

to normalize aggregate quarterly output to one.

Life-cycle transition parameters are set to imply a life expectancy of 60 years, with an expected 40 years of working life and expected 20 years of retirement. Accordingly, we set $\rho_o = 0.0063$ and $\rho_x = 0.0125$ which imply $\nu = 0.6677$. The depreciation rate of durables, δ , is set to 0.04 following Baxter (1996). The initial level of government debt is set to sixty percent of annual output. For simplicity we assume that the Central Bank starts off without any bond holdings or debt. The shock process parameter ξ is set to 0.2 which implies that the half life of the response for the nominal interest rate is about 2.5 years. Parameter values are presented in Table 1.

<<Table 1 here>>

4.2 The dynamic effects of open market operations

Figure 3 presents the responses to an expansionary monetary policy shock, implemented using open market operations. The magnitude of the shock is scaled to imply a reduction in the nominal interest rate of about 75 basis points. The shock increases the price level as well as aggregate output on impact. The responses of durables and non-durables make clear that this increase in output is entirely driven by an increase in expenditures on durables. Non-durables decline on impact, although the magnitude of the response is an order of magnitude smaller than the response of durables; this is consistent with the responses based on sign restrictions. (We come back to this point in Section 4.4.) Finally, the shock leads to a moderate decline in the (expected) real interest rate.

<<Figure 3 here>>

In the periods after the initial shock, the nominal interest rate and the price level gradually revert back to their initial levels and the same holds for non-durable purchases. This happens as a result of the reversion in the monetary policy rule. The booms in durables and output are also gradually reversed but the responses overshoot and turn into a bust several quarters after the shock, qualitatively in line with the empirical impulse responses. The overshooting in durable purchases and output is related to the large degree of endogenous persistence in the model, which is also reflected in the response of the real interest rate which continues to decline in the year following the shock and remains low quite persistently.

Figure 4 plots several variables that provide insight into the impact of monetary policy shocks as well as their endogenous propagation over time. The top left panel plots the response of real

money balances which increase on impact and then gradually revert back to the steady state, akin to the responses of the nominal interest rate and the price level. From the positive response of the price level it can be seen that nominal money balances increase as well. The top right panel plots the transfer to the young households, which increases by about 0.6 percent of output one year following the shock and gradually revert back in the years after.

<<Figure 4 here>>

The middle two panels show the responses of consumption by the young, whereas the bottom panels show the consumption responses of the young vis-à-vis the old agents. Relative to the old, consumption of durables and non-durables by the young increases, both on impact and during later periods. All households face a reduction in their real wealth due to the increase in prices, but the old are not compensated by an increase in transfers; hence, they lose relative to the young.²⁹ In absolute terms, consumption of durables by the young increases as well. This response can be understood by noting that a young agent benefits from the increase in transfers, but only up to the period in which she retires. This implies a steepening of the decline in the agent's income profile over the life cycle which increases the desire to save for retirement. In equilibrium, aggregate bond holdings are zero; moreover, the supply of money is determined largely by monetary factors and therefore cannot easily accommodate an increased aggregate desire to save. Bringing forward durable purchases, however, is an alternative way of saving that is not much restricted by supply factors since production can be shifted from non-durables towards durables. Indeed, the absolute level of non-durable purchases by the young declines on impact, driving the decline in aggregate non-durable purchases, though the size of this response is negligible when compared to the response of durable goods, as is the case in the data.

Several quarters after the shock, the response of non-durable purchases by the young turns from negative to positive. This sign switch is important to understand the bust in output that follows several periods after the shock. To see this, note that under the assumed preferences, the young agents' labor supply equation directly links output and the non-durable consumption of the young, as it can be written as:

$$\frac{1}{c_t^y} = \frac{y_t}{\nu}.$$

²⁹ Additionally, for old agents wealth is the only source of income, whereas the young agents also receive wage income, which in real terms is not directly affected by inflation. This is another reason why the young agents are less vulnerable to inflation.

Thus, under these preferences output and non-durable purchases necessarily co-move negatively. Intuitively, leisure is a normal good and the urge to buy durables makes young agents willing to forego leisure and non-durable consumption in the initial periods of the expansion. While remaining young, however, the young agents increasingly reap the benefits from the redistributive effects of the monetary policy, which increases their lifetime wealth and thus their demand for leisure and non-durables. The latter “pure wealth effect” starts to dominate several quarters after the initial shock and the non-durables response turns from negative to positive. The above discussion makes clear that this effect is also behind the boom-bust response of aggregate output. In Section 4.4 we discuss the negative comovement in light of the empirical results—it is consistent with the evidence based on sign restrictions, but not with that based on the recursive assumption; we also discuss how the negative comovement can be reversed within the model.

4.3 Helicopter drops

We now contrast the effects of open market operations to the effects of shocks in an alternative economy in which monetary policy is implemented using “helicopter drops” of money. By a helicopter drop we mean an expansion in the money supply that is not accompanied by an increase in Central Bank bond holdings, but rather an outright transfer to the Treasury.³⁰ It then follows that the total transfer from the Treasury to the households is given by its interest earnings on bond holdings (which can be negative) plus the change in the money supply. In real terms, the transfer to the households becomes:

$$\frac{r_{t-1}b_{t-1}^{\mathbf{g}}}{1 + \pi_t} + m_t - \frac{m_{t-1}}{1 + \pi_t} = \nu\rho_o\tau_t^{\mathbf{n}} + \nu(1 - \rho_o)\tau_t^{\mathbf{y}} + (1 - \nu)\tau_t^{\mathbf{o}} \quad (26)$$

We assume again that helicopter drops are gradually reversed after the initial shock, following the same feedback rule as used in the economy with market operations.³¹

Figures 5 and 6 plot the responses for the economy with helicopter drops, together with those for our benchmark economy with open market operations. Note first that the response of the nominal interest rate is virtually the same as it was before in the case of OMO. The figures show that although responses of prices and real money balances to helicopter drops are comparable to those in our benchmark economy with OMO, the effects on real economic outcomes are drastically different. In particular, with helicopter drops output and durable expenditures *decline* following

³⁰Consequently, $b_t^{\mathbf{cb}}$ remains zero at all times.

³¹For comparability, we do not re-scale the magnitude of the shock relative to the benchmark model.

an expansion of the money supply, whereas the real interest rate *increases* several periods after the shock. Thus, the transmission of monetary policy depends importantly on the operating procedures of the Central Bank and the associated monetary-fiscal arrangements.

<<**Figure 5 here**>>

<<**Figure 6 here**>>

The responses plotted in Figure 6 reveal why the effects of a monetary expansion are so different in the two economies. First of all, the response of the government transfer to households is very different when helicopter drops are used. Rather than a persistent increase in these transfers, there is a large increase after one year with a magnitude of about 2 percent of annual GDP. In later periods, there is a persistent decline in government transfers, relative to the steady state. Thus, in the economy with open market operations a monetary expansion is relatively favorable for those households in their working life long after the initial shock, which includes generations yet unborn in the initial period of the shock. The impact of a helicopter drop, by contrast, is more similar to a one-time redistribution between retired agents and agents in their working life. Hence the pure wealth effect dominates immediately following the initial helicopter drop and young households increase both leisure and non-durable consumption, as well as durables. As all three utility components are increased simultaneously, however, the response of durables is weaker than in the economy with OMO. Hence, the increase in durable purchases by the young is insufficient to offset the decline in durable purchases by the old, resulting in a decline in aggregate durable purchases.

4.4 The response of non-durables and the role of risk aversion

Although most of the responses implied by our benchmark model are in line with both the recursive and sign-restriction VAR evidence, the model predicts a decline in non-durables whereas the recursive VAR predicts a very small but nonetheless positive response. Interestingly, however, in the VAR with sign restrictions, a majority of the responses for nondurables show a decline, as in our model, and in contrast with the recursive VAR results. This is indeed a case in which the two identification strategies lead to different responses. Without taking a strong position on which one is the correct one, we explore next whether a plausible reparametrization of the model can be helpful in bringing the model closer to the recursive VAR responses.

Figure 7 plots the response to a monetary expansion implemented using OMOs, comparing the benchmark model to a version in which the coefficient of risk aversion, σ , is lowered from 1

to 0.6.³² The figure shows that under this parametrization, the output increase becomes more persistent. Whereas in the benchmark model the output turns negative after about one and a half year, this is increased to 2.5 years under lower risk aversion, close to the time of the sign switch in the empirical response. Also, the model now predicts a joint increase in non-durables, durables and output following a monetary expansion. This is a result that is difficult to obtain in sticky-price models, see Barsky, House and Kimball (2007). The response of durables is markedly smaller under the alternative parameterization, but still much larger than the response of non-durable consumption.

<<Figure 7 here>>

4.5 Other Extensions of the Model

We have explored before the role of risk aversion and how this parameter controls the response of nondurables to monetary policy shocks. In the web Appendix we study two additional extensions, briefly described here in the interest of space.

The first extension considers labour market rigidities by introducing a search and matching friction into the baseline model. A main result here is that a monetary policy expansion, by lowering real interest rates, increases the investment rate of firms. Firms invest by posting vacancies, leading to additional matches with workers. To firms –which are owned by the households– these matches are essentially durables, since they deliver profits over prolonged periods of time. Thus, for the precise same reason that durable expenditures increase following a monetary expansion, hiring increases. This in turn leads to a reduction in unemployment and an increase in output. The increase in output is still largely driven by durable consumption expenditures, despite the fact that in this model agents have another form of investment at their disposal (posting vacancies).

The second extension further enriches the model: we replace the stochastic ageing structure with a more realistic ageing setup. In this version, all agents live for a fixed number of 60 years, out of which they spend the final 20 years in retirement. Given that the model period is one quarter, the population consists of 240 generations at each point in time. Since the distribution of wealth across these generations affects prices and interest rates, the number of variables needed to fully characterize the state of the economy is large (though finite). Nonetheless, it is possible to solve this model using perturbation methods. In equilibrium, households are net

³²We also recalibrate the model to match the steady-state targets described previously.

borrowers during the initial years of their lives, as in the data. Moreover, the responses to an expansionary monetary policy shock are qualitatively in line with our baseline model, that is, the model predicts an increase in output driven by durables.

5 Concluding remarks

We study the effects of open market operations (OMO) in a general equilibrium model with a parsimonious life cycle structure and no nominal rigidities. We show that monetary expansions implemented through OMO generate negative wealth effects in the economy, with a more negative impact on old agents whose income stems from financial assets. Working agents respond to the monetary expansion by working and saving more and by accumulating durable goods. This causes a boom in output driven by the durable good sector. This response, together with the distributional effects embedded in the model are consistent with the empirical evidence on the effects of monetary interventions in the US economy.

Our model thus offers a setting consistent with i) the way in which Central Banks affects the policy rate, i.e., mostly through OMO; ii) empirical estimates on how such changes affects the macroeconomy and more specifically, the durable good sector; and iii) empirical evidence on the distributional effects of monetary policy. Our results address the challenge posed by Barsky, House and Kimball (2003, 2007), who pointed out a key counterfactual prediction of the standard New Keynesian representative-agent model with durable goods. The mechanism emphasized in our model can thus be used to complement the workhorse New Keynesian model in monetary policy analyses. The model can be easily extended to feature search and matching frictions, real wage rigidities, and deterministic ageing, all extensions we explore and discuss in depth in the web Appendix.

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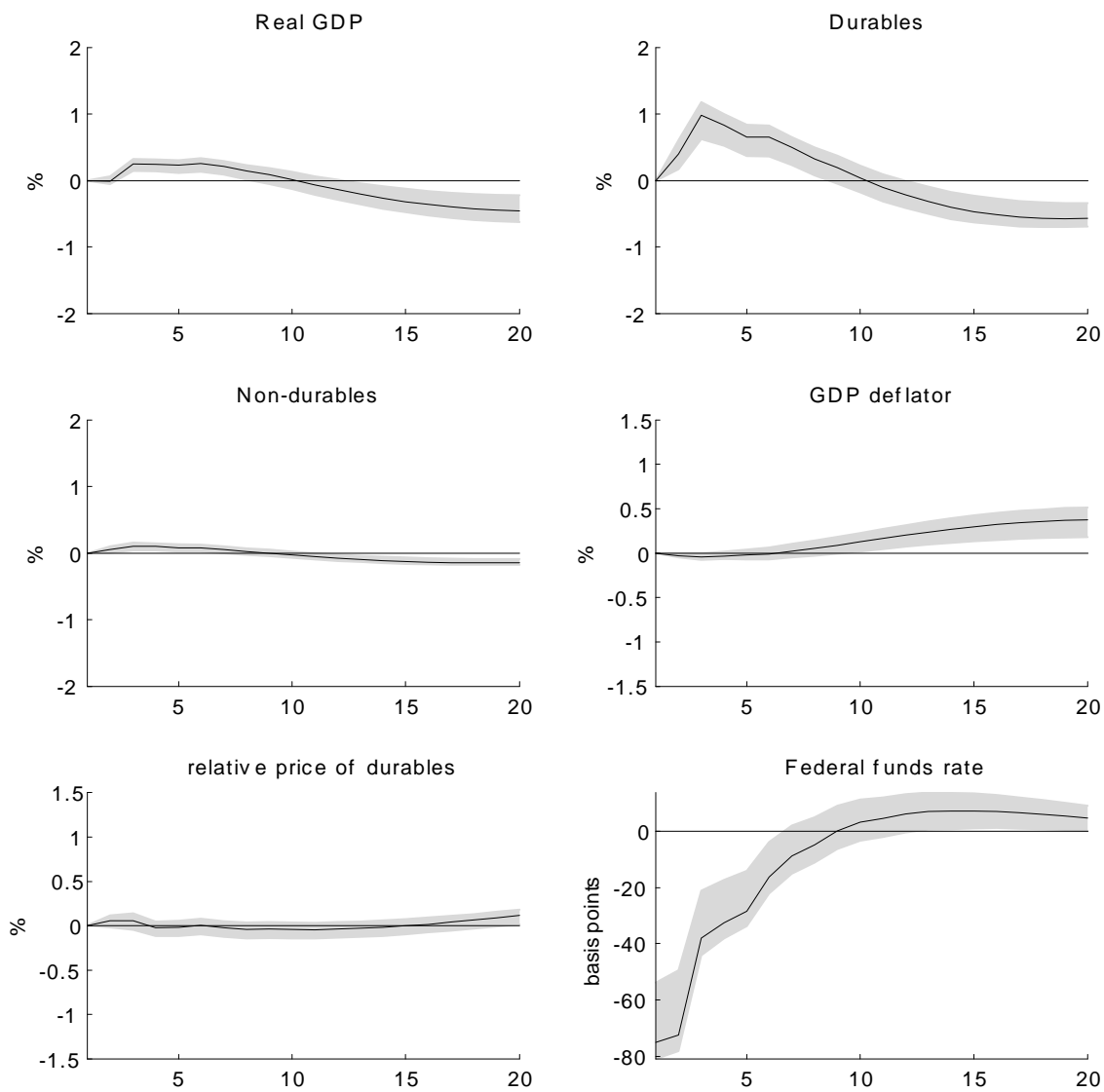
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Tables and Figures

Table 1. Parameter values for the baseline model.

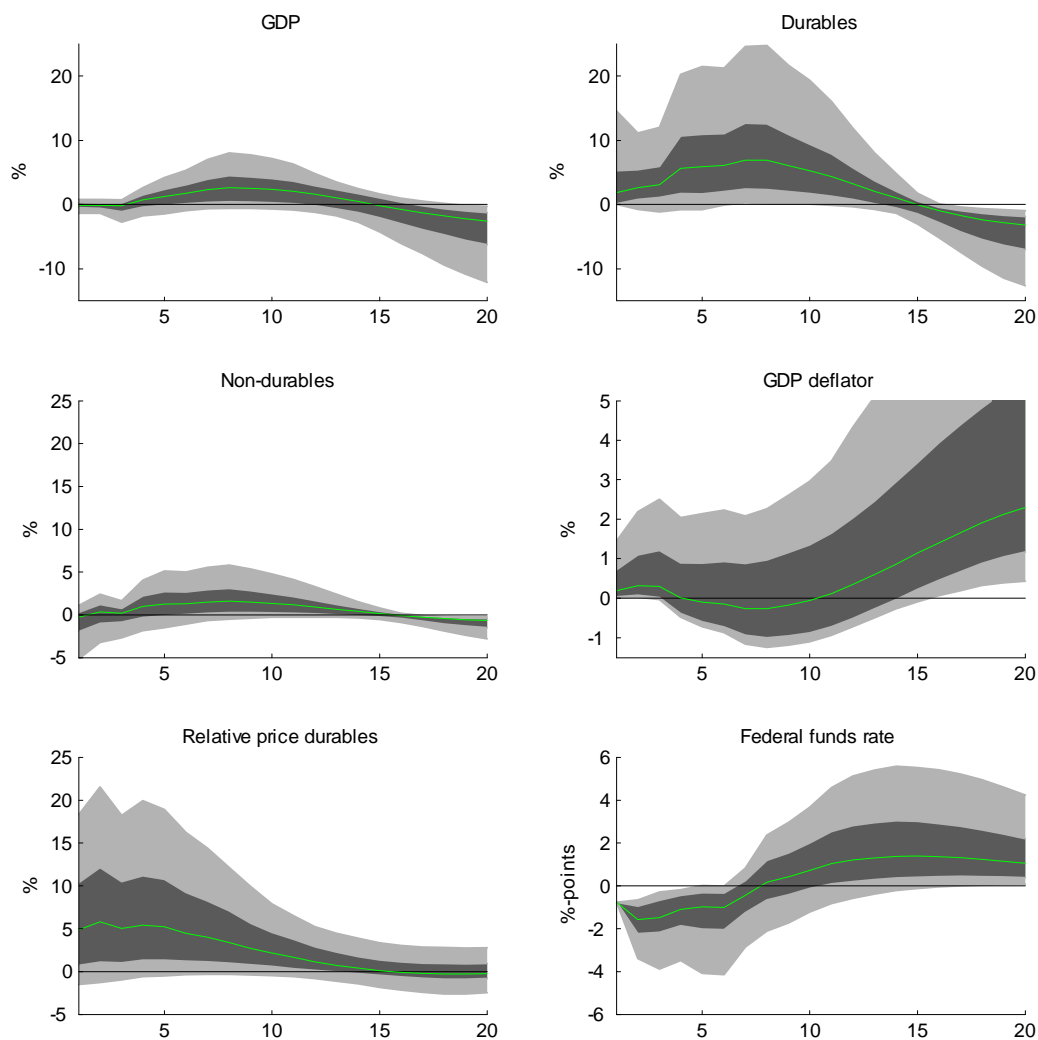
	value	description	motivation
β	0.9732	subjective discount factor	target 4% s.s. annual interest rate
η	0.31	durable preference param.	target 20% s.s. spending on durables (NIPA)
μ	0.0068	money preference param.	target 1.8 s.s. M2 velocity ($\frac{y}{m}$) (FRB/NIPA)
σ	1	coefficient of relative risk aversion	convention macro literature
ϵ	1	intra-temporal elast. of substitution	convention macro literature
κ	1	inv. elasticity labor supply	convention macro literature
ζ	0.5795	disutility of labor	normalize aggregate quarterly output to one
ρ_o	0.0063	ageing probability	average duration working life 40 years
ρ_x	0.0125	death probability	average duration retirement 20 years
δ	0.04	depreciation rate durables	Baxter (1996)
b_0^g	-2.4	initial bond holdings Treasury	government debt 60% of annual output
b_0^{cb}	0	initial bond holdings Central Bank	no initial central bank debt/bonds
ξ	0.2	coefficient monetary rule	half life response nominal interest rate 2.5 years

Figure 1: Impulse response function of headline variables to a monetary policy shock using a recursive VAR



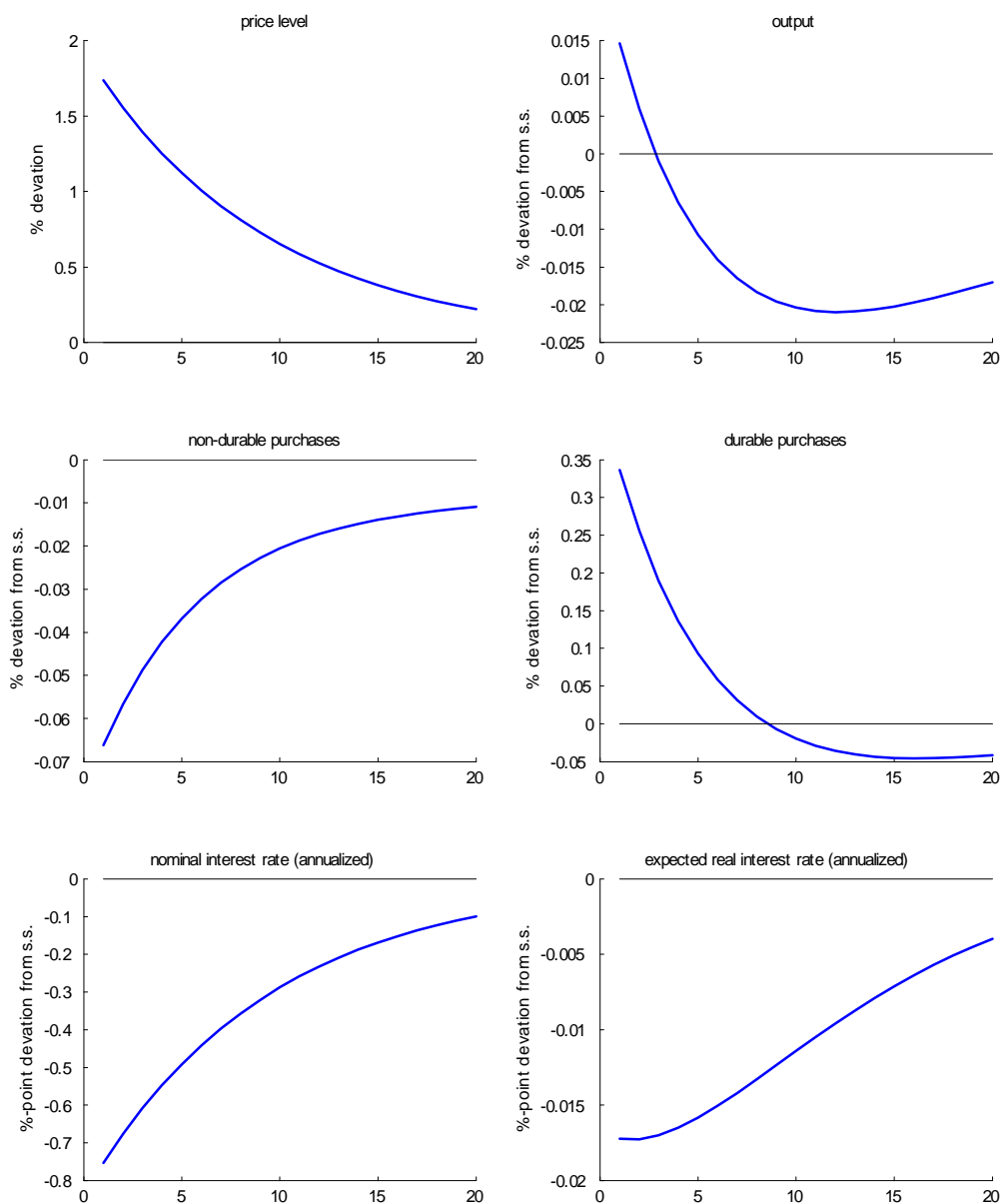
Notes: responses to an unanticipated monetary expansion, associated with a decline in the Federal funds rate of 75 basis points. Grey areas denote 95 percent error bands which were generated using a bootstrap procedure. The horizontal axes denote quarters after the shock.

Figure 2: Impulse response function of headline variables to a monetary policy shock in the sign-restrictions VARs.



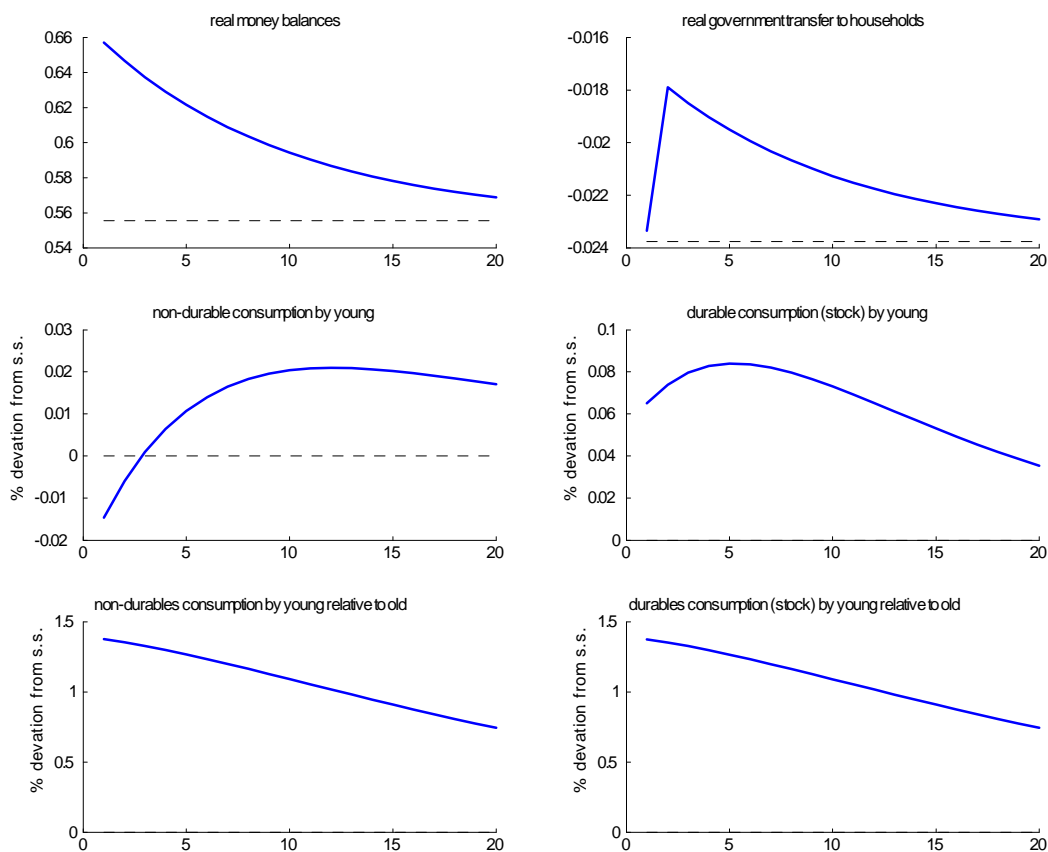
Notes: responses to an unanticipated monetary expansion in the sign-restrictions VAR. Responses are scaled to imply an initial decline in the Federal funds rate of 75 basis points. Light grey areas denote 70 percent bands of the identified responses, whereas dark grey areas denote 90 percent bands. The green line is the average over the 0.1 percent of the responses which are closest to the median response of the price level in the initial period of the shock. Horizontal axes denote quarters following the shock. No statistical confidence bands are plotted.

Figure 3: Model responses of aggregate variables to expansionary OMO.



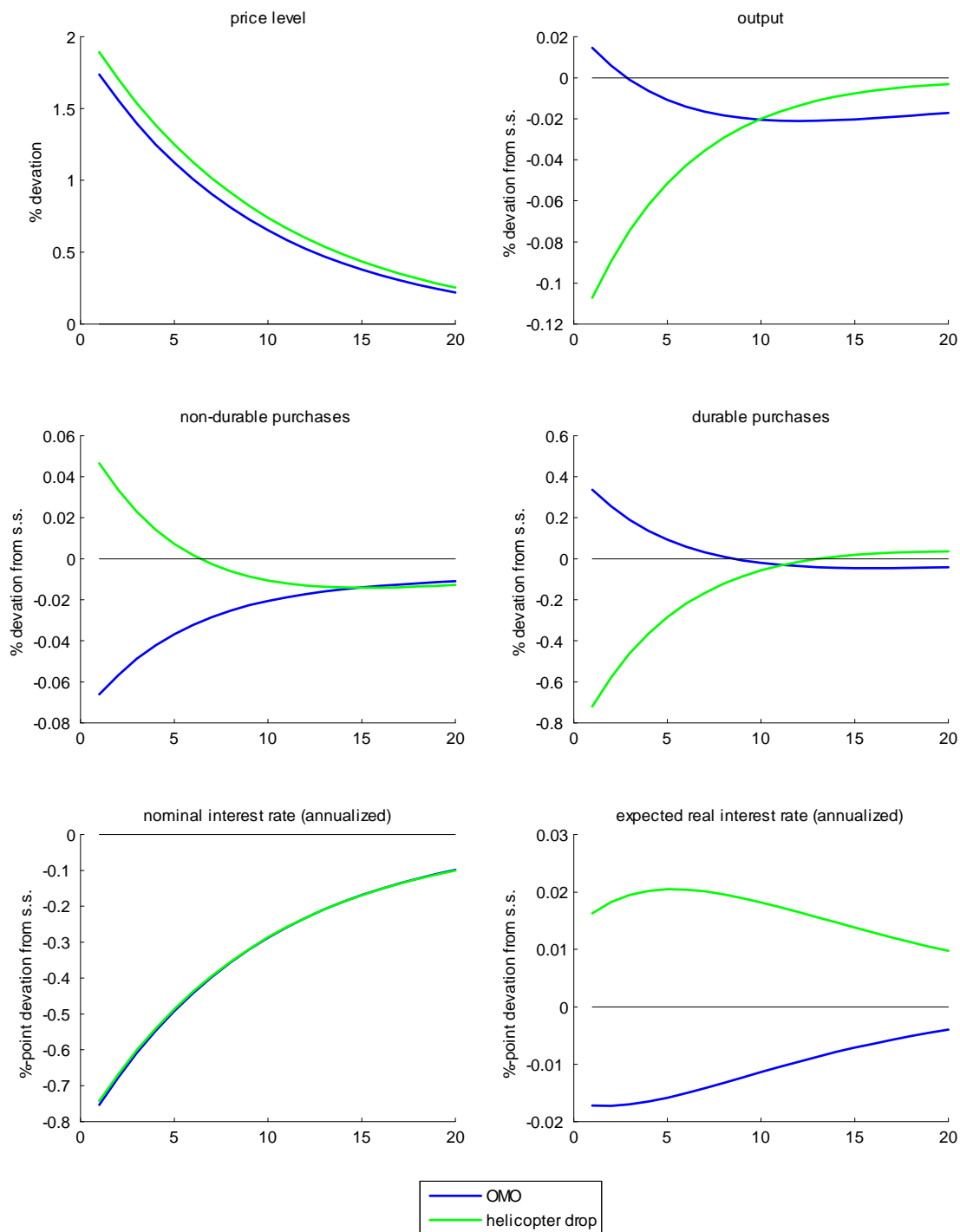
Notes: responses to an expansionary monetary policy shock –implemented using open market operations– in the baseline model. Horizontal axes denote quarters following the shock.

Figure 4: Model responses of money, transfers, and relative consumption to expansionary OMO.



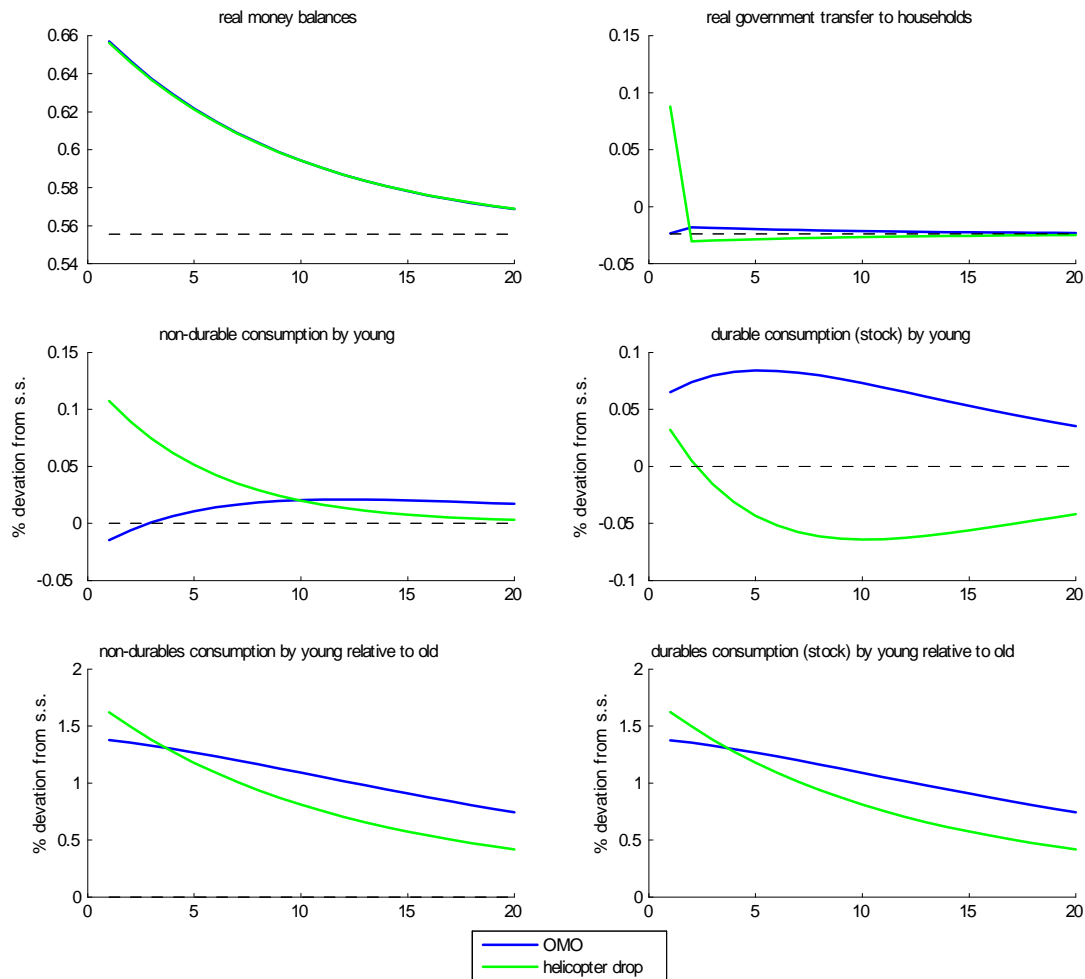
Notes: responses to an expansionary monetary policy shock –implemented using open market operations– in the baseline model. Vertical axes in the top panels denote levels, whereas those in the bottom panels denote percentage deviations from the steady state. Horizontal axes denote quarters following the shock.

Figure 5: Model responses to expansionary OMO versus helicopter drop.



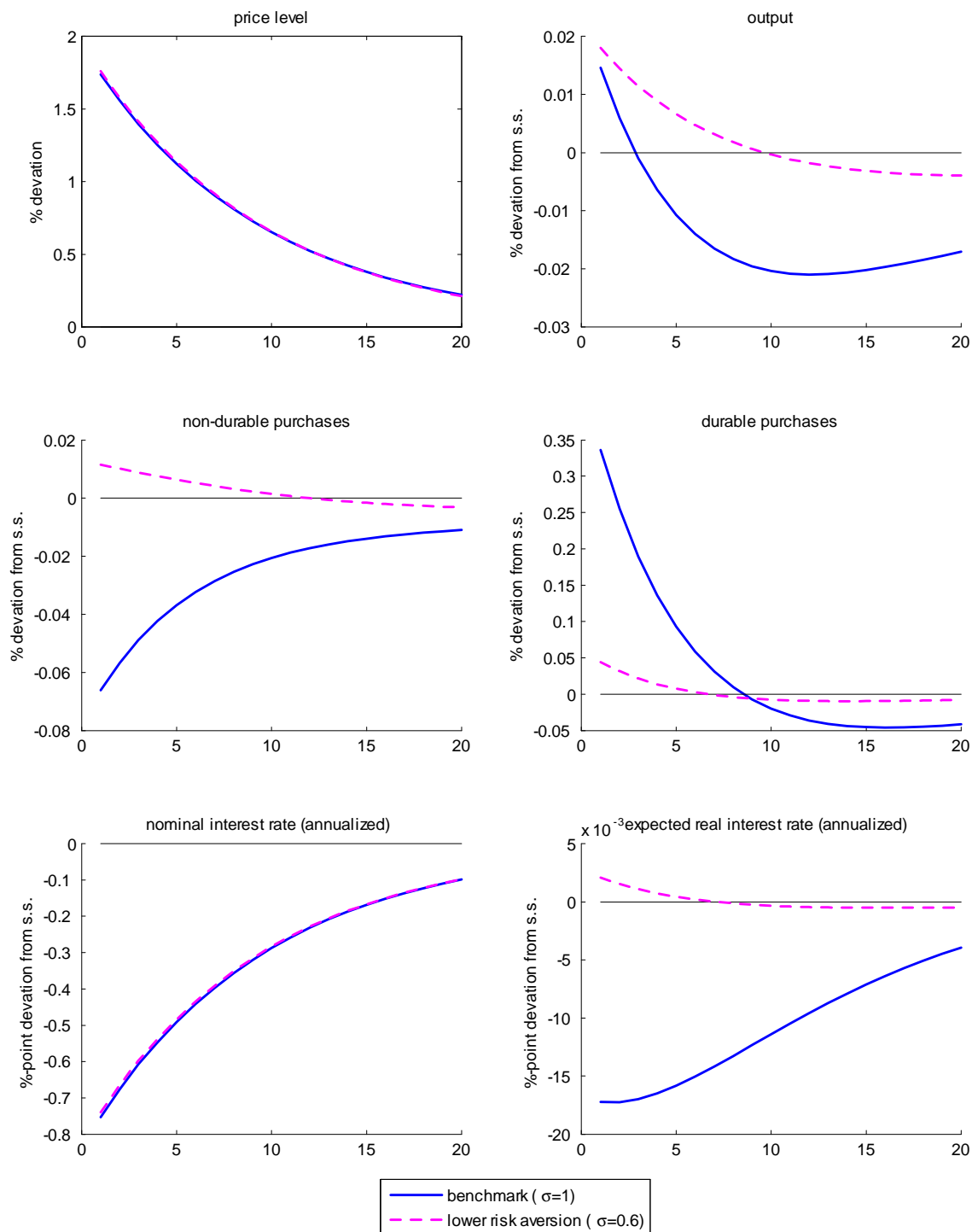
Notes: responses to an expansionary monetary policy shocks –implemented using open market operations (blue line) or helicopter drops (green line)– in the baseline model. Horizontal axes denote quarters following the shock.

Figure 6: Model responses to expansionary OMO versus helicopter drop.



Notes: responses to an expansionary monetary policy shocks –implemented using open market operations (blue line) or helicopter drops (green line)– in the baseline model. Vertical axes in the top panels denote levels, whereas those in the bottom panels denote percentage deviations. Horizontal axes denote quarters following the shock.

Figure 7: Model responses to expansionary OMO; benchmark versus lower risk aversion



Notes: responses to an expansionary monetary policy shock –implemented using open market operations– in the baseline model ($\sigma = 1$, blue solid line) as well as in a model with lower risk aversion ($\sigma = 0.6$, purple dashed line). Horizontal axes denote quarters following the shock.

Web Appendix not intended for publication: The Transmission of Monetary Policy Operations through Redistributions and Durables Purchases

by Vincent Sterk and Silvana Tenreyro

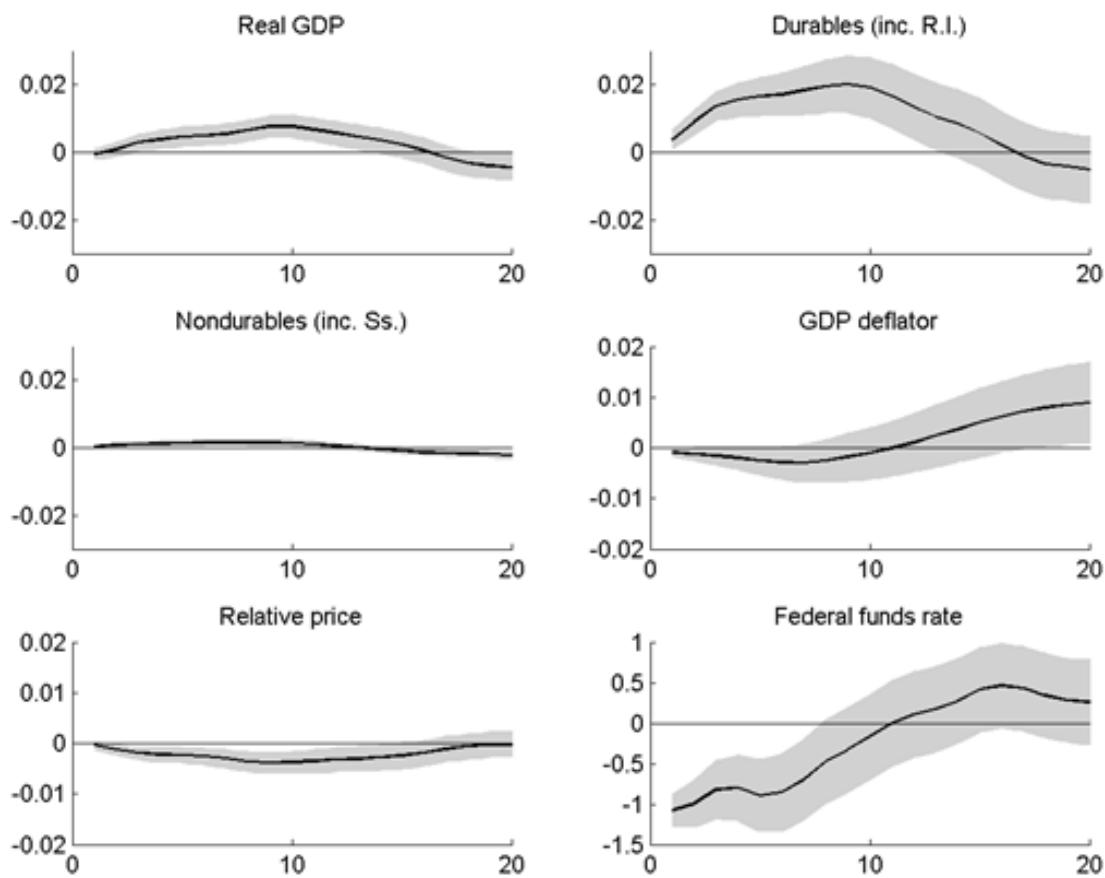
In this web Appendix we present additional evidence supplementing the empirical results, provide full derivation of the model equations and study extensions of the model that allow for search and matching frictions in the labour market as well as wage rigidity.

A1. Alternative Estimation Approach

In this section, we show the responses of key economic aggregates to monetary policy expansions using an alternative specification strategies and including taxes in the specification. Figure A1 shows the empirical response of the same macroeconomic variables as in Figure 1 when the identification of shocks relies on Romer and Romer (2004). As illustrated in the Figure, the results are qualitatively similar to those obtained from a recursive VAR, and remarkably close also from a quantitative point of view. The key difference from the results based on the identification with sign restrictions is in the response of prices. As discussed earlier, the response of the GDP deflator to a monetary policy expansion can be significantly positive when minimal restrictions are imposed.

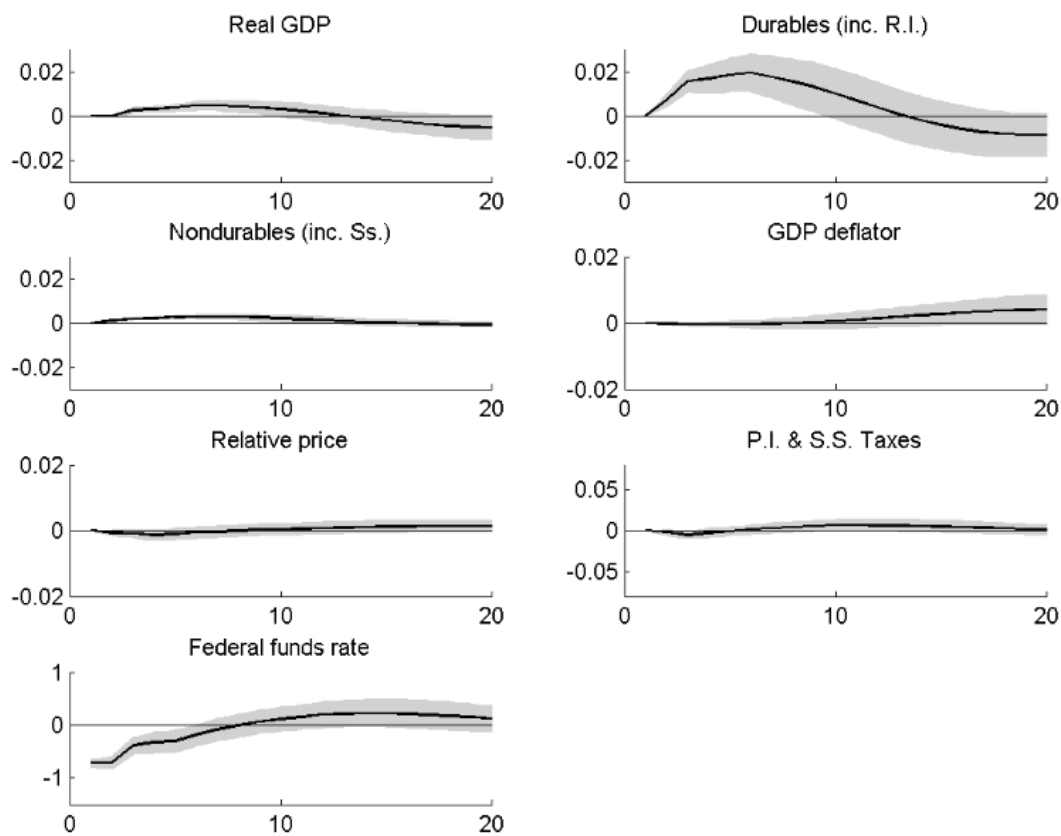
Figure A2 shows the empirical response of the macroeconomic variables using the recursive VAR identification as in Figure 1, but including personal income and social security taxes in the system. Figure A3 shows the same plots using the Romer and Romer identification. The responses of all variables is similar to those resulting from the baseline estimations without taxes. Interestingly, however taxes decrease following the monetary expansion. This is consistent with our model: with a monetary expansion, the bond holdings by the Central Bank increase, which leads to higher transfers from the Central Bank to the Treasury. This, in turn, leads to lower taxes (higher transfers) to individuals. The effect on taxes is quite persistent.

Figure A1: Impulse response function of headline variables to monetary policy shock Romer&Romer



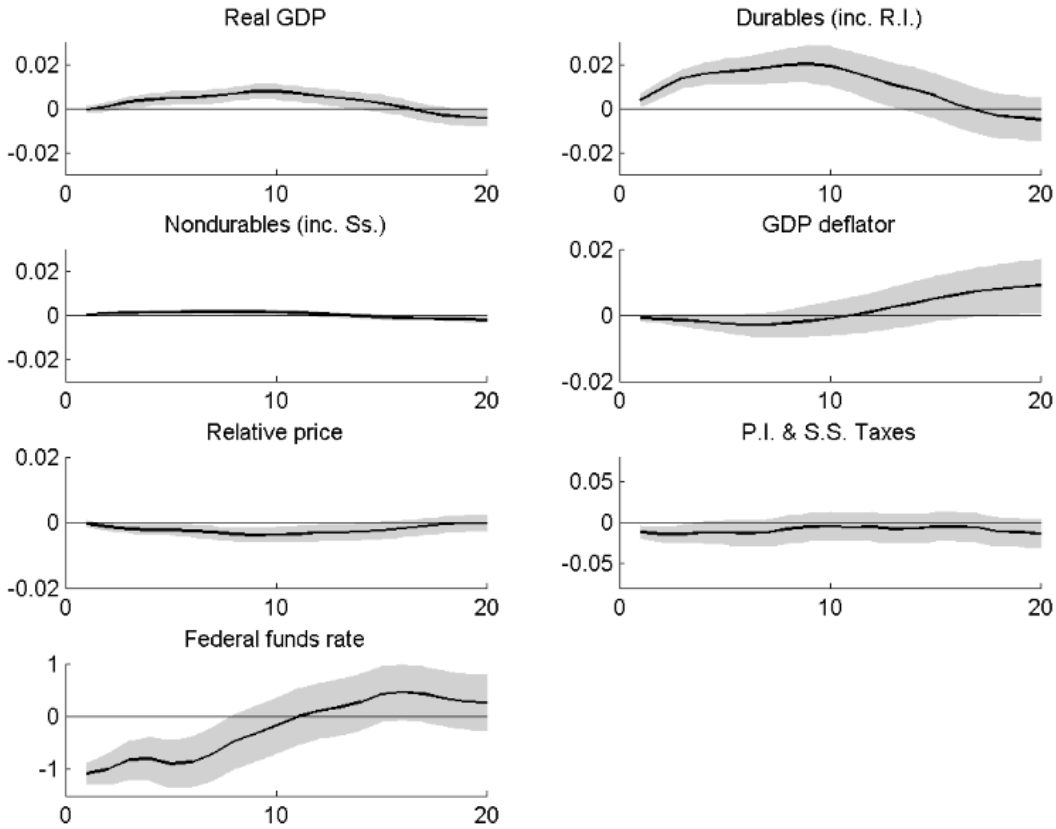
Notes: responses to an unanticipated monetary expansion, associated with a decline in the Federal funds rate of 75 basis points. Grey areas denote 95 percent error bands which were generated using a bootstrap procedure. The horizontal axes denote quarters after the shock. Romer& Romer identification.

Figure A2: Impulse response function of headline variables to monetary policy shock VAR approach



Notes: responses to an unanticipated monetary expansion, associated with a decline in the Federal funds rate of 75 basis points. Grey areas denote 95 percent error bands which were generated using a bootstrap procedure. The horizontal axes denote quarters after the shock. Recursive identification.

Figure A3: Impulse response function of headline variables to monetary policy shock Romer&Romer adding fiscal variables



Notes: responses to an unanticipated monetary expansion, associated with a decline in the Federal funds rate of 75 basis points. Grey areas denote 95 percent error bands which were generated using a bootstrap procedure. The horizontal axes denote quarters after the shock. Romer & Romer identification.

A2. The response of different demographic groups

Wong (2014) explores the response of expenditures to monetary policy shocks by different demographic groups. In this Section, we replicate her results and decompose expenditures in durables and nondurables to check the soundness of our model. We find that nondurable expenditures respond very little to monetary policy expansions, as in the aggregate results—and consistent with our model. This is true for all demographic groups. Durable good expenditures are the key variable responding to monetary interventions. Consistent with our model, we find that the increase in expenditures during a monetary expansion is almost entirely driven by the response

of young people; hence as in the model we present, the relative response of durables by the young vis-à-vis the old, increases significantly following an expansion. In what follows, we describe the data and approach.

A2.1. Data

The longitudinal data is based on the microdata of the consumer expenditure survey obtained from the ICPSR at the University of Michigan for years 1980-2007³³. Each household is surveyed for 4 subsequent quarters, where they report monthly expenditures at a disaggregated level. Information about the household demographics and finances are also available. Following Aguiar and Hurst (2012), only households that respond for all quarters (that is, with at least 12 months of data) are kept. Moreover, we keep only urban households, as rural households were not surveyed in the first covered years. This leaves a total of about 80,000 households.

Our measure of durables includes residential investments and other long-term expenditures such as vehicle purchases (new cars, parts) or recreational equipment. Nondurables include services as well as food, alcohol, tobacco, clothing, fossils consumption, and other miscellaneous categories.

The identified monetary policy shocks were obtained through the methodology of Romer and Romer (2004), extended up to 2007.

A2.2. Method

We run the following regression:

$$\Delta \ln y_{it} = \sum_{s=0}^{20} \beta_s mps_{t-s} + \text{Dummies} + e_{it}$$

Where the left hand-side is the log-change in consumption for household i at time t , mps_{t-s} is the monetary policy shock at t with s lags, and dummies include household and cohort (the year of birth) fixed-effects, as well as family size, the only demographic variable whose coefficient is significant. As we work in log-changes, we compute the cumulative IRF, that is, the cumulative sum of the beta coefficients. For the 95 percent confidence interval band, we follow Romer & Romer (2004)'s Monte Carlo approach in that we draw 10,000 coefficients from a multivariate normal distribution with mean vector and variance-covariance matrix from the OLS regression.

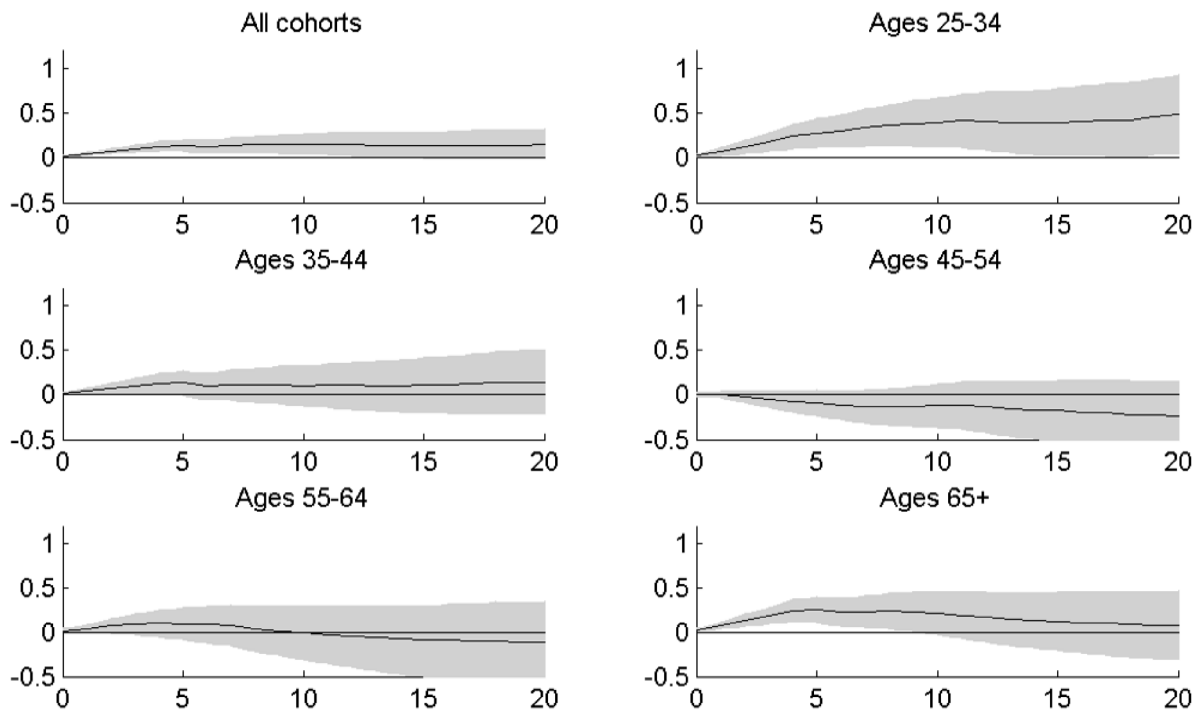
³³The CEX data is available up to 2012, but the analysis here is restricted to take place before 2007.

For each of these draws, the cumulative IRF is computed, and the 2.5 and 97.5 quantiles are kept to produce the bands of the confidence interval. Then, this regression is run for the entire cohort, and for different age groups.

A2.3. Results

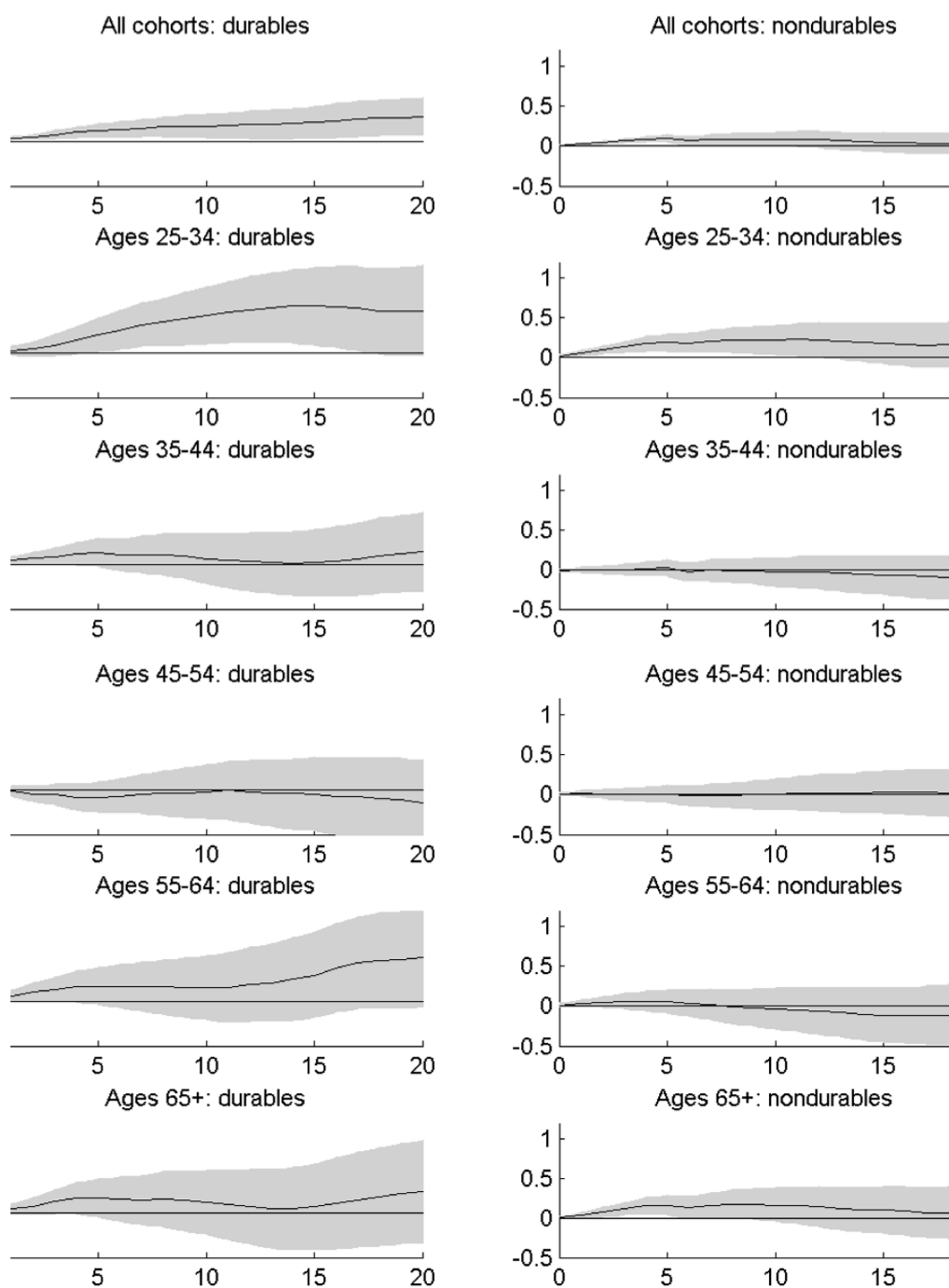
The responses of household expenditures to expansionary monetary policy shocks suggest that the increase in consumption is triggered mainly by young households (25-34), as is shown in Figure A4. More specifically, the increase is preliminary due to the durable good response, as Figure A5 indicates.

Figure A4. Response of Total Expenditures by Age group



Note: The plots show the response of total expenditures to an identified monetary policy shock using Romer&Romer dates. Shaded areas show 90 percent confidence bands.

Figure A5. Response of Durable and Non-durable Good Expenditures by Age group



Note: The plots show the response of durable and non-durable good expenditures to an identified monetary policy shock using Romer&Romer dates. Shaded areas show 90 percent confidence bands.

A3. Model derivations

This Section derives the present-value budget constraint of the government and provides details on the model and the solution strategy.

A3.1 The government's budget constraint

The consolidated government budget constraint in real terms can be written as:

$$b_t^{\mathbf{g}} + b_t^{\mathbf{cb}} - m_t = \frac{1 + r_{t-1}}{1 + \pi_t} (b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}}) - \frac{m_{t-1}}{1 + \pi_t} - \tau_t^{\mathbf{g}}$$

where $\tau_t^{\mathbf{g}} \equiv \nu \rho_o \tau_t^{\mathbf{n}} + \nu (1 - \rho_o) \tau_t^{\mathbf{y}} + (1 - \nu) \tau_t^{\mathbf{o}}$ is the total transfer to the households. We now derive the present-value government budget constraint, see also Ireland (2005). Define:

$$\varpi_{t+1} \equiv \frac{1 + r_t}{1 + \pi_{t+1}} (b_t^{\mathbf{g}} + b_t^{\mathbf{cb}}) - \frac{m_t}{1 + \pi_{t+1}}$$

and use this definition to express the period- t budget constraint as:

$$\begin{aligned} \varpi_{t+1} &= \frac{1 + r_t}{1 + \pi_{t+1}} \left(\frac{1 + r_{t-1}}{1 + \pi_t} (b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}}) - \frac{m_{t-1}}{1 + \pi_t} - \tau_t^{\mathbf{g}} + \frac{r_t}{1 + r_t} m_t \right), \\ &= \frac{1 + r_t}{1 + \pi_{t+1}} \left(\varpi_t - \tau_t^{\mathbf{g}} + \frac{r_t}{1 + r_t} m_t \right). \end{aligned}$$

Also, define D_s as in the main text note that $\frac{1+r_s}{1+\pi_{s+1}} D_{s+1} = D_s$. Consider budget constraint for period s and multiply both sides by D_{s+1} :

$$D_{s+1} \varpi_{s+1} = D_s \left(\varpi_s - \tau_s^{\mathbf{g}} + \frac{r_s}{1 + r_s} m_s \right).$$

Sum all constraints from period t to infinity:

$$\sum_{s=t}^{\infty} D_{s+1} \varpi_{s+1} = \sum_{s=t}^{\infty} D_s \left(\varpi_s - \tau_s^{\mathbf{g}} + \frac{r_s}{1 + r_s} m_s \right),$$

where we impose the limit condition $\sum_{s \rightarrow \infty} D_s \varpi_s = 0$. Finally, rearrange to obtain:

$$\sum_{s=t}^{\infty} D_s \left(\frac{r_s}{1 + r_s} m_s - \tau_s^{\mathbf{g}} \right) = \frac{m_{t-1} - (1 + r_{t-1}) (b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}})}{1 + \pi_t}$$

Furthermore, in the representative agent version of the model we can express the household's first-order condition for money and bonds, respectively, as:

$$\begin{aligned} U_{c,t} &= U_{m,t} + \tilde{\beta} \mathbb{E}_t \frac{1}{1 + \pi_{t+1}} U_{c,t+1} \\ U_{c,t} &= \tilde{\beta} \mathbb{E}_t \frac{1 + r_t}{1 + \pi_{t+1}} U_{c,t+1} \end{aligned}$$

which can be combined as:

$$U_{c,t} = \frac{1 + r_t}{r_t} U_{m,t}$$

Under the logarithmic preferences assumed in Section 3.1.8 this equation becomes $\mu c_t = \frac{r_t}{1+r_t} m_t$. Given that non-durable consumption is not affected by monetary policy in the representative agent version, it follows that $\frac{r_t}{1+r_t} m_t$ is not affected either.

A3.2 Solving the model

The model is solved using first-order perturbation (linearization). This part of the Appendix describes the first-order conditions for the optimization problems of the individuals and discusses aggregation of the individuals' choices.

Old agents and aggregation. Although the model features a representative young agent, there is wealth heterogeneity among the old agents. Typically, dynamic models with a large number of heterogeneous agents are challenging to solve. For our model, however, it turns out that the decision rules of the old are linear in wealth, which implies that aggregation is straightforward. Hence we can solve for aggregates without reference to the distribution of wealth among old agents. Wealth heterogeneity between young and old agents, however, is a key factor driving aggregate dynamics.

We exploit that the use of first-order perturbation implies certainty equivalence (see Schmitt-Grohé and Uribe (2004)). As a consequence, first-order approximations to the equilibrium laws of motion of the model coincide with those obtained for a version without aggregate uncertainty.³⁴ In what follows, we therefore omit expectations operators.³⁵

The first-order conditions for the choices of durables, money and bonds by an old household

³⁴Both versions preserve *idiosyncratic* uncertainty.

³⁵Alternatively, one could first linearize the model equations and then perform the steps described below.

i can be written, respectively, as:

$$\begin{aligned} U_{c,i,t} &= U_{d,i,t} + \beta(1 - \rho_x)(1 - \delta)U_{c,i,t+1}, \\ U_{c,i,t} &= U_{m,i,t} + \frac{\beta(1 - \rho_x)}{1 + \pi_{t+1}}U_{c,i,t+1}, \\ U_{c,i,t} &= \frac{\beta(1 - \rho_x)(1 + r_t)}{1 + \pi_{t+1}}U_{c,i,t+1}. \end{aligned}$$

Now introduce four auxiliary variables $\gamma_{c,i,t} \equiv \frac{c_{i,t}}{a_{i,t}}$, $\gamma_{d,i,t} \equiv \frac{d_{i,t}}{a_{i,t}}$, $\gamma_{m,i,t} \equiv \frac{m_{i,t}}{a_{i,t}}$ and $\gamma_{b,i,t} \equiv \frac{b_{i,t}}{a_{i,t}}$. The crucial step is to show that there are four restrictions that pin down $\gamma_{c,i,t}$, $\gamma_{d,i,t}$, $\gamma_{m,i,t}$ and $\gamma_{b,i,t}$ as functions of *only* aggregate variables. To find these coefficients, first combine the first-order conditions to obtain:

$$\begin{aligned} U_{c,i,t} &= U_{d,i,t} + (1 - \delta)(1 + \pi_{t+1})(U_{c,i,t} - U_{m,i,t}) \\ U_{c,i,t} &= (1 + r_t)(U_{c,i,t} - U_{m,i,t}) \end{aligned}$$

Under the assumed nested CES preferences we obtain:

$$\begin{aligned} U_{c,i,t} &= x_{i,t}^{-\sigma} \frac{\epsilon}{\epsilon - 1} \left[c_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \eta d_{i,t}^{\frac{\epsilon-1}{\epsilon}} + \mu m_{i,t}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}-1} \frac{\epsilon - 1}{\epsilon} c_{i,t}^{\frac{\epsilon-1}{\epsilon}-1}, \\ &= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}} c_{i,t}^{\frac{-1}{\epsilon}}, \\ U_{d,i,t} &= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}} \eta d_{i,t}^{\frac{-1}{\epsilon}}, \\ U_{m,i,t} &= x_{i,t}^{\frac{-\sigma\epsilon+1}{\epsilon}} \mu m_{i,t}^{\frac{-1}{\epsilon}}. \end{aligned}$$

The combined first-order conditions become:

$$\gamma_{c,i,t}^{\frac{-1}{\epsilon}} = \eta \gamma_{d,i,t}^{\frac{-1}{\epsilon}} + (1 - \delta)(1 + \pi_{t+1}) \left(\gamma_{c,i,t}^{\frac{-1}{\epsilon}} - \mu \gamma_{m,i,t}^{\frac{-1}{\epsilon}} \right) \quad (27)$$

$$\gamma_{c,i,t}^{\frac{-1}{\epsilon}} = (1 + r_t) \left(\gamma_{c,i,t}^{\frac{-1}{\epsilon}} - \mu \gamma_{m,i,t}^{\frac{-1}{\epsilon}} \right) \quad (28)$$

To get the third restriction, consider the Euler equation for bonds, which can be written as:

$$\left(\frac{x_{i,t}}{x_{i,t+1}} \right)^{\frac{-\sigma\epsilon+1}{\epsilon}} \left(\frac{c_{i,t}}{c_{i,t+1}} \right)^{\frac{-1}{\epsilon}} = \frac{\beta(1 - \rho_x)(1 + r_t)}{(1 + \pi_{t+1})} \quad (29)$$

and use the fact that $a_{i,t+1} = \left((1 - \delta) \gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}} \right) a_{i,t}$ to write:

$$\begin{aligned} \frac{c_{i,t}}{c_{i,t+1}} &= \frac{\gamma_{c,i,t}}{\gamma_{c,i,t+1} \left((1 - \delta) \gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}} \right)} \\ \frac{x_{i,t}}{x_{i,t+1}} &= \left[\left(\frac{\frac{\epsilon-1}{\epsilon} \gamma_{c,i,t} + \eta \frac{\epsilon-1}{\epsilon} \gamma_{d,i,t} + \mu \frac{\epsilon-1}{\epsilon} \gamma_{m,i,t}}{\frac{\epsilon-1}{\epsilon} \gamma_{c,i,t+1} + \eta \frac{\epsilon-1}{\epsilon} \gamma_{d,i,t+1} + \mu \frac{\epsilon-1}{\epsilon} \gamma_{m,i,t+1}} \right) \right]^{\frac{\epsilon}{\epsilon-1}} \frac{1}{(1 - \delta) \gamma_{d,i,t} + \frac{\gamma_{m,i,t}}{1+\pi_{t+1}} + \frac{(1+r_t)\gamma_{b,i,t}}{1+\pi_{t+1}}} \end{aligned}$$

The budget constraint gives the fourth restriction since it can be written as:

$$\gamma_{c,i,t} a_{i,t} + \gamma_{d,i,t} a_{i,t} + \gamma_{m,i,t} a_{i,t} + \gamma_{b,i,t} a_{i,t} = a_{i,t}$$

or:

$$\gamma_{c,i,t} + \gamma_{d,i,t} + \gamma_{m,i,t} + \gamma_{b,i,t} = 1 \quad (30)$$

Equations (27)-(30) pin down $\gamma_{c,i,t}$, $\gamma_{d,i,t}$, $\gamma_{m,i,t}$ and $\gamma_{b,i,t}$ as functions of only aggregate variables,

as we have substituted out individual wealth from all the equations. Hence we can omit individual i -subscripts for these variables. Given the average wealth level among old agents, a_t^o , we can now compute averages for the old agents' decision variables as $c_t^o = \gamma_{c,t} a_t^o$, $d_t^o = \gamma_{d,t} a_t^o$, $m_t^o = \gamma_{m,t} a_t^o$ and $b_t^o = \gamma_{b,t} a_t^o$. Note that these objects do not depend on the distribution of wealth among old agents. Finally, we can express aggregate wealth owned by the old agents as:

$$\begin{aligned} a_t^o &= (1 - \rho_x) \left((1 - \delta) d_{t-1}^o + \frac{m_{t-1}^o + (1 + r_{t-1}) b_{t-1}^o}{1 + \pi_t} \right) \\ &\quad + \rho_o (1 - \rho_x) \frac{\nu}{1 - \nu} \left[(1 - \delta) d_{t-1}^y + \frac{m_{t-1}^y + (1 + r_{t-1}) b_{t-1}^y}{1 + \pi_t} \right]. \end{aligned}$$

Young agents. As discussed in the main text there is effectively a representative young agent. Its first-order conditions for the choices of labour, durables, money and bonds can be written, respectively, as:

$$\begin{aligned} U_{c,t}^y &= \zeta h_t^\kappa \\ U_{c,t}^y &= U_{d,t}^y + \beta (1 - \rho_o) (1 - \delta) U_{c,t+1}^y + \beta \rho_o (1 - \rho_x) (1 - \delta) U_{c,t+1}^{yo}, \\ U_{c,t}^y &= U_{m,t}^y + \beta \left(\frac{1 - \rho_o}{1 + \pi_{t+1}} \right) U_{c,t+1}^y + \beta \frac{\rho_o (1 - \rho_x)}{1 + \pi_{t+1}} U_{c,t+1}^{yo}, \\ \frac{U_{c,t}^y}{(1 + r_t)} &= \beta \frac{1 - \rho_o}{1 + \pi_{t+1}} U_{c,t+1}^y + \beta \frac{\rho_o (1 - \rho_x)}{1 + \pi_{t+1}} U_{c,t+1}^{yo}. \end{aligned}$$

Here, $U_{c,t}^{\mathbf{y}}$ and $U_{c,t}^{\mathbf{yo}}$ are the marginal utility of non-durables of the young and newly retired agents, respectively, which satisfy:

$$\begin{aligned} U_{c,t}^{\mathbf{y}} &= (x_t^{\mathbf{y}})^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} \\ U_{d,t}^{\mathbf{y}} &= (x_t^{\mathbf{y}})^{\frac{-\sigma\epsilon+1}{\epsilon}} \eta (d_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} \\ U_{m,t}^{\mathbf{y}} &= (x_t^{\mathbf{y}})^{\frac{-\sigma\epsilon+1}{\epsilon}} \mu (m_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} \\ U_{c,t}^{\mathbf{yo}} &= (x_t^{\mathbf{yo}})^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_t^{\mathbf{yo}})^{\frac{-1}{\epsilon}} \end{aligned}$$

where $x_t^{\mathbf{yo}} = \left[(c_t^{\mathbf{yo}})^{\frac{\epsilon-1}{\epsilon}} + \eta (d_t^{\mathbf{yo}})^{\frac{\epsilon-1}{\epsilon}} + \mu (m_t^{\mathbf{yo}})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$. Note that for the newly retired agents it holds that $c_t^{\mathbf{yo}} = \gamma_{c,t} a_t^{\mathbf{y}}$. Finally, the wealth of a young agent can be expressed as:

$$\begin{aligned} a_t^{\mathbf{y}} &= (1 - \rho_o + \rho_o \rho_x) \left((1 - \delta) d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}} + (1 + r_{t-1}) b_{t-1}^{\mathbf{y}}}{1 + \pi_t} \right) \\ &\quad + \frac{1 - \nu}{\nu} \rho_x \left((1 - \delta) d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}} + (1 + r_{t-1}) b_{t-1}^{\mathbf{o}}}{1 + \pi_t} \right). \end{aligned}$$

The full system. We are now ready to collect the equations and summarize the entire model. Old agents:

$$\gamma_{c,t}^{\frac{-1}{\epsilon}} = \eta \gamma_{d,t}^{\frac{-1}{\epsilon}} + (1 - \delta) (1 + \pi_{t+1}) \left(\gamma_{c,t}^{\frac{-1}{\epsilon}} - \mu \gamma_{m,t}^{\frac{-1}{\epsilon}} \right) \quad (31)$$

$$\gamma_{c,t}^{\frac{-1}{\epsilon}} = (1 + r_t) \left(\gamma_{c,t}^{\frac{-1}{\epsilon}} - \mu \gamma_{m,t}^{\frac{-1}{\epsilon}} \right) \quad (32)$$

$$\frac{\beta (1 - \rho_x) (1 + r_t)}{(1 + \pi_{t+1})} = (\Phi_t)^{\frac{-\sigma\epsilon+1}{\epsilon}} \left(\frac{\gamma_{c,t+1}}{\gamma_{c,t}} \left((1 - \delta) \gamma_{d,t} + \frac{\gamma_{m,t}}{1 + \pi_{t+1}} + \frac{(1 + r_t) \gamma_{b,t}}{1 + \pi_{t+1}} \right) \right)^{\frac{1}{\epsilon}} \quad (33)$$

$$\Phi_t = \left[\left(\frac{\gamma_{c,t}^{\frac{\epsilon-1}{\epsilon}} + \eta \gamma_{d,t}^{\frac{\epsilon-1}{\epsilon}} + \mu \gamma_{m,t}^{\frac{\epsilon-1}{\epsilon}}}{\gamma_{c,t+1}^{\frac{\epsilon-1}{\epsilon}} + \eta \gamma_{d,t+1}^{\frac{\epsilon-1}{\epsilon}} + \mu \gamma_{m,t+1}^{\frac{\epsilon-1}{\epsilon}}} \right) \right]^{\frac{\epsilon}{\epsilon-1}} \frac{1}{(1 - \delta) \gamma_{d,t} + \frac{\gamma_{m,t}}{1 + \pi_{t+1}} + \frac{(1 + r_t) \gamma_{b,t}}{1 + \pi_{t+1}}} \quad (34)$$

$$c_t^{\mathbf{o}} = \gamma_{c,t} a_t^{\mathbf{o}} \quad (35)$$

$$d_t^{\mathbf{o}} = \gamma_{d,t} a_t^{\mathbf{o}} \quad (36)$$

$$m_t^{\mathbf{o}} = \gamma_{m,t} a_t^{\mathbf{o}} \quad (37)$$

$$b_t^{\mathbf{o}} = \gamma_{b,t} a_t^{\mathbf{o}} \quad (38)$$

$$a_t^{\mathbf{o}} = (1 - \rho_x) \left((1 - \delta) d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}} + (1 + r_{t-1}) b_{t-1}^{\mathbf{o}}}{1 + \pi_t} \right) \quad (39)$$

$$+ \rho_o (1 - \rho_x) \frac{\nu}{1 - \nu} \left[(1 - \delta) d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}} + (1 + r_{t-1}) b_{t-1}^{\mathbf{y}}}{1 + \pi_t} \right] \quad (40)$$

$$a_t^{\mathbf{o}} = c_t^{\mathbf{o}} + d_t^{\mathbf{o}} + m_t^{\mathbf{o}} + b_t^{\mathbf{o}} \quad (41)$$

Young agents:

$$(x_t^{\mathbf{y}})^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} = \zeta h_t^\kappa \quad (42)$$

$$(c_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} = \eta (d_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} + \beta (1 - \rho_o) (1 - \delta) \left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}} \right)^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_{t+1}^{\mathbf{y}})^{\frac{-1}{\epsilon}} \\ + \beta \rho_o (1 - \rho_x) (1 - \delta) \left(\frac{x_{t+1}^{\mathbf{y}\circ}}{x_t^{\mathbf{y}}} \right)^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_{t+1}^{\mathbf{y}\circ})^{\frac{-1}{\epsilon}}, \quad (43)$$

$$(c_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} = \mu (m_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} + \beta \left(\frac{1 - \rho_o}{1 + \pi_{t+1}} \right) \left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}} \right)^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_{t+1}^{\mathbf{y}})^{\frac{-1}{\epsilon}} \\ + \beta \frac{\rho_o (1 - \rho_x)}{1 + \pi_{t+1}} \left(\frac{x_{t+1}^{\mathbf{y}\circ}}{x_t^{\mathbf{y}}} \right)^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_{t+1}^{\mathbf{y}\circ})^{\frac{-1}{\epsilon}}, \quad (44)$$

$$(c_t^{\mathbf{y}})^{\frac{-1}{\epsilon}} = \beta \frac{(1 - \rho_o) (1 + r_t)}{1 + \pi_{t+1}} \left(\frac{x_{t+1}^{\mathbf{y}}}{x_t^{\mathbf{y}}} \right)^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_{t+1}^{\mathbf{y}})^{\frac{-1}{\epsilon}} \\ + \beta \frac{\rho_o (1 - \rho_x) (1 + r_t)}{1 + \pi_{t+1}} \left(\frac{x_{t+1}^{\mathbf{y}\circ}}{x_t^{\mathbf{y}}} \right)^{\frac{-\sigma\epsilon+1}{\epsilon}} (c_{t+1}^{\mathbf{y}\circ})^{\frac{-1}{\epsilon}}. \quad (45)$$

$$a_t^{\mathbf{y}} = (1 - \rho_o + \rho_o \rho_x) \left((1 - \delta) d_{t-1}^{\mathbf{y}} + \frac{m_{t-1}^{\mathbf{y}} + (1 + r_{t-1}) b_{t-1}^{\mathbf{y}}}{1 + \pi_t} \right) \\ + \frac{1 - \nu}{\nu} \rho_x \left((1 - \delta) d_{t-1}^{\mathbf{o}} + \frac{m_{t-1}^{\mathbf{o}} + (1 + r_{t-1}) b_{t-1}^{\mathbf{o}}}{1 + \pi_t} \right) \quad (46)$$

$$c_t^{\mathbf{y}} + d_t^{\mathbf{y}} + m_t^{\mathbf{y}} + b_t^{\mathbf{y}} = a_t^{\mathbf{y}} + h_t^{\mathbf{y}} + \tau_t^{\mathbf{s}} \quad (47)$$

$$c_t^{\mathbf{y}\circ} = \gamma_{c,t} a_t^{\mathbf{y}} \quad (48)$$

$$x_t^{\mathbf{y}\circ} = \left[(\gamma_{c,t} a_t^{\mathbf{y}})^{\frac{\epsilon-1}{\epsilon}} + \eta (\gamma_{d,t} a_t^{\mathbf{y}})^{\frac{\epsilon-1}{\epsilon}} + \mu (\gamma_{m,t} a_t^{\mathbf{y}})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (49)$$

$$x_t^{\mathbf{y}} = \left[(c_t^{\mathbf{y}})^{\frac{\epsilon-1}{\epsilon}} + \eta (d_t^{\mathbf{y}})^{\frac{\epsilon-1}{\epsilon}} + \mu (m_t^{\mathbf{y}})^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (50)$$

Government policy:

$$\frac{r_{t-1} (b_{t-1}^{\mathbf{g}} + b_{t-1}^{\mathbf{cb}})}{1 + \pi_t} = \nu (1 - \rho_o) \tau_t^{\mathbf{s}} \quad (51)$$

$$\frac{m_t}{m_{t-1}} (1 + \pi_t) = 1 + z_t \quad (52)$$

$$z_t = \xi (\bar{m} - m_{t-1}) + \varepsilon_t \quad (53)$$

Market clearing:

$$c_t + d_t = \nu h_t^y + (1 - \delta) d_{t-1} \quad (54)$$

$$c_t = \nu c_t^y + (1 - \nu) c_t^o \quad (55)$$

$$d_t = \nu d_t^y + (1 - \nu) d_t^o \quad (56)$$

$$m_t = \nu m_t^y + (1 - \nu) m_t^o \quad (57)$$

$$0 = b_t^g + b_t^{cb} + \nu b_t^y + (1 - \nu) b_t^o \quad (58)$$

These are 28 equations in 28 variables, being $c_t, c_t^o, c_t^{yo}, c_t^y, d_t, d_t^o, d_t^y, m_t, m_t^o, m_t^y, b_t^o, b_t^y, b_t^g, b_t^{cb}, x_t^y, x_t^{yo}, \Phi_t, \gamma_{c,t}, \gamma_{d,t}, \gamma_{m,t}, \gamma_{b,t}, h_t^y, r_t, \pi_t, \tau_t^s, z_t, a_t^o$, and a_t^{yo} . We leave out the government's budget constraint, which is redundant by Walras' law.

Special cases We present two simplifying special cases of the model.

Special case 1 ($\epsilon = 1$). When the utility elasticity ϵ equals one, the utility function becomes a Cobb-Douglas basket nested in a CRRA function:

$$U(c_{i,t}, d_{i,t}, m_{i,t}) = \frac{(c_{i,t} d_{i,t}^\eta m_{i,t}^\mu)^{1-\sigma} - 1}{1 - \sigma}$$

and the marginal utilities become $U_{c,i,t} = \frac{x_{i,t}^{1-\sigma}}{c_{i,t}}$, $U_{d,i,t} = \eta \frac{x_{i,t}^{1-\sigma}}{d_{i,t}}$ and $U_{m,i,t} = \mu \frac{x_{i,t}^{1-\sigma}}{m_{i,t}}$. In the system to be solved, we correspondingly set:

$$\begin{aligned} x_t^y &= (c_t^y) (d_t^y)^\eta (m_t^y)^\mu \\ x_t^{yo} &= (c_t^{yo}) (d_t^{yo})^\eta (m_t^{yo})^\mu \\ \Phi_t &= \left(\frac{\gamma_{c,t}}{\gamma_{c,t+1}} \right) \left(\frac{\gamma_{d,t}}{\gamma_{d,t+1}} \right)^\eta \left(\frac{\gamma_{m,t}}{\gamma_{m,t+1}} \right)^\mu \left((1 - \delta) \gamma_{d,t} + \frac{\gamma_{m,t}}{1 + \pi_{t+1}} + \frac{(1 + r_t) \gamma_{b,t}}{1 + \pi_{t+1}} \right)^{-(1+\eta+\mu)} \end{aligned}$$

Special case 2 ($\sigma = \epsilon = 1$). When both the risk aversion coefficient σ and the utility elasticity ϵ are unity, the utility function further simplifies to:

$$U(c_{i,t}, d_{i,t}, m_{i,t}) = \ln c_{i,t} + \eta \ln d_{i,t} + \mu \ln m_{i,t}$$

and the marginal utilities become $U_{c,i,t} = \frac{1}{c_{i,t}}$, $U_{d,i,t} = \frac{\eta}{d_{i,t}}$ and $U_{m,i,t} = \frac{\mu}{m_{i,t}}$. We can therefore set $(x_t^y)^{\frac{-\sigma\epsilon+1}{\epsilon}} = (x_t^{yo})^{\frac{-\sigma\epsilon+1}{\epsilon}} = (\Phi_t)^{\frac{-\sigma\epsilon+1}{\epsilon}} = 1$.

A3.3 Extensions of the model: unemployment and wage rigidities

In this Section we introduce frictions in the labor market to the model. In particular, we model a simple search and matching friction between workers and firms following the approach of Diamond, Mortensen and Pissarides.

Young agents can be either unemployed or matched with a firm.³⁶ A worker-firm pair produces $\theta > 0$ units of output. A separation between a worker and a firm takes place if the worker retires. If the worker does not retire, the match dissolves with an exogenous probability ρ_s . The overall separation rate, denoted $\tilde{\rho}_s$, is therefore given by $\tilde{\rho}_s = \rho_o + (1 - \rho_o) \rho_s$. Newborn agents enter the workforce as unemployed. It follows that the number of job searchers in the economy, which we denote s_t , is given by $s_t = \rho_o \nu + (1 - \rho_o) \rho_s n_{t-1}$.

Following Merz (1995) and many others, we assume that there is full income sharing among workers. Hence, we preserve our setup without heterogeneity among young agents. Matching in the labor market takes place at the beginning of the period, after aggregate and individual shocks have realized, but before production takes place. The evolution of the employment rate among young agents, denoted n_t , is given by:

$$n_t = (1 - \tilde{\rho}_s) n_{t-1} + g_t,$$

where g_t denotes the number of new hires in period t .

The asset value of a firm matched with a worker is given by:

$$V_t = \theta - w_t + (1 - \tilde{\rho}_s) \mathbb{E}_t \Lambda_{t,t+1} V_{t+1},$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor of the owner of the firm. For simplicity we assume that only young agents are able to run firms.^{37,38} Unmatched firms are enabled to search on the labor market after paying a vacancy cost χ . There is free entry of firms, which implies that

$$\chi = \lambda_t V_t,$$

where $\lambda_t \equiv \frac{g_t}{v_t}$ is the probability of finding a worker, where v_t is the total number of vacancies

³⁶We set $\zeta = 0$ in this model version, i.e. there is no disutility from work. We do not model unemployment benefits.

³⁷Thus, upon retirement agents are forced to sell off the ownership of the firm to a young agent.

³⁸It follows that $\mathbb{E}_t \Lambda_{t,t+1} V_{t+1} = \beta (1 - \rho_o) \mathbb{E}_t \frac{U_{c,t+1}^y}{U_{c,t}^y} V_{t+1} + \beta \rho_o (1 - \rho_x) \mathbb{E}_t \frac{U_{c,t+1}^{yo}}{U_{c,t}^{yo}} V_{t+1}$

posted in the economy. Given a number of vacancies and a number of searchers, the total number of new matches follows from an aggregate matching function given by:

$$g_t = \nu s_t^\alpha v_t^{1-\alpha}.$$

where ν is a scaling's parameter and α is the elasticity of the number of new matches with respect to the number of searchers.

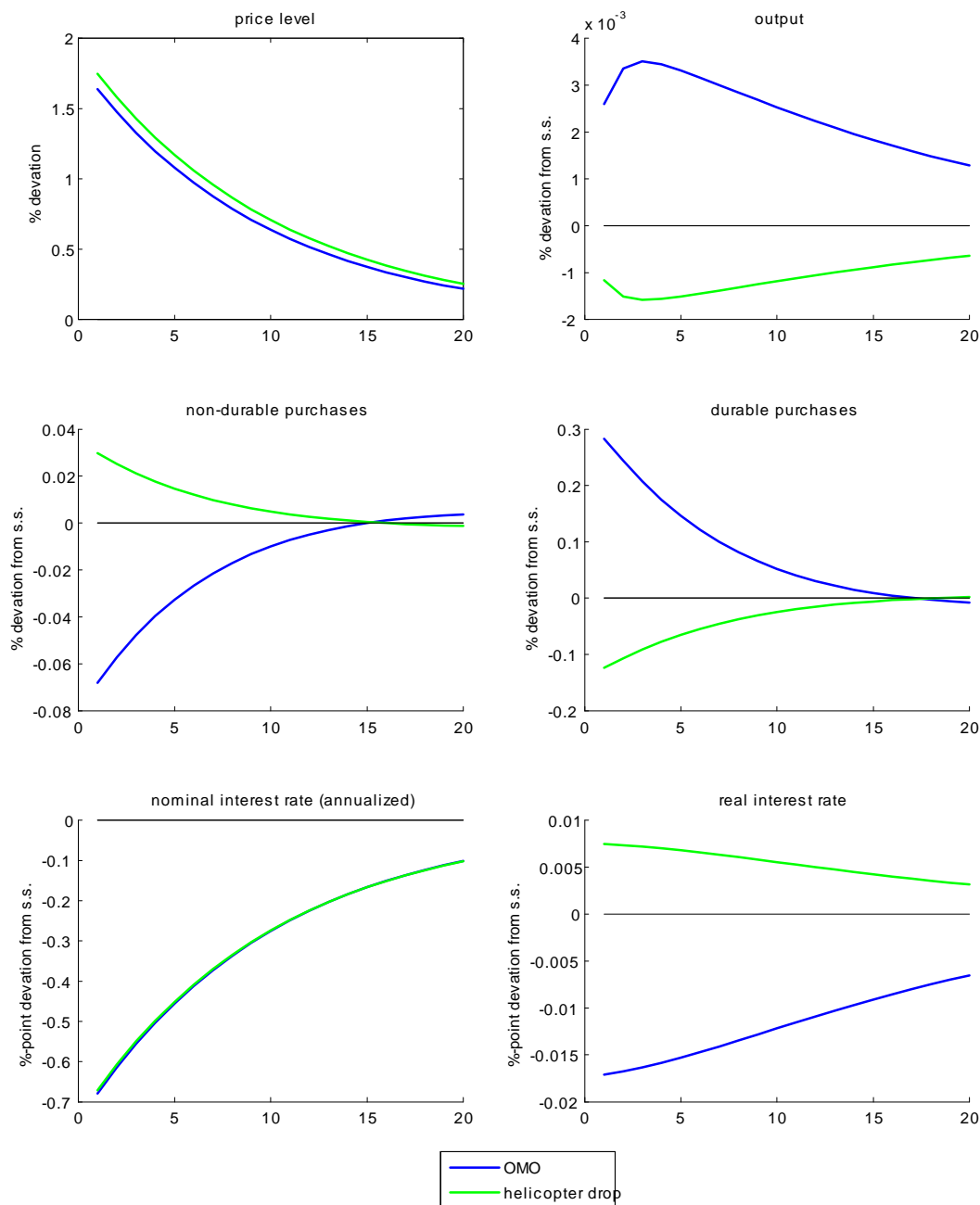
We assume the real wage is fixed, i.e. $w_t = w$, where we normalize w to one, which is consistent with equilibrium.³⁹ The parameter values of the model can be found in column A of Table 2. Calibration targets include a steady-state unemployment rate of 5 percent.

The blue line in Figure A3 plots the responses to a monetary expansion implemented using OMOs. Like in the model with a Walrasian market, durables increase on impact, whereas non-durables decline somewhat. Output declines marginally initially, but quickly rises above its steady state level, showing a much more persistent increase than in the model with a Walrasian labor market. Underlying is an effect that is not present in the model with a Walrasian labor market: with matching frictions, firm investment serves as an additional way of saving. Hence, the increased desire to save following the monetary expansion is not only reflected in a surge in durable purchases, but also in an increase in firm investment, leading to a persistent increase in vacancy posting. The latter in turn results in a gradual increase in output.

Figure A6 also plots the responses to a monetary shock implemented using helicopter drops. As in the baseline model, output and durables decline following an expansionary helicopter drop, whereas the real interest rate increases. Thus, the implementation of monetary policy continues to affect real outcomes when the labor market is subject to search and matching frictions.

³⁹One can verify the real wage is always inside the bargaining set in our simulations.

Figure A6: Model with unemployment: expansionary OMO versus helicopter drop.



Notes: responses to an expansionary monetary policy shocks –implemented using open market operations (blue line) or helicopter drops (green line)– in the model with search and matching frictions. Horizontal axes denote quarters following the shock.

Table 2. Parameter values.models with search and matching frictions

	A. stoch ageing	B. det. ageing	description	motivation
β	0.9755	0.9845	subjective discount factor	target 4% s.s. annual interest rate
η	0.31	0.31	durable preference param.	target 20% s.s. spending on durables (NIPA)
μ	0.0067	0.0075	money preference param.	target 1.8 s.s. M2 velocity ($\frac{\mu}{m}$) (FRB/NIPA)
θ	1.0005	1.005	worker productivity	target 5 % unemployment rate
ν	0.70	0.70	scaling parameter matching function	target vacancy filling rate 70%
χ	0.0037	0.032	vacancy cost	target 0.5% s.s. spending on vacancies
σ	1	1	coefficient of relative risk aversion	convention macro literature
ϵ	1	1	intratemporal elast. of substitution	convention macro literature
α	0.5	0.5	matching function elasticity	convention macro literature
ρ_s	0.1	0.1	separation rate	monthly separation rate of 3.4 %
δ	0.04	0.04	depreciation rate durables	Baxter (1996)
\tilde{A}	—	240	lifetime	duration working life 40 years
A	—	160	working life	duration retirement 20 years
ρ_o	0.0063	—	ageing probability	average duration working life 40 years
ρ_x	0.0125	—	death probability	average duration retirement 20 years
b_0^g	-2.4	-2.4	initial bond holdings Treasury	government debt 60% of annual output
b_0^{cb}	0	0	initial bond holdings Central Bank	no initial central bank debt/bonds
ξ	0.3	0.17	coefficient monetary rule	half life response nominal interest rate 2.5 years

.0.1 A.3.4. Extensions of the model: deterministic ageing

This appendix introduces deterministic ageing to the model with search and matching frictions. Much of the model remains unchanged and we discuss only the parts that are affected by the ageing structure.

Rather than ageing/dying stochastically, each household now lives for a deterministic number of A periods and retires deterministically after $\tilde{A} < A$ periods. Retired households are excluded from the labor force. Households below the retirement age are either employed or unemployed. When employed, a household inelastically supplies one unit of labor to a firm in return for a fixed wage w , which is independent of age.⁴⁰ When unemployed, a household receives no income. For simplicity, we assume that all generations own equal amounts of firm ownership so that firm profits are equally distributed among agents.

Financial markets are incomplete. We allow households to write insurance contracts against idiosyncratic labor income shocks (but not aggregate risk). As a result, consumption is equalized among households within each generation. Under this premise, there is a representative household for each generation, but there is heterogeneity across generations. The monetary and fiscal arrangements are the same as in our baseline model. Also, agents derive no utility from bequests and the wealth of the deceased agents is collected by the government. Government transfers affect only productive agents.

Let the age index of households be denoted by $j = 1, 2, \dots, A$. The size of the total population is normalized to one so the measure of each generation is $1/A$. We denote the employment rate within generation j in period t by $n_{j,t}$.

Households below the retirement age that are not matched to a firm search for a job. The matching process between workers and firms takes place at the beginning of each period and is random. This implies that firms cannot direct their search towards agents of particular age categories and that job finding rates are equal across age groups. Households enter the economy without a job. Worker-firm relationships break up with an exogenous probability ρ_s at the end of a period. A match also breaks up when a household retires. The fraction of searchers among agents of age $j < \tilde{A}$, denoted $s_{j,t}$, is given by:

$$s_{j,t} = \begin{cases} 1 & \text{for } j = 1 \\ 1 - n_{j-1,t-1} + \rho_s n_{j-1,t-1} & \text{for } 2 \leq j < \tilde{A} \end{cases},$$

⁴⁰We calibrate the wage such that it is always in the bargaining set.

where a prime denote a variable for the next period. The total measure of searchers is denoted $s_t \equiv \frac{1}{A} \sum_{j=1}^{\tilde{A}} s_{j,t}$. Given a number of vacancies and a number of searchers, the total number of new matches again follows from an aggregate matching function given by $g_t = \nu s_t^\alpha v_t^{1-\alpha}$. Age-dependent employment rates satisfy:

$$\begin{aligned} n_{j,t} &= \frac{g_t}{s_t} \text{ for } j = 1 \\ n_{j,t} &= \frac{g_t}{s_t} s_{j,t} + (1 - \rho_s) n_{j-1,t-1} \text{ for } 1 < j < \tilde{A} \\ n_{j,t} &= 0 \text{ for } j \geq \tilde{A} \end{aligned}$$

As in our baseline model, households can invest money, bonds and durables. The very oldest agents, however, are not allowed to borrow, that is, $b_A \geq 0$. This condition replaces the transversality condition of our infinite-horizon problem. We can express the decision problem of a representative household of age j recursively and in real terms as:

$$\begin{aligned} V_j &= \max_{c_j, d_j, m_j, b_j} U(c_j, d_j, m_j) + \beta \mathbb{E} V'_{j+1} \\ \text{s.t.} & \\ c_j + d_j + m_j + b_j &= a_j + w n_j + T_j + \Pi \\ a'_{j+1} &\equiv (1 - \delta) d_j + \frac{m_j}{1 + \pi'} + \frac{(1 + r_b) b_j}{1 + \pi'} \\ c, d, m, b_A &\geq 0. \end{aligned} \tag{59}$$

where V_j is the household's value of households of age j and Π denotes profits of the firms, net of the costs of new vacancies. In addition to the above constraint, agents take as given the laws of motion for prices, interest rate, transfers and profits, as well as for the age-dependent employment rates.

A worker-firm pair produces θ units of goods. The asset value to a firm of a match with a worker of age $j < \tilde{A}$ is therefore given by:

$$V_{j,t} = \theta - w + (1 - \rho_s) \Lambda_{t,t+1} \mathbb{E} V'_{j+1,t+1}$$

where $V_j = 0$ for $j \geq \tilde{A}$ due to retirement. Here, $\Lambda_{t,t+1}$ is the firms' stochastic discount factor between the current and the next period.⁴¹

⁴¹The linearized stochastic discount factor is the same across generations below the maximum age, hence we

Free vacancy posting implies that the cost of posting a vacancy, denoted χ , equal the expected benefit:

$$\chi = \sum_{j=1}^{\tilde{A}-1} \lambda_{j,t} V_{j,t}$$

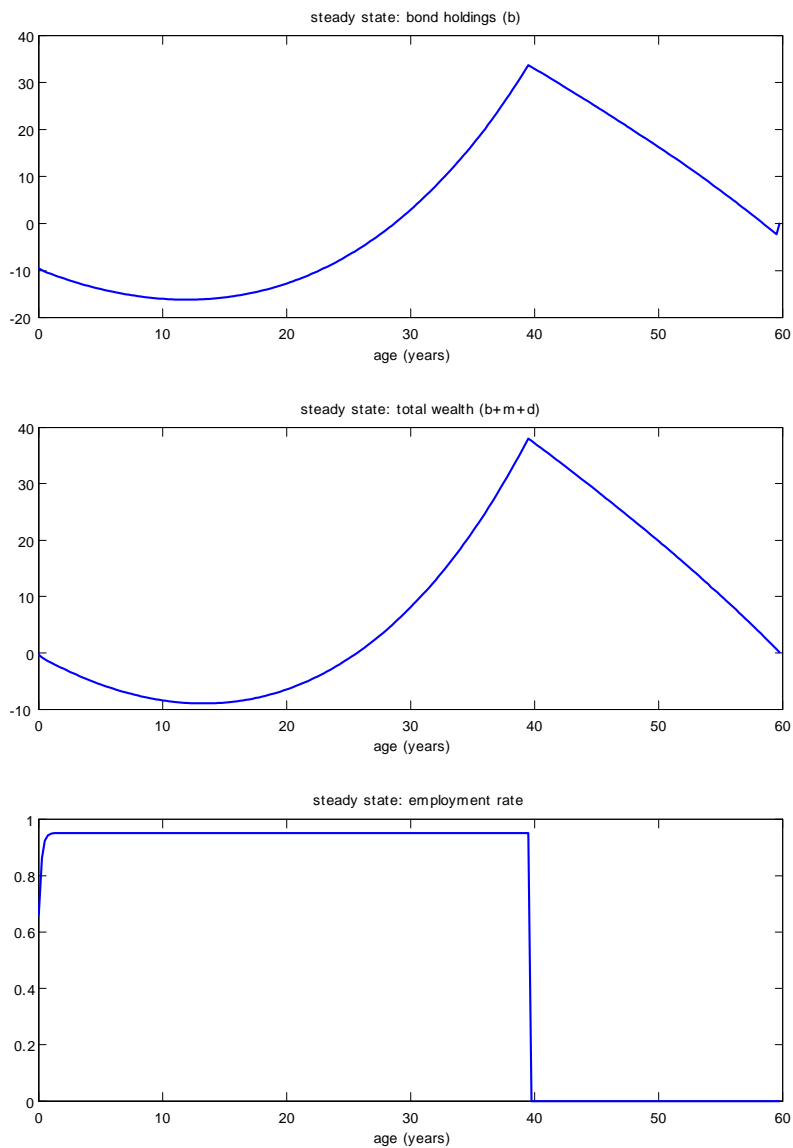
where λ_j is the probability of meeting a worker of age j , given by $\lambda_{j,t} = \frac{q_t}{v_t} \frac{s_{j,t}}{\tilde{A} s_t}$.

Parameter values of the model are displayed in column B of Table 2. The model period is one quarter, as in the baseline. Figure A7 plots steady-state bond holdings and total wealth by age. As in the data, the youngest households are borrowers. After about 15 years they start paying off their debt and obtain positive net worth after about 25 years. Upon retirement, agents start depleting their wealth. Figure 8 also plots employment rates by age. Since all agents start off as unemployed, employment rates are relatively low for the youngest households. At age 40 agents retire and no longer supply labor.

Figure A8 plots the responses to an expansionary monetary policy shock implemented using OMO. As in the previous versions of the model, there is a durables-driven increase in output. Thus, introducing deterministic ageing does not affect our main qualitative results. The increase in durable purchases is more persistent than in the models with stochastic ageing, however. Figure A8 also plots responses to a monetary expansion implemented using helicopter drops. Quantitatively, OMO and helicopter drops generate different effects, although the responses are more similar than in the model with stochastic ageing.

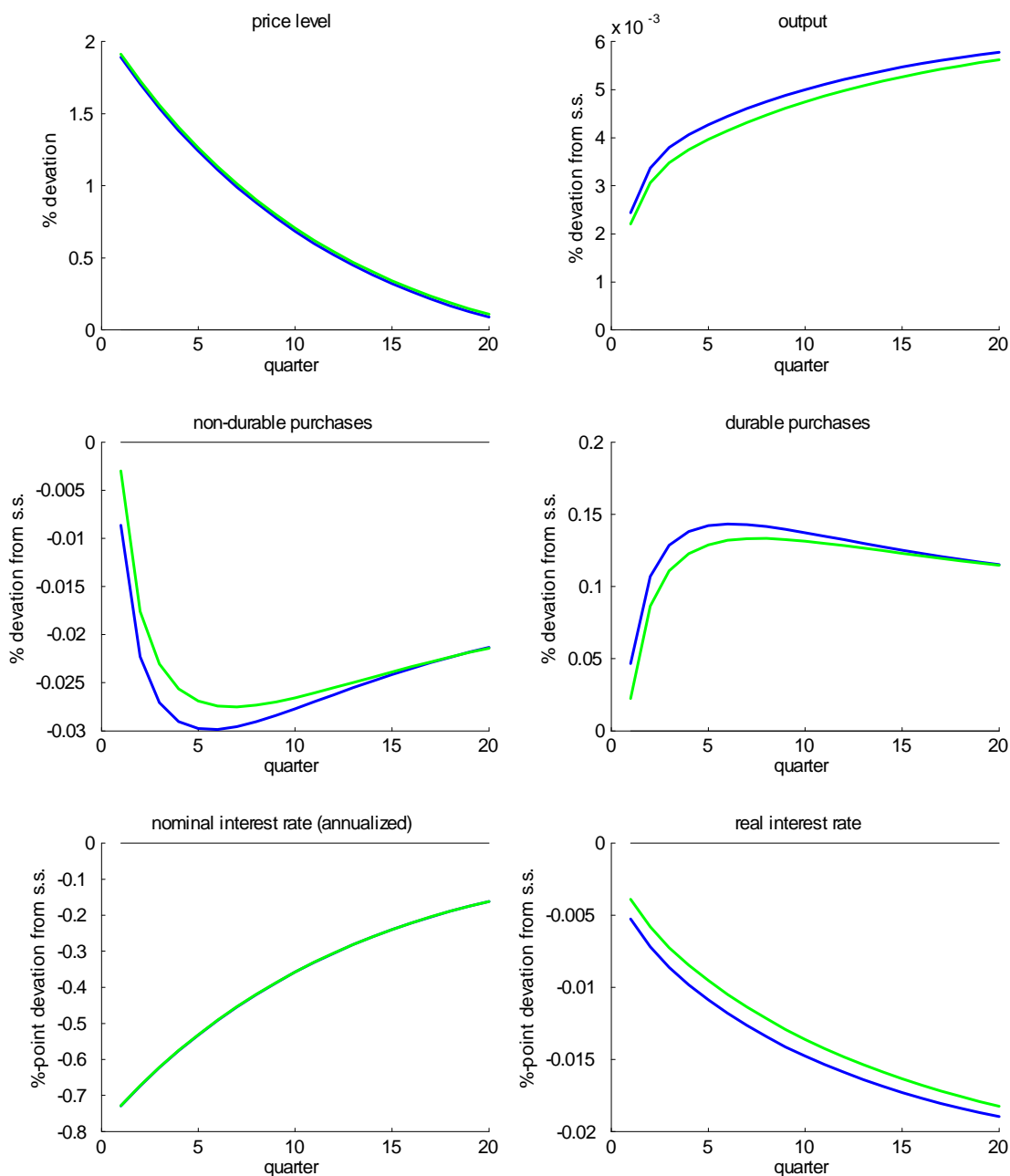
can set $\Lambda = \beta \frac{c_{j,t}}{c_{j,t+1}}$ for arbitrary $j < A$. Another implication is that –under linearization– households would be indifferent between bonds and equity if we allowed firm ownership to be tradable. Thus, the portfolio choice between bonds and firm ownership would not be determined under the linearized solution.

Figure A7: Model with unemployment and deterministic ageing: steady-state outcomes by age.



Notes: The figure plots steady-state variables by age. Bond holdings (top panel) and total wealth (middle panel) have been scaled by aggregate output per capita. Agents retire at age 40.

Figure A8: Model with unemployment and deterministic ageing: expansionary OMO versus helicopter drops



Notes: responses to an expansionary monetary policy shocks –implemented using open market operations (blue line) or helicopter drops (green line)– in the model with search and matching frictions and deterministic ageing. Horizontal axes denote quarters following the shock.

A Additional References

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