Efficiently Inefficient Markets for Assets and Asset Management*

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Abstract

We consider a model where investors can invest directly or search for an asset manager, information about assets is costly, and managers charge an endogenous fee. The efficiency of asset prices is linked to the efficiency of the asset management market: if investors can find managers more easily, more money is allocated to active management, fees are lower, and asset prices are more efficient. Informed managers outperform after fees, uninformed managers underperform after fees, and the net performance of the average manager depends on the number of "noise allocators." Small investors should be passive, but large and sophisticated investors benefit from searching for informed active managers since their search cost is low relative to capital. Hence, managers with larger and more sophisticated investors are expected to outperform.

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Asset managers play a central role in making financial markets efficient as their size allows them to spend significant resources on acquiring and processing information. The asset management market is subject to its own frictions, however, since investors must search for informed asset managers. Indeed, institutional investors literally fly around the world to examine asset managers in person, assessing their investment process, quality of the investment professionals, trading infrastructure, risk management, back office, valuation practices, custody of the assets, IT security, and so on. Similarly, individual investors search for asset managers, some via local branches of financial institutions, others via the internet or otherwise.

How does this search for asset managers affect capital allocation and the efficiency of the underlying security market? How large of an outperformance can investors expect from asset managers before and after fees? What type of manager can be expected to outperform? Which type of investors should use active, rather than passive, investing?

We seek to address these questions in a model with two levels of frictions: investors' costs of searching for informed asset managers and asset managers' cost of collecting information about assets. Despite this apparent complexity, the model is very tractable and delivers several new predictions that link the levels of inefficiency in the security market and the market for asset management: (1) If investors can find managers more easily, more money is allocated to active management, fees are lower, and security prices are more efficient; (2) As search costs diminish, asset prices become efficient in the limit, even if information-collection costs remain large; (3) Managers of complex assets earn larger fees and are fewer, and such assets are less efficiently priced; (4) Informed managers outperform after fees, uninformed managers underperform after fees, and the net performance of the average manager depends on the number of "noise allocators," who allocate to randomly chosen managers; (5) Searching for informed active managers is attractive for large or sophisticated investors with small search cost, while small or unsophisticated investors should be passive; (6) Managers with larger and more sophisticated investors are expected to outperform; (7) Finally, we discuss the economic magnitude of our predictions and welfare considerations.

As a way of background, the key benchmark is that security markets are perfectly efficient (Fama (1970)), but this leads to two paradoxes: First, no one has an incentive to collect information in an efficient market, so how does the market become efficient (Grossman and Stiglitz (1980))? Second, if asset markets are efficient, then positive fees to active managers implies inefficient markets for asset management (Pedersen (2015)).

Grossman and Stiglitz (1980) show that the first paradox can be addressed by considering informed investing in a model with noisy supply, but, when an agent has collected information about securities, she can invest on this information on behalf of others, so professional asset managers arise naturally as a result of the returns to scale in collecting and trading on information (Admati and Pfleiderer (1988), Ross (2005), Garcia and Vanden (2009)). Therefore, we introduce professional asset managers into the Grossman-Stiglitz model, allowing us to study the efficiency of asset markets jointly with the efficiency of the markets for asset management.

One benchmark for the efficiency of asset management is provided by Berk and Green (2004), who consider the implications of perfectly efficient asset-management markets (in the context of exogenous and inefficient asset prices). In contrast, we consider an imperfect market for asset management due to search frictions, consistent with the empirical evidence of Sirri and Tufano (1998), Jain and Wu (2000), Hortaçsu and Syverson (2004), and Choi et al. (2010). We focus on investors' incentive to search for informed managers and managers' incentives to acquire information about assets with endogenous prices, abstracting from how agency problems and imperfect contracting can distort asset prices (Shleifer and Vishny (1997), Stein (2005), Cuoco and Kaniel (2011), Buffa et al. (2014)).

We employ the term *efficiently inefficient* to refer to the equilibrium level of inefficiency given the two layers of frictions in the spirit of Grossman-Stiglitz's notion of "an equilibrium degree of disequilibrium." Paraphrasing Grossman-Stiglitz, prices in efficiently inefficient markets reflect information, but only partially, so that some managers have an incentive to expend resources to obtain information, but not all, so investors have an incentive to expend resources to find informed managers.

Our equilibrium works as follows. Among the group of asset managers, an endogenous number decide to acquire information about a security. Investors must decide whether to expend search costs to find one of the informed asset managers. In an interior equilibrium, investors are indifferent between passive investing (i.e., investing that does not rely on information collection) and searching for an informed asset manager. Investors do not collect information on their own, since the costs of doing so are higher than the benefits to an individual due to the relatively high equilibrium efficiency of the asset markets. This high equilibrium efficiency arises from investors' ability to essentially "share" information collection costs by investing through an asset manager. When an investor meets an asset manager, they negotiate a fee, and asset prices are set in a competitive noisy rational expectations market. The economy also features a group of "noise traders" (or "liquidity traders") who take random security positions as in Grossman-Stiglitz. Likewise, we introduce a group of "noise allocators" who allocate capital to a random group of asset managers, e.g., because they place trust in these managers as modeled by Gennaioli et al. (2015).

We solve for the equilibrium number of investors who invest through managers, the equilibrium number of informed asset managers, the equilibrium management fee, and the equilibrium asset prices. The model features both search and information frictions, but the solution is surprisingly simple and yields a number of clear new results.

First, we show that informed managers outperform before and after fees, while uninformed managers naturally underperform after fees. Investors who search for asset managers must be compensated for their search and due diligence costs, and this compensation comes in the form of expected outperformance after fees. Investors are indifferent between passive and active investing in an interior equilibrium, so a larger search cost must be associated with a larger outperformance by active investors. Noise allocators invest partly with uninformed managers and therefore may experience underperformance after fees. The value-weighted average manager (which equals their average investor) outperforms after fees if the number of noise allocators is small, and underperforms if many noise allocators exist.

The model helps explain a number of empirical regularities on the performance of asset managers that are puzzling in light of the existing literature. Indeed, while the "old consensus" in the literature was that the average mutual fund has no skill (Fama (1970), Carhart (1997)), a "new consensus" has emerged that the average hides a significant cross-sectional variation in manager skill among mutual funds, hedge funds, private equity, and venture capital.[1] For instance, Kosowski et al. (2006) conclude that *"a sizable minority of managers pick stocks well enough to more than cover their costs."* In our model, this outperformance after fees is expected as compensation for investors' search costs, but it is puzzling in light of the prediction of Fama (1970) that all managers underperform after fees, and the prediction of Berk and Green (2004) that all managers deliver zero outperformance after fees. Further, the fact that top hedge funds and private equity managers deliver larger outperformance than top mutual funds is consistent with our model under the assumption that investors face larger search costs in these segments.

While the data support our novel prediction that some managers outperform others, we can test the model at a deeper level by examining whether it can also explain *who* outperforms. To do this, we extend the model by considering investors and asset managers who differ in their size or sophistication. We show that large and sophisticated investors benefit from searching for an informed manager since their search cost is low relative to their capital. In contrast, small unsophisticated investors are better served by passive investing. As a result, active investors who are small must be noise allocators, while large active investors could be rational searching investors (or noise allocators). Hence, we predict that large investors perform better than small investors on average, because large investors are more likely to find informed managers. This prediction is consistent with the findings of Dyck and Pomorski (2015), who report that large institutional investors select managers who outperform those of small investors.

We also predict that asset managers who have larger and more sophisticated investors outperform those serving small unsophisticated investors. Consistent with this prediction,

[1]Evidence on mutual funds is provided by Grinblatt and Titman (1989), Wermers (2000), Kacperczyk et al. (2008), Fama and French (2010), Berk and Binsbergen (2012), and Kacperczyk et al. (2014)), on hedge funds by Kosowski et al. (2007), Fung et al. (2008), Jagannathan et al. (2010), and on private equity and venture capital by Kaplan and Schoar (2005).

managers of institutional investors outperform those of retail investors (Evans and Fahlenbrach (2012), Dyck et al. (2013), Gerakos et al. (2014)).

The model also generates a number of implications of cross-sectional and time-series variation in search costs. The important observation is that, if search costs are lower such that investors more easily can identify informed managers, then more money is allocated to active management, fees are lower, and security markets are more efficient. If investors' search costs go to zero and the pool of potential investors is large, then the asset market becomes efficient in the limit. Indeed, as search costs diminish, fewer and fewer asset managers with more and more asset under management collect smaller and smaller fees (both per investor and in total), and this evolution makes asset prices more and more efficient even though information-collection costs remain constant (and potentially large). It may appear surprising (and counter to the result of Grossman and Stiglitz (1980)) that markets can become close to efficient despite large information collection costs, but this result is driven by the fact that the costs are shared by investors through an increasingly consolidated group of asset managers.

We discuss how these model-implied effects of changing search costs can help explain cross-sector, cross-country, and time-series evidence on the efficiency, fees, and asset management industry for mutual funds, hedge funds, and private equity and gives rise to new tests. For instance, if search costs have diminished over time as information technology has improved, markets should have become more efficient, consistent with the evidence of Wurgler (2000) and Bai et al. (2013), and linked to the amount of assets managed by professional traders (Rosch et al. (2015)).

The related theoretical literature includes, beside the papers already cited, noisy rational expectations models (Grossman (1976), Hellwig (1980), Diamond and Verrecchia (1981), Admati (1985)), other models of informed trading (Glosten and Milgrom (1985), Kyle (1985)), information acquisition (Van Nieuwerburgh and Veldkamp (2010), Kacperczyk et al. (2014)), search models in finance (Duffie et al. (2005), Lagos (2010)), and models of asset management (Pastor and Stambaugh (2012), Vayanos and Woolley (2013), Stambaugh (2014)). We complement the literature by combining a model of search costs and information costs, by solving a general equilibrium for asset markets and asset management markets, and by deriving new predictions on performance and its relation to investor and manager characteristics, on the role of search costs for market efficiency, and on the industrial organization of asset management.

The next section lays out the basic model, Section 2 provides the solution, and Section 3 derives the key properties of the equilibrium. Section 4 extends the model to allow small and large investors and asset managers. Section 5 considers further applications of the framework, including the economic magnitude of the predicted effects, multiple assets, and welfare considerations. Section 6 discusses our empirical predictions in the context of the empirical literature. Section 7 concludes. Appendix A describes the real-world issues related to search and due diligence of asset managers and Appendix B contains proofs.

1 Model of Assets and Asset Managers

1.1 Investors and Asset Managers

The economy features several types of competitive agents trading in a financial market, as illustrated by Figure 1. Searching investors trade directly or through asset managers, asset managers trade on behalf of groups of investors, noise allocators make random allocations to asset managers, and noise traders make random trades in financial markets.

Specifically, the economy has $\bar{A}$ searching investors (or "allocators"), each of whom can either (i) invest directly in asset markets after having acquired a signal s at cost k, (ii) invest directly in asset markets without the signal, or (iii) invest through an asset manager. Due to economies of scale, a natural equilibrium outcome is that investors do not acquire the signal, but, rather, invest as uninformed or through a manager. We highlight below some weak conditions under which all realistic equilibria take this form, and we therefore rule out that investors acquire the signal.[2] Consequently, we focus on the number A of investors who make

[2]One could consider an extension with investors with different abilities, in which case some investors may

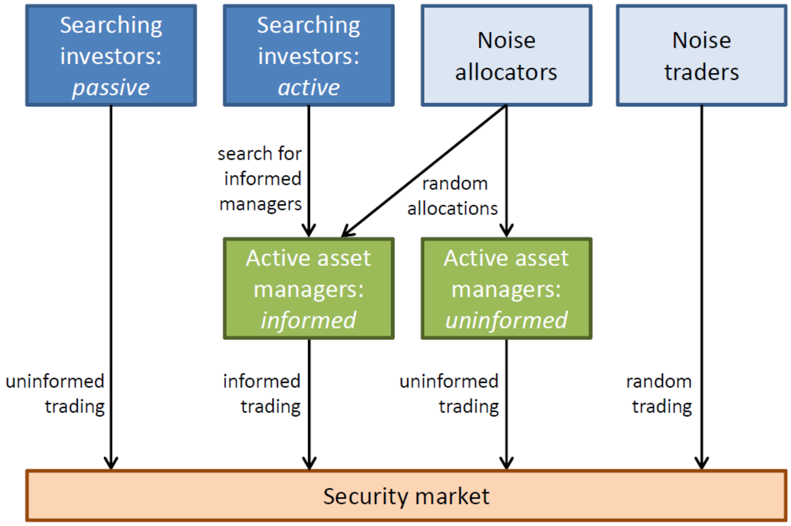

Figure 1: **Model Overview.**

informed investments through a manager, inferring the number of uninformed investors as the residual, $\bar{A} - A$.

The economy has $\bar{M}$ risk-neutral asset-management firms.[3] Of these asset managers, only M elect to pay a cost k to acquire the signal s and thereby become informed asset managers. The remaining $\bar{M} - M$ managers seek to collect asset management fees and invest without information. The number of informed asset managers is determined as part of the equilibrium. We note that we think of the sets of managers and investors as continua (e.g., M is the mass of informed managers).[4]

To invest with an informed asset manager, investors must search for, and vet, managers, which is a costly activity. Specifically, the cost of finding an informed manager and confirming that she has the signal (i.e., performing due diligence) is $c(M, A)$, which depends on both the number of informed asset managers M and the number of investors A in these asset

collect information on their own.

[3]The total number of asset managers $\bar{M}$ can be endogenized based on an entry cost k^u for being an uninformed manager. Such an endogenous entry leaves the other equilibrium conditions unaffected when we interpret the information cost k as the additional cost that informed managers must incur, i.e., their total cost is $k^u + k$. Asset management firms are risk-neutral as they face only idiosyncratic risk that can be diversified away by their owners.

[4]Treating agents as a continuum keeps the exposition as simple as possible, but the model's properties also obtain in a limit of a finite-investor model.

management firms. We consider a general continuous search cost function,[5] but finding an informed manager is naturally impossible if none exists, so $c(0, A) = \infty$ for all A. The search cost c captures the realistic feature that most investors spend significant resources finding an asset manager they trust with their money, as described in detail in Appendix A. In the real world, there is also a (small) cost c^u of uninformed investing. For simplicity, we start by assuming that investors can make passive allocations at no cost, $c^u = 0$, but Section 5.5 considers the general case.

We assume that all investors have constant absolute risk aversion (CARA) utility over end-of-period consumption with risk-aversion parameter γ (following Grossman and Stiglitz (1980)). For convenience, we express the utility as certainty-equivalent wealth — hence, with end-date wealth $\tilde{W}$, an investor's utility is $-\frac{1}{\gamma}\log(E(e^{-\gamma\tilde{W}}))$. Each investor is endowed with an initial wealth W.

When an investor has found an asset manager and confirmed that the manager has the technology to obtain the signal, they negotiate the asset management fee f. The fee is set through Nash bargaining and, at this bargaining stage, all costs are sunk — both the manager's information acquisition cost and the investor's search cost.

Lastly, the economy features a group of "noise traders" and one of "noise allocators." As in Grossman and Stiglitz (1980), noise traders buy an exogenous number of shares of the security, $\bar{q} - q$, as described below. Noise traders create uncertainty about the supply of shares and are used in the literature to capture that it can be difficult to infer fundamentals from prices. Noise traders are also called "liquidity traders" in some papers and their demand can be justified by a liquidity need, hedging demand, or behavioral reasons. They are characterized by the fact that their trades are not solely motivated by informational issues.

Following the tradition of noise traders, we introduce the concept of "noise allocators," of total mass N, who allocate their funds across randomly chosen asset managers. Noise allocators play a similar role in the market for asset management to noise traders in the market for assets — specifically, noise allocators can make it difficult for searching investors

[5] We require continuity of c only on $[0, \infty)^2 \setminus \{(0, 0)\}$. A few of our results on comparative statics rely on further properties of the function c and we state the desired assumptions when needed.

to determine whether a manager is informed by looking at whether she has other investors (although we don't explicitly model this feature). Further, since noise allocators partly invest with uninformed asset managers, they change the performance characteristics in the distributions of managers and investors.

Noise allocators pay the general fee f, which we can view as an assumption for simplicity. However, we show that such behavior by noise traders can be derived as an outcome by incorporating the following two features into the model. First, noise allocators use a poor search technology. They pay the same cost c to be matched with a manager, but they find a random manager, not necessarily an informed one. Second, noise allocators face a high cost c^u of investing on their own even without information, a cost so high that they always search for a manager. Our results do not depend on whether noise allocators believe that the manager is informed or not, so noise allocators can be viewed as fully rational or biased. Our assumptions can be seen as capturing the idea that noise allocators invest based on trust as proposed by Gennaioli et al. (2015). We note that by taking a fixed number of noise allocators we rule out, for simplicity, that managers exploit behavioral biases to affect the number of noise allocators.

1.2 Assets and Information

We adopt the asset-market structure of Grossman and Stiglitz (1980), aiming to focus on the consequences of introducing asset managers into this framework.[6] Specifically, there exists a risk-free asset normalized to deliver a zero net return, and a risky asset with payoff v distributed normally with mean $\bar{v}$ and standard deviation σ_v. Agents can obtain a signal s of the payoff, where

$$s = v + \varepsilon. \tag{1}$$

[6]We follow Grossman and Stiglitz (1980) in considering a one-period setting, but we note that several new and interesting features could arise in a dynamic model with investors searching for asset managers. In that case, an active investor may face the dynamic tradeoff between a frequent incidence of switching costs and trading based on stale information (technology). Future research should consider the robustness of our results to such a multi-period setting as well as novel dynamic predictions.

The noise ε has mean zero and standard deviation σ_ε, is independent of v, and is normally distributed.

The risky asset is available in a stochastic supply given by q, which is jointly normally distributed with, and independent of, the other exogenous random variables. The mean supply is $\bar{q}$ and the standard deviation of the supply is σ_q. We think of the noisy supply as the number of shares outstanding $\bar{q}$ plus the supply $q - \bar{q}$ from the noise traders.

Given this asset market, uninformed investors buy a number of shares x_u as a function of the observed price p, to maximize their utility u_u (certainty-equivalent wealth), taking into account that the price p may reflect information about the value:

$$u_u(W) = -\frac{1}{\gamma} \log \left(\mathrm{E} \left[\max_{x_u} \mathrm{E} \left(e^{-\gamma(W + x_u(v-p))} | p \right) \right] \right) = W + u_u(0) \equiv W + u_u. \tag{2}$$

We see that, because of the CARA utility function, an investor's wealth level simply shifts his utility function and does not affect his optimal behavior. Therefore, we define the scalar u_u as the wealth-independent part of the utility function (a scalar that naturally depends on the asset-market equilibrium, in particular the price efficiency).

Asset managers observe the signal and invest in the best interest of their investors. This informed investing gives rise to the gross utility u_i of an active investor (i.e., not taking into account his search cost and the asset management fee — we study those, and specify their impact on the ex-ante utility, later):

$$u_i(W) = -\frac{1}{\gamma} \log \left(\mathrm{E} \left[\max_{x_i} \mathrm{E} \left(e^{-\gamma(W + x_i(v-p))} | p, s \right) \right] \right) = W + u_i(0) \equiv W + u_i. \tag{3}$$

As above, we define the scalar u_i as the wealth-independent part of the utility function. The gross utility of an active investor differs from that of an uninformed via conditioning on the signal s.

1.3 Equilibrium Concept

We first consider the (partial) equilibrium in the asset market given the numbers of informed and uninformed investors. We denote the mass of informed investors by I and note that it is the sum of the number A of rational investors who decide to search for a manager and the number of the noise allocators who happen to find an informed manager, where the latter is the total number N of noise allocators times the fraction $M/\bar{M}$ of informed managers (using the law of large numbers):

$$I = A + N\frac{M}{\bar{M}}. \tag{4}$$

Clearly, the remaining investors, $\bar{A} + N - I$, invest as uninformed, either directly or via an uninformed manager.

An *asset-market equilibrium* is an asset price p such that the asset market clears:

$$q = Ix_i + (\bar{A} + N - I)x_u, \tag{5}$$

where x_i is the demand that maximizes the utility of informed investors (3) given p and the signal s, and x_u is the demand of uninformed investors (2). The market clearing condition equates the noisy supply q with the total demand from all informed and uninformed investors.

Second, we define a *general equilibrium for assets and asset management* as a number of informed asset managers M, a number of active investors A, an asset price p, and asset management fees f such that (i) no manager would like to change her decision of whether to acquire information, (ii) no investor would like to switch status from active (with an associated utility of $W + u_i - c - f$) to passive (conferring utility $W + u_u$) or vice-versa, (iii) the price is an asset-market equilibrium, and (iv) the asset management fees are the outcome of Nash bargaining.

2 Solving the Model

2.1 Asset-Market Equilibrium

We first derive the asset-market equilibrium. The price p of the risky asset is determined as in a market in which I investors are informed (because their portfolios are chosen by informed managers) and the remaining $\bar{A} + N - I$ investors are uninformed. We first take I as given by (4), and then later solve for the equilibrium number of informed investors and managers. For a given I, the linear asset-market equilibrium is as in Grossman and Stiglitz (1980), but for completeness we record the main results here.[7]

In the linear equilibrium, an informed agent's demand for the asset is a linear function of prices and signals and the price is a linear function of the signal and the noisy supply:

$$p = \theta_0 + \theta_s \left((s - \bar{v}) - \theta_q (q - \bar{q}) \right), \tag{6}$$

where, as we show in the appendix, the coefficients are given by

$$\theta_0 = \bar{v} - \frac{\gamma \bar{q} \operatorname{var}(v|s)}{I + (\bar{A} + N - I) \frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)}} \tag{7}$$

$$\theta_s = \frac{I \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2} + (\bar{A} + N - I) \frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)} \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2 \sigma_q^2}}{I + (\bar{A} + N - I) \frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)}} \tag{8}$$

$$\theta_q = \gamma \frac{\sigma_\varepsilon^2}{I}. \tag{9}$$

As we see, the equilibrium price depends on the ratio $\frac{\operatorname{var}(v|s)}{\operatorname{var}(v|p)}$, which is given explicitly in Proposition 1 and has an important interpretation. Indeed, following Grossman and Stiglitz (1980), we define the *efficiency* (or informativeness) of asset prices based on this ratio. For

[7] Our setup differs from the one of Grossman and Stiglitz (1980) by a change of variables, which leads to some superficial differences in the results. Palvolgyi and Venter (2014) derive interesting non-linear equilibria in the Grossman and Stiglitz (1980) model.

convenience, we concentrate on the quantity

$$\eta \equiv \log\left(\frac{\sigma_{v|p}}{\sigma_{v|s}}\right) = \frac{1}{2}\log\left(\frac{\mathrm{var}(v|p)}{\mathrm{var}(v|s)}\right), \tag{10}$$

which represents the price *inefficiency*. This quantity records the amount of uncertainty about the asset value for someone who only knows the price p, relative to the uncertainty remaining when one knows the signal s. The price inefficiency is a positive number, $\eta \geq 0$, since the price is a noisy version of the signal, $\mathrm{var}(v|p) \geq \mathrm{var}(v|p, s) = \mathrm{var}(v|s)$. Naturally, a higher η corresponds to a more inefficient asset market and a zero inefficiency corresponds to a price that fully reveals the signal. This definition of inefficiency relates naturally to the concept of relative entropy in information theory, as we shall see in Section 5.3.

The relative utility of investing based on the manager's information versus investing as uninformed, $u_i - u_u \geq 0$, also plays a central role in the remainder of the paper. We can also think of it as a measure of the outperformance of informed investors relative to uninformed ones. As we shall see, the relative utility is central for our analysis for several reasons: It affects investors' incentive to search for managers, the equilibrium asset management fee, and managers' incentive to acquire information. Importantly, in equilibrium, investors' relative utility is linked to the asset price inefficiency η, and both depend on the number of informed investors as described in the following proposition.

Proposition 1 *There exists a unique linear asset-market equilibrium given by* (6)–(9)*. In the linear asset-market equilibrium, the utility differential between informed and uninformed investors,* $u_i - u_u$*, is given by the inefficiency of the price,* η*:*

$$\gamma(u_i - u_u) = \eta. \tag{11}$$

Further, η *is decreasing in the number of informed investors* I *and can be written as*

$$\eta = -\frac{1}{2}\log\left(1 - \frac{\sigma_q^2\sigma_\varepsilon^2}{I^2/\gamma^2 + \sigma_q^2\sigma_\varepsilon^2}\frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2}\right) \quad \in (0, \infty). \tag{12}$$

Naturally, when there are more informed investors I, asset prices become more efficient (lower η), implying that informed and uninformed investors receive more similar utility (lower $u_i - u_u$). We note that the asset price efficiency does not depend directly on the the number of asset managers M. What determines the asset price efficiency is the risk-bearing capacity of agents investing based on the signal, and this risk-bearing capacity is ultimately determined by the number of active investors (not the number of managers they invest through). The number of asset managers does affect asset price efficiency *indirectly*, however, since M affects I as seen in (4), and, importantly, since the number of searching investors A and the number of asset manages are determined jointly in equilibrium as we shall see.

2.2 Asset Management Fee

The asset-management fee is set through Nash bargaining between an investor and a manager. The bargaining outcome depends on each agent's utility in the events of agreement vs. no agreement (the latter is called the "outside option"). For the investor, the utility in an agreement of a fee of f is $W - c - f + u_i$. If no agreement is reached, the investor's outside option is to invest as uninformed with his remaining wealth, yielding a utility of $W - c + u_u$ as the cost c is already sunk. Hence, the investor's gain from agreement is $u_i - u_u - f$.

The investor's outside option is equal to the utility of searching again for another manager in an interior equilibrium. Hence, we can think of the investor's bargaining threat as walking away to invest on his own or to find another manager. In other words, in a search market, managers engage in imperfect competition which determines the fee and the equilibrium entry.

Similarly, the asset manager's gain from agreement is the fee f. This is true because the manager's information cost k is sunk and there is no marginal cost to taking on the investor.

The bargaining outcome maximizes the product of the utility gains from agreement:[8]

$$\max_{f} \left(u_i - u_u - f\right) f \tag{13}$$

The solution is the equilibrium asset management fee f given by

$$f = \frac{1}{2}\frac{\eta}{\gamma}, \qquad \text{[equilibrium asset management fee]} \tag{14}$$

using that $u_i - u_u = \eta/\gamma$ based on Equation (11). This equilibrium fee is simple and intuitive: The fee would naturally have to be zero if asset markets were perfectly efficient, so that no benefit of information existed ($\eta = 0$), and it increases in the size of the market inefficiency. Indeed, active asset management fees can be viewed as evidence that investors believe that security markets are less than fully efficient.

We next derive the investors' and managers' decisions in an equally straightforward manner. Indeed, an attractive feature of this model is that it is very simple to solve, yet provides powerful results.

2.3 Investors' Decision to Search for Asset Managers

An investor optimally decides to look for an informed manager as long as

$$u_i - c - f \geq u_u, \tag{15}$$

or, recalling the equality $\eta = \gamma(u_i - u_u)$,

$$\eta \geq \gamma(c + f). \tag{16}$$

This relation must hold with equality in an "interior" equilibrium (i.e., an equilibrium in which strictly positive amounts of investors decide to invest as uninformed and through

[8]Note that we specify the bargaining objective in terms of certainty-equivalent wealth, which is natural and tractable.

asset managers — as opposed to all investors making the same decision). Inserting the equilibrium asset management fee (14), we have already derived the investor's indifference condition: $\gamma c = \frac{1}{2}\eta$.

Using similar straightforward arguments, we see that an investor would prefer using an asset manager to acquiring the signal singlehandedly provided $k \geq c + f$. Using the equilibrium asset management fee derived in Equation (14), the condition that asset management is preferred to buying the signal can be written as $k \geq 2c$. In other words, finding an asset manager should cost at most half as much as actually being one, which seems to be a condition that is clearly satisfied in the real world. We can also make use of (17) to express this condition equivalently as $A \geq 2M$, i.e., there must be at least two searching investors for every manager, another realistic implication.

2.4 Noise Allocators

Since noise allocators face a high cost of investing on their own, they all search for an asset manager. A fraction $M/\bar{M}$ randomly find an informed manager while the rest find uninformed managers. Since noise allocators cannot tell the difference between informed and uninformed managers, they pay the same fee either way. What specific fee they pay is not central to our results, but we can model the bargaining as above.

Noise allocators receive a utility from investing with a random active manager that we denote by u_n. Given her unattractive option of investing on their own, noise allocators' alternative to investing with the current manager is paying the cost c again to find another manager and investing with him at an expected fee of $\bar{f}$. The gains from agreeing to pay the current manager a fee of f is therefore $W + u_n - c - f - (W + u_n - 2c - \bar{f}) = c + \bar{f} - f$.

The manager has a gain from agreement of f so the equilibrium fee maximizes $(c+\bar{f}-f)f$, which under $\bar{f} = f$ gives $f = c$. As seen from (14) and (16), the fee paid by noise allocators is the same as the fee paid by other investors in an interior equilibrium.

2.5 Entry of Informed Asset Managers

A prospective informed asset manager must pay the cost k to acquire information. Becoming an informed manager has the benefit that the manager can expect to receive the capital of A/M searching investors, in addition to the capital from noise allocators $N/\bar{M}$ that she receives regardless. Therefore, she chooses to become informed provided that the total extra fee revenue covers the cost of operations:

$$f\,A/M \geq k. \tag{17}$$

This manager condition must hold with equality for an interior equilibrium, and we can easily insert the equilibrium fee (14) to get $M = \frac{\eta A}{2\gamma k}$.

2.6 General Equilibrium for Assets and Asset Management

We have arrived at following two indifference conditions:

$$\frac{\eta(I)}{2\gamma} = c\,(M, A) \qquad \text{[investors' indifference condition]} \tag{18}$$

$$\frac{\eta(I)}{2\gamma} = \frac{M}{A}k, \qquad \text{[asset managers' indifference condition]} \tag{19}$$

where η is a function of $I = A + N\frac{M}{\bar{M}}$ given explicitly by (12). Hence, solving the general equilibrium comes down to solving these two explicit equations in two unknowns (A, M). Recall that a general equilibrium for assets and asset management is a four-tuple (p, f, A, M), but we have eliminated p by deriving the market efficiency η in a (partial) asset market equilibrium and we have eliminated f by expressing it in terms of η. We can solve equations (18)–(19) explicitly when the search-cost function c is specified appropriately as we show in the following example, but the remainder of the paper provides general results and intuition for general search-cost functions.

Example: Closed-Form Solution. A cost specification motivated by the search literature is

$$c\left(M, A\right) = \bar{c}\left(\frac{A}{M}\right)^{\alpha} \text{ for } M > 0 \quad \text{ and } \quad c(M, A) = \infty \text{ for } M = 0, \tag{20}$$

where the constants $\alpha > 0$ and $\bar{c} > 0$ control the nature and magnitude of search frictions. The idea is that informed asset managers are easier to find if a larger fraction of all asset managers are informed, while performing due diligence (which requires the asset manager's time and cooperation) is more difficult in a tighter market with a larger number of searching investors. With this search cost function, equations (18)–(19) can be combined to yield

$$\eta = 2\gamma\left(\bar{c}k^{\alpha}\right)^{\frac{1}{1+\alpha}}, \tag{21}$$

which shows how search costs and information costs determine market inefficiency η. We then derive the equilibrium number of informed investors I from (12):

$$I = \gamma\sigma_q\sigma_\varepsilon\sqrt{\frac{\sigma_v^2}{\sigma_\varepsilon^2+\sigma_v^2}\frac{1}{1-e^{-2\eta}}-1} = \gamma\sigma_q\sigma_\varepsilon\sqrt{\frac{\sigma_v^2}{\sigma_\varepsilon^2+\sigma_v^2}\frac{1}{1-e^{-4\gamma\left(\bar{c}k^{\alpha}\right)^{\frac{1}{1+\alpha}}}}-1}, \tag{22}$$

The number of informed managers can be linearly related to the number of searching investors based on (19) and (21)

$$M = \frac{\eta}{2\gamma k}A = \left(\frac{\bar{c}}{k}\right)^{\frac{1}{1+\alpha}}A, \tag{23}$$

so the number of managers per investor M/A depends on the magnitude of the search cost $\bar{c}$ relative to the information cost k. Combining (23) with the identity $I = A + M\frac{N}{\bar{M}}$ yields the solution for A

$$A = I\left(1+\frac{N}{\bar{M}}\left(\frac{\bar{c}}{k}\right)^{\frac{1}{1+\alpha}}\right)^{-1}, \tag{24}$$

concentrating on parameters for which $A < \bar{A}$.

When η is small — a reasonable value is $\eta = 6\%$, as we show in Section 5.1 — we can approximate the number of informed investors more simply as

$$I \cong \frac{\gamma}{(2\eta)^{1/2}} \frac{\sigma_q \sigma_\varepsilon \sigma_v}{(\sigma_\varepsilon^2 + \sigma_v^2)^{1/2}} = \frac{\gamma^{1/2}}{2(\bar{c}k^\alpha)^{\frac{1}{2(1+\alpha)}}} \frac{\sigma_q \sigma_\varepsilon \sigma_v}{(\sigma_\varepsilon^2 + \sigma_v^2)^{1/2}}, \tag{25}$$

illustrating more directly how search costs $\bar{c}$ and information costs k lower the number of informed investors, while risk aversion γ and noise trading σ_q raise I.

Figure 2 provides a graphical illustration of the determination of equilibrium as the intersection of the managers' and investors' indifference curves. The figure is plotted based on the parametric example above,[9] but it also illustrates the derivation of equilibrium for a general search function $c(M, A)$.

Specifically, Figure 2 shows various possible combinations of the numbers of active investors, A, and asset managers, M. The solid blue line indicates investors' indifference condition (18). When (A, M) is to the North-West of the solid blue line, investors prefer to search for asset managers because managers are easy to find and attractive to find due to the limited efficiency of the asset market. In contrast, when (A, M) is South-East of the blue line, investors prefer to be passive as the costs of finding a manager is not outweighed by the benefits. The indifference condition is naturally increasing as investors are more willing to be active when there are more asset managers.

Similarly, the dashed red line shows the managers' indifference condition (19). When (A, M) is above the red line, managers prefer not to incur the information cost k since too many managers are seeking to service the investors. Below the red line, managers want to become informed asset managers. Interestingly, the manager indifference condition is hump shaped for the following reason: When the number of active investors increases from zero, the

[9] We use the following parameters. Starting with the investors, the total number of optimizing investors is $\bar{A} = 10^8$, the number of noise allocators is $N = 2 \times 10^7$, the absolute risk aversion is $\gamma = 3 \times 10^{-5}$, corresponding to a relative risk aversion $\gamma^R = 3$ and an average invested wealth of $W = 10^5$. Turning to asset markets, the number of shares outstanding is normalized to $\bar{q} = 1$, the expected final value of the asset equals total wealth $\bar{v} = (\bar{A} + N)W = 12 \times 10^{12}$, the asset volatility is 20% meaning that $\sigma_v = 0.2\bar{v}$, the signal about the asset has a 30% noise, $\sigma_\varepsilon = 0.3\bar{v}$, and the noise in the supply is 20% of shares outstanding, $\sigma_q = 0.2$. Lastly, the frictions are given by the cost of being an informed asset manager $k = 2 \times 10^7$ and the search cost parameters $\alpha = 0.8$ and $\bar{c} = 0.3$. This is a one-asset example, but Section 5.1 shows that the key quantitative implications are the same for a multi-asset, "stock-picking" version of the model.

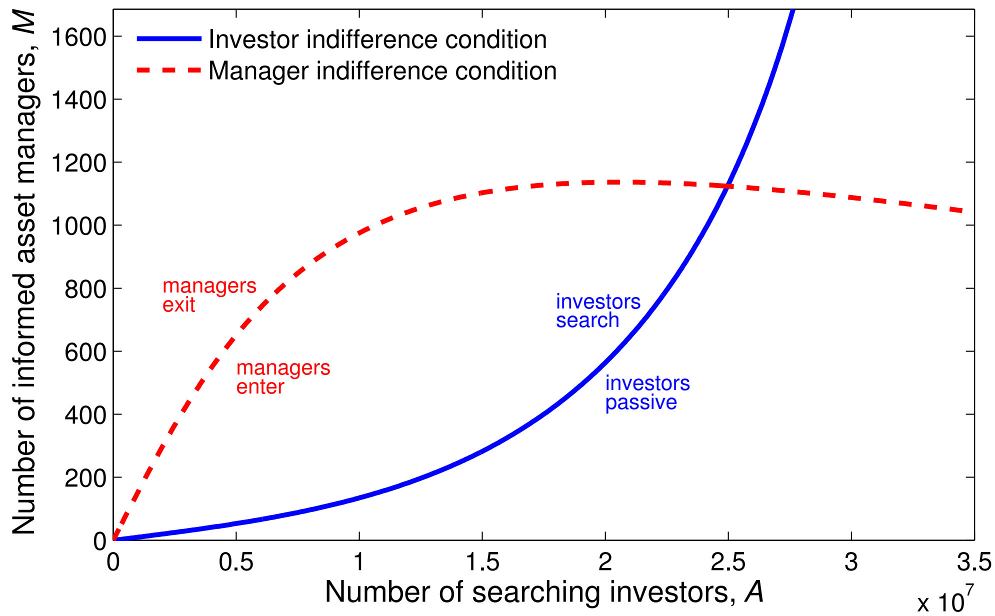

Figure 2: **Equilibrium for assets and asset management.** Illustration of the equilibrium determination of the number of searching investors A and the number of informed asset managers M. Each investor decides whether to search for an asset manager or be passive depending on the actions (A, M) of everyone else, and, similarly, managers decide whether or not to acquire information. The right-most crossing of the indifference conditions is an interior equilibrium.

number of informed managers also increases from zero, since the managers are encouraged to earn the fees paid by searching investors. However, the total fee revenue is the product of the number of active investors A and the fee f. The equilibrium asset management fee decreases with number of active investors because active investment increases the asset-market efficiency, thus reducing the value of the asset management service. Hence, when so many investors have become active that this fee-reduction dominates, additional active investment decreases the number of informed managers.

The economy in Figure 2 has two equilibria. One equilibrium is that there is no asset management: $(A, M) = (0, 0)$. In this equilibrium, no investor searches for asset managers as there is no one to be found, and no asset manager sets up operation because there are no investors. We naturally focus on the more interesting equilibrium with $A > 0$ and $M > 0$.

Figure 2 also helps illustrate the set of equilibria more generally. First, if the search and information frictions c and k are strong enough, then the blue line is initially steeper than the red line and the two lines only cross at $(A, M) = (0, 0)$, meaning that this equilibrium is unique due to the severe frictions. Second, if frictions c and k are mild enough, then the blue line ends up below the red line at the right-hand side of the graph with $A = \bar{A}$. In this case, all investors being active is an equilibrium. Lastly, when frictions are intermediate — as in Figure 2 — the largest equilibrium is an interior equilibrium, i.e., $A < \bar{A}$ and $M < \bar{M}$. We focus on such interior equilibria since they are the most realistic and interesting ones. We note that, while Figure 2 has only a single interior equilibrium, more interior equilibria may exist for other specifications of the search cost function (e.g., because the investor indifference condition starts above the origin, or because it can in principle "wiggle" enough to create additional crossings of the two lines).

3 Equilibrium Properties

We now turn to our central results on how the frictions in the market for money management interact with the efficiency of the asset market. We say that the asset price is *fully efficient* if $\eta = 0$, meaning that the price fully reflects the signal. In equilibrium, asset prices always involve some degree of inefficiency ($\eta > 0$), but efficiency can arise as a limit as we shall see. We employ the term *efficiently inefficient* to refer to the equilibrium level of inefficiency given the frictions (as discussed in the introduction).

We start by considering some basic properties of performance in efficiently inefficient markets. We use the term outperformance to mean that an informed investor's performance yields a higher expected utility than that of an uninformed, and vice versa for underperformance. We note that an investor's expected utility is directly linked to his (squared) Sharpe ratio (see Proposition 8), whose expectation is in turn proportional to the expected return. (We derive this basic result for a mean-variance framework in the proof of Proposition 7.)

Proposition 2 (Performance) *In a general equilibrium for assets and asset management:*

(i) Informed asset managers outperform passive investing before and after fees, $u_i - f > u_u$*.*

(ii) Uninformed asset managers underperform after fees.

(iii) Searching investors' outperformance net of fees just compensates their search costs in an interior equilibrium, $u_i - f - c = u_u$.

(iv) Larger equilibrium search frictions means higher net outperformance for informed managers.

(v) The value-weighted average manager (or, equivalently, the value-weighted average investor) outperforms after fees if and only if the number N *of noise allocators is small relatively to the number* A *of searching investors,* $A \geq N\left(1 - 2\frac{M}{\bar{M}}\right)$.[10]

These results follow from the fact that investors must have an incentive to incur search costs to find an asset manager and pay the asset-management fees. Investors who have incurred a search cost can effectively predict manager performance. Interestingly, this performance predictability is larger in an asset management market with larger search costs.

To the extent the search costs are larger for hedge funds than mutual funds, larger for international equity funds than domestic ones, larger for insurance products than mutual funds, and larger for private equity than public equity funds, this result can explain why the former asset management funds may deliver larger outperformance and why the markets they invest in are less efficient.

Next, we consider the other implications of the search cost for finding informed asset managers. To analyze comparative statics, we focus on the equilibrium with the largest value of I (which is the equilibrium featuring the highest price efficiency).

Proposition 3 (Search for asset management)

(i) Consider two search cost functions, c_1 *and* c_2*, with* $c_1 > c_2$ *and the corresponding largest-I equilibria. In the equilibrium with the lower search costs* c_2*, the numbers of active investors* A *and of informed investors* I *are larger, the number of managers* M

[10]Outperformance obtains if and only if N is below a cut-off that depends only on the other exogenous parameters under the additional conditions $c_M \leq 0$ and $c_A \geq 0$. These conditions are discussed following Proposition 4 below.

may be higher or lower, the asset price is more efficient, the asset management fee f is lower, and the total fee revenue $f(A+N)$ may be either higher or lower.

(ii) If $\{c_j\}_{j=1,2,3,...}$ is a decreasing series of cost functions that converges to zero at every point, then $A = \bar{A}$ when the cost is sufficiently low, that is, all rational agents search for managers. If the number of investors $\{\bar{A}_j\}$ increases towards infinity as j goes to infinity, then η goes to zero (full price efficiency in the limit), the asset management fee f goes to zero, the number of asset managers M goes to zero, the number of investors per manager goes to infinity, and the total fee revenue of all asset managers $f(A+N)$ goes to zero.

This proposition provides several intuitive results, which we illustrate in Figure 3. As seen in the figure, a lower search costs means that the investor indifference curve moves down, leading to a larger number of active investors in equilibrium. This result is natural, since investors have stronger incentives to enter when their cost of doing so is lower.

The number of asset managers can increase or decrease (as in the figure), depending on the location of the hump in the manager indifference curve. This ambiguous change in M is due to two countervailing effects. On the one hand, a larger number of active investors increases the total management revenue that can be earned given the fee. On the other hand, more active investors means more efficient asset markets, leading to lower asset management fees. When the search cost is low enough, the latter effect dominates and the number of managers starts falling as seen in part (ii) of Proposition 3.

As search costs continue to fall, the asset-management industry becomes increasingly concentrated, with fewer and fewer asset managers managing the money of more and more investors. This leads to an increasingly efficient asset market and market for asset management. Specifically, the asset-management fee and the total fee revenue decrease toward zero, and increasingly fewer resources are spent on information collection as only a few managers incur the cost k, but invest on behalf of an increasing number of investors.

We next consider the effect of changing the cost of acquiring information.

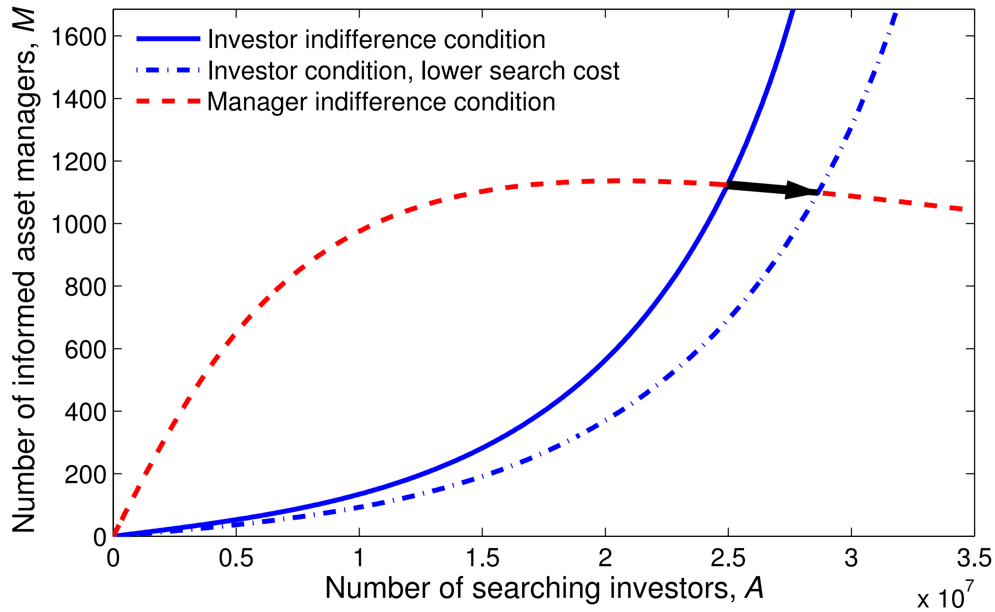

Figure 3: **Equilibrium effect of lower investor search costs.** The figure illustrates that lower costs of finding asset managers implies more active investors in equilibrium and, hence, increased asset-market efficiency.

Proposition 4 (Information cost) *Suppose that c satisfies $c_M \leq 0$ and $c_A \geq 0$. As the cost of information k decreases, the largest equilibrium changes as follows: The number of informed investors I increases, the number of asset managers M increases, the asset-price efficiency increases, and the asset-management fee f goes down. The number of active investors A may increase or decrease.*

The proposition relies on a regularity condition on the search cost function c, namely that finding an informed manager is easier if a larger fraction of all managers are informed, $c_M \leq 0$, and more challenging if more other investors are competing for the asset manager's attention, $c_A \geq 0$. This condition is satisfied for the search cost function considered in our example in equation (20).

The results of this proposition are illustrated in Figure 4. As seen in the figure, a lower information cost for asset managers moves their indifference curve out. This leads to a higher number of asset managers and informed investors in equilibrium, which increases the

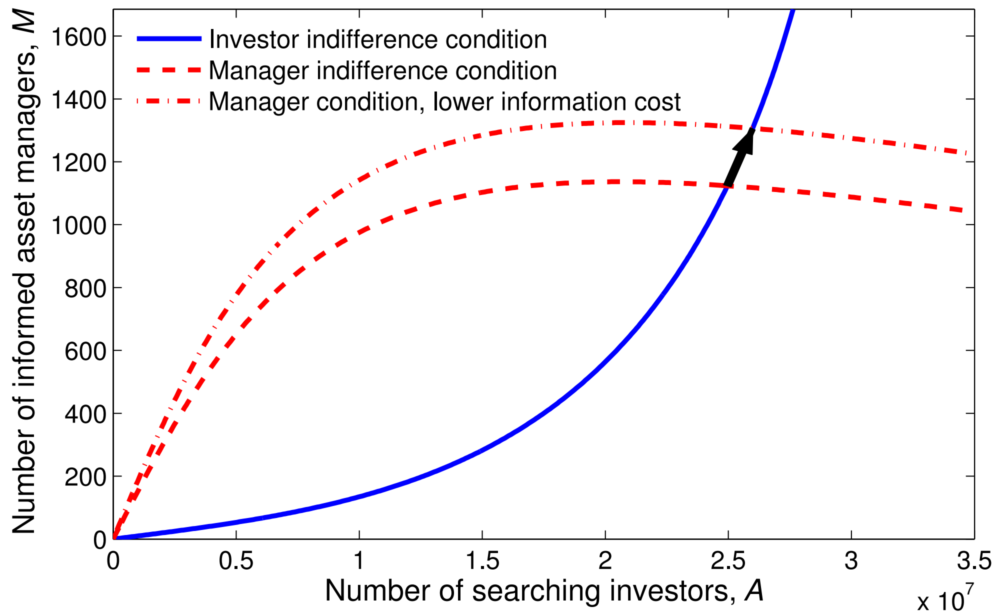

Figure 4: **Equilibrium effect of lower information acquisition costs.** The figure illustrates that lower costs of getting information about assets implies more active investors and more asset managers in equilibrium and, hence, increased asset-market efficiency.

asset-price efficiency. Naturally, less "complex" assets — assets with lower k — are priced more efficiently than more complex ones, and the more complex ones have fewer managers, higher fees, and fewer investors. We note that the larger efficiency is equivalent to lower gains from searching for a manager, which explains why the number of searching investors, A, can decrease.

We also consider the importance of fundamental asset risk and noise trader risk in the determination of the equilibrium. An increase in risk exacerbates the disadvantage of investing uninformed, which attracts more investors to active management and more informed managers to service them.

Proposition 5 (Risk) *Suppose that c satisfies $c_M \leq 0$ and $c_A \geq 0$. An increase in the fundamental volatility σ_v or in the noise-trading volatility σ_q leads to a larger number of active investors A, informed investors I, and informed asset managers M. The effect on the efficiency of asset prices and the asset-management fee f, as well as the total fee revenues*

$f(A+N)$, is ambiguous. The same results obtain with a proportional increase in $(\sigma_v, \sigma_\varepsilon)$ or in all risks $(\sigma_v, \sigma_\varepsilon, \sigma_q)$.

4 Small and Large Investors and Asset Managers

So far, we have considered an economy in which all investors and managers are identical ex ante, but, in the real world, investors differ in their wealth and financial sophistication and managers differ in their education and investment approach. Should large asset owners such as high-net-worth families, pension funds, or insurance companies invest differently than small retail investors? If so, how does the decision to be active depend on the amount of capital invested and the financial sophistication? What type of asset managers are more likely to be informed and what type of manager can be expected to outperform?

To address these issues, we extend the model to capture different types of investors and managers. For the former, each investor $a \in [0, \bar{A}]$ has an investor-specific search cost c_a, where a smaller search cost corresponds to greater sophistication. Further, investors have different levels of absolute risk aversion, γ_a. We can interpret these as arising from different wealth W_a or different relative risk aversions γ_a^R, corresponding to a constant absolute risk aversion of $\gamma_a = \gamma_a^R / W_a$. Since wealth levels vary a lot more than relative risk aversions, variation in γ_a is mostly driven by wealth differences in the real world. In any event, c_a, γ_a^R, and W_a are drawn randomly, independently of each other and across agents. Also, noise allocators $n \in [0, N]$ have (c_n, γ_n^R, W_n) drawn independently from the same distribution.[11]

To capture different types of asset managers, we assume that each manager $m \in [0, \bar{M}]$ has a manager-specific cost k_m of becoming informed — one can think of this feature as skill heterogeneity — and that they are ordered according to this cost. Hence, managers with lower index m have lower costs, that is, the function $k : [0, \bar{M}] \to \mathbb{R}$ is increasing.

[11] These independence assumptions only affect our performance results and we note that these results would only be strengthened under the realistic assumptions that high sophistication (low c) correlates with high wealth W, or if noise allocators are more likely to have low sophistication and wealth.

4.1 Who Should be Active vs. Passive?

We solve the model as before, but now different investors have different portfolio choices, asset management fees, and optimal search decisions. Clearly, each investor's portfolio choice is as before, just scaled with the individual risk tolerance, $1/\gamma_a$. Turning to the negotiation of the asset management fee, a searching investor's best outside option is now paying the cost c again to find another manager, rather than investing on her own (because such agents prefer searching for a manager to investing on their own). Hence, the fee is determined just as for the noise allocators in Section 2.4, implying that $f_a = c_a$.To complete the description of investor behavior, note that each investor prefers to search for a manager if $u_{i,a} - c_a - f_a \geq u_{u,a}$. Inserting the fee and $\gamma_a(u_{i,a} - u_{u,a}) = \eta$, we get the first result in the following proposition.

Proposition 6 (Who should be active/passive)

(i) An investor a should invest with an active manager if he has a large wealth W_a, low relative risk aversion γ_a^R, or low cost c_a of finding and assessing the manager, all relative to the asset-market inefficiency η, that is, if

$$\frac{\gamma_a^R c_a}{W_a} = \gamma_a c_a \leq \frac{1}{2}\eta, \tag{26}$$

and otherwise should be passive.

(ii) An asset manager m should acquire information if her information cost is low, $k_m \leq k_M$, and otherwise remain uninformed.

This first result is intuitive and consistent with the idea that the active investors should be those who have a comparative advantage in asset allocation, either large investors who can hire a serious manager-selection team or sophisticated investors with special insights on asset managers. Indeed, the cost of finding and vetting an informed asset manager is a smaller fraction of the investment for large investors as is captured by equation (26).

The second result is also natural. Asset managers are more likely to have success in informed trading if they are well educated, experienced, and have access to an existing research infrastructure, while other managers who find it more difficult to collect useful information might prefer to limit their costs.

Having characterized the optimal behavior of investors and managers, we turn to the determination of the equilibrium price efficiency η and the number of informed managers M. The price efficiency depends on the aggregate risk tolerance of all investors with informed managers,

$$\tau = \bar{A}\, E\left(\frac{1}{\gamma_a} 1_{\{\gamma_a c_a \leq \frac{1}{2}\eta\}}\right) + N \frac{M}{\bar{M}}\, E\left(\frac{1}{\gamma_n}\right). \tag{27}$$

Here, the first part is the total risk tolerance of searching investors (those who decide to search based on (26)), and the second term is the total risk tolerance of all the noise allocators who happen to find an informed manager. Given the total risk-bearing capacity of investors with informed managers, equation (12) which determines price inefficiency η is modified by replacing I/γ with τ:

$$\eta(\tau) = -\frac{1}{2}\log\left(1 - \frac{\sigma_q^2 \sigma_\varepsilon^2}{\tau^2 + \sigma_q^2 \sigma_\varepsilon^2} \frac{\sigma_v^2}{\sigma_\varepsilon^2 + \sigma_v^2}\right). \tag{28}$$

Since τ increases in η in (27) and, vice versa, η decreases in τ in (28), there exists a unique solution to these equations for each number of informed mangers M. Finally, the indifference condition for the marginal asset manager M states that the fee revenue from searching investors per manager covers her cost k_M:

$$\frac{\bar{A}}{M}\, E\left(c_a\, 1_{\{\gamma_a c_a \leq \frac{1}{2}\eta\}}\right) = k_M. \tag{29}$$

Hence, a general equilibrium with many types of investors and managers is characterized by η, τ, and M that satisfy (27)–(29) and such an equilibrium exists.[12]

[12]While the indifference condition (29) applies with equality in an interior equilibrium, corner solutions are characterized by either $M = 0$ and the right-hand side being greater or $M = \bar{M}$ and the left-hand side being greater.

4.2 How Size and Sophistication Affect Performance

The model makes clear predictions about the expected performance differences across different types of investors and asset managers. Investors who are more wealthy (measured by W_a) and more sophisticated (measured by $1/c_a$) are more likely to search for an informed manager and, as a result, such investors allocate to better managers on average.

To state these performance predictions in terms of percentage returns, we suppose, without loss of generality, that a manager scales the portfolio such that any investor with a relative risk aversion of $\bar{\gamma}^R$ optimally invest his entire wealth with the manager, say W. Then we can define the manager's return as her dollar profit per capital committed W. An investor with twice the relative risk aversion $\gamma_a^R = 2\bar{\gamma}^R$ naturally invests only half his wealth with the manager and earns the same percentage return (before fees) on the committed capital.

Proposition 7 (Size, sophistication, and performance)

(i) Investors with larger wealth W earn higher expected returns on their investments with asset managers (before and after fees) than smaller ones, on average. Similarly, more sophisticated investors (i.e., with low c) earn higher expected returns on their investment with managers than unsophisticated ones, on average.

(ii) Returns and average investor size covary positively across managers. Similarly, returns and average sophistication covary positively.

(iii) Asset managers with a comparative advantage in collecting information ($k_m \leq k_M$) earn higher expected returns (before and after fees) than those with large information costs.

These results are intuitive and give rise to several testable predictions that we confront with the existing evidence in Section 6. Part (i) shows that, since large and sophisticated investors can better afford to spend resources on finding an informed manager, they are more likely to find one and, as a result, they expect to earn higher returns. The higher returns are partly a compensation for the search costs that these agents incur, but they can even outperform after search costs when inequality (26) is strict.

Said differently, if a small investor with no special knowledge of asset managers (that is, an investor for whom (26) is not satisfied) invests with an active manager, then he must be a noise allocator. Since noise allocators pay fees even to uninformed managers, such investors are expected to earn lower returns.

On the other hand, noise allocators are under-represented among large sophisticated investors. We note that the model-implied effect is not linear in that, as investors become very large (or sophisticated), they search for a manager almost surely, and therefore an even larger size has a negligible effect on their expected performance.

Part (ii) makes a similar statement from the perspective of the asset manager. Asset managers with larger and more sophisticated investors are more likely to have investors who have performed due diligence and confirmed that they are informed about security markets. These managers, being more likely to have passed a screening, should deliver higher expected return on average (even though some of them can still be uninformed as some large investors can also be noise allocators). Other measures proxying for the type of a manager's clientele, such as the proportion of large investors (i.e., with wealth above a given threshold), would work as well.

Part (iii) shows that managers who find it easier to collect information will be likely to do it. Indeed, for the marginal manager, the cost of information equals the benefit so anyone with higher costs will not acquire information. Hence, an asset manager may be more likely to be informed if she is well educated, experienced, and benefits from firm-wide investment research as part of an investment firm with multiple funds. Hence, investors' search process might partly consist of examining whether an asset manager has such qualities, as discussed further in Appendix A.

5 Further Applications of the Framework

5.1 Understanding the Economic Magnitude

To illustrate the economic magnitudes of some of the interesting properties of the model in a simple way, it is helpful to write our predictions is relative terms. Specifically, as seen in Section 4, investors' preferences can be written in terms of the relative risk aversion γ^R and wealth W such that $\gamma = \gamma^R/W$. Further, the asset management fee can be viewed as a fixed proportion of the investment size and we define the proportional fee as $f^{\%} = f/W$. With these definitions, we get the following predictions on the economic magnitude of the market inefficiency, asset management fee, and improvement in gross Sharpe ratio (i.e., before fees and search costs) for investors allocating to informed managers relative to uninformed managers.[13]

Proposition 8 (Economic magnitude) *The market inefficiency η is linked to the proportional asset management fee and relative risk aversion,*

$$\eta = 2f^{\%}\gamma^R, \tag{30}$$

and can be characterized by the difference in squared gross Sharpe ratios attainable by informed (SR_i) vs. uninformed (SR_u) investors using a log-linear approximation:

$$\eta \cong \frac{1}{2}\left(E(SR_i^2) - E(SR_u^2)\right). \tag{31}$$

To illustrate these results, suppose that all investors have relative risk aversion of $\gamma^R = 3$ and that the equilibrium percentage asset management fee is $f^{\%} = 1\%$. Then we have the following relation for the asset market inefficiency η based on (30):

$$\eta = 2f^{\%}\gamma^R = 2 \cdot 1\% \cdot 3 = 6\%. \tag{32}$$

[13]Since each type of investor $n = i, u$ chooses a position of $x = \frac{\mathrm{E}_n(v)-p}{\gamma \mathrm{Var}_n(v)}$, the investor's conditional Sharpe ratio is $SR_n = \frac{|\mathrm{E}_n(v)-p|}{\sqrt{\mathrm{Var}_n(v)}}$ (where E_n and Var_n are the mean and variance conditional on n's information).

In other words, the standard deviation of the true asset value from the perspective of a trader who knows the signal is $e^{-6\%} \approx 94\%$ of that of a trader who only observes the price. Further, we see that the Sharpe ratios must satisfy

$$E(SR_i^2) - E(SR_u^2) = 4f^{\%}\gamma^R = 4 \cdot 1\% \cdot 3 = 0.12.$$

Hence, if uninformed investing yields an expected squared Sharpe ratio of 0.4^2 (similar to that of the market portfolio), informed investing must yield an expected Sharpe ratio around 0.53^2 (i.e., $0.53^2 - 0.4^2 = 0.12$). Hence, at this realistic fee level, the implied difference in Sharpe ratios between informed and uninformed managers is relatively small and hard to detect empirically.

We note that the same calculations linking inefficiency, fees, and Sharpe ratios go through in a multi-asset market as shown in Section 5.3 (given the appropriate generalization of the formula for efficiency η.) Informed strategies can therefore be thought of as cross-sectional, "stock-picking" strategies, with the same implications on the quantitative realism of the link between performance and fees.

5.2 Welfare and Market Liquidity

It is interesting to consider welfare implications of the model, although many welfare effects are non-monotonic and ambiguous as is often the case in welfare analysis. We consider a welfare function that is simply the sum of all agents' utilities:

$$\begin{aligned}\text{welfare} = {} & A\,(u_i - c - f) + (\bar{A} - A)u_u + \frac{M}{\bar{M}}N(u_i - c - f) + \left(1 - \frac{M}{\bar{M}}\right)N(u_u - c - f) \\ & + M\left(\frac{fA}{M} - k\right) + Nf + \nu, \end{aligned} \tag{33}$$

namely the utilities of the A active investors, the $\bar{A} - A$ passive investors, the A noise allocators, the M informed asset managers, the uninformed asset managers who earn the fees from the noise allocators, and the utility of the noise traders ν. To define the utility of the noise traders, we proceed in the spirit of Leland (1992) and endow them with risk-neutral preferences over their proceeds, $\nu = \mathrm{E}\left[(q - \bar{q})p\right]$. We could also include the utility of the original securities owners, but for simplicity we set the supply of shares to be $\bar{q} = 0$.

In the real world, the welfare benefits of efficient markets also derive from a better allocation of resources due to real investment decisions, better labor market allocations, improved incentives of corporate officers, and many other effects not captured by our model. A complete study of all such welfare effects is beyond the scope of this paper so we limit ourself to showing that even this limited welfare function yields complex results.[14]

We can simplify the welfare function as follows:

$$\text{welfare} = (\bar{A} + N)u_u + I(u_i - u_u) - (A + N)c - Mk + \nu, \tag{34}$$

which makes clear that the central welfare costs are the resources spent on search, namely $(A + N)c$, and the resources spent on information collection, Mk. These costs are offset by the investment benefits (u_i, u_u, and ν) of resources spent on information and matching.

In an interior equilibrium, investors are indifferent towards being active or passive and managers break even. These two observations allow us to further simplify the welfare function as

$$\text{welfare} = (\bar{A} + N)u_u + N\frac{M}{\bar{M}}(u_i - u_u) - Nc + \nu, \tag{35}$$

that is, the welfare is the same as if all agents receive the utility they would have as uninformed agents in a market characterized exogenously by the equilibrium market efficiency η, with the additional benefit of allowing some noise allocators to invest based on information. (Achieving this efficiency level endogenously, of course, requires that a certain number of agents be active.)

Interestingly, the utility of the noise traders, ν, is closely linked to the equilibrium market liquidity. To see this link, we define market illiquidity as the equivalent of Kyle's lambda in

[14]The complexity of the welfare analysis in noisy-REE frameworks is apparent, for instance, in Leland (1992), who studies the desirability of banning insider trading.

our model,

$$\lambda \equiv -\frac{dp}{dq} = \theta_s \theta_q. \tag{36}$$

where θ_s and θ_q are given in Equations (8)–(9). In other words, λ measures the market impact of trading. Since noise traders move prices against themselves despite their lack of information, a higher market illiquidity is associated with lower utility:

$$\nu = -\lambda \sigma_q^2. \tag{37}$$

We see that a higher market illiquidity λ and more noise trading σ_q^2 both lower the utility of the noise traders.

We are interested in the dependence of welfare on the search cost. While the overall welfare depends on search costs in a complex way, the model yields some nice results regarding liquidity and noise trader utility. Indeed, as we have seen, noise trader utility depends on the market liquidity, but λ is not monotonic in the search cost or the number of active investors in general. In particular, a lower cost, leading to a higher number of informed agents I, increases the potential for adverse selection and may therefore accentuate the reaction of the price to the noise traders' position. However, under certain conditions, market liquidity is at its highest when search costs are low, as the following proposition states.

Proposition 9 (Welfare and liquidity) *When the search cost c and I are small enough, a decrease in c reduces Kyle's lambda λ, and this improvement in market liquidity increases the welfare of the noise traders. The total welfare can increase or decrease as a result of the lower c depending on the parameters.*

5.3 Multiple Assets: Security Selection and Information Theory

We next extend the model to cover multiple assets, and show that the earlier results continue to obtain subject to very minor changes. Hence, our results are not only about managers seeking to market time a single market, but they are also about the more common case where a manager performs security selection, e.g., selecting among a large number of stocks.

The economy now has $n \geq 1$ assets and the payoff v is consequently a normally distributed vector of dimension n with mean $\bar{v}$ and variance-covariance matrix Σ_v, which we write as $v \sim \mathcal{N}(\bar{v}, \Sigma_v)$. Asset managers can acquire various signals about all the assets. We collect all the signals in a vector of dimension n that we denote $s = v + \varepsilon$, where $\varepsilon \sim \mathcal{N}(0, \Sigma_\varepsilon)$ is the noise in the signal.[15] Lastly, the noisy supply $q \sim \mathcal{N}(\bar{q}, \Sigma_q)$ is now also of dimension n and all other assumptions are the same.

The solution of the model is as before. We first conjecture and verify a linear pricing function:

$$p = \theta_0 + \theta_s^\top \left((s - \bar{v}) + \theta_q^\top (q - \bar{q}) \right). \tag{38}$$

The resulting optimal demands are linear, as well. We generalize our definition (10) of market inefficiency η as follows:

$$\eta = \frac{1}{2} \log \left(\frac{\det(\mathrm{var}(v|p))}{\det(\mathrm{var}(v|s))} \right). \tag{39}$$

As before, the market is considered less efficient if the signal s contains more information than the price p. The informativeness can be measured using entropy and, as is known from information theory, the entropy of a multivariate normal is a half times the log-determinent of the variance-covariance matrix (plus a constant). In other words, the market efficiency is the difference in entropy, which can also be seen to be the expected Kullback-Leibler divergence:

Proposition 10 (Efficiency, Entropy, and the Value of Information) *A unique linear equilibrium exists for general $n \geq 1$. In this equilibrium, market inefficiency η given by* (39) *equals the difference in entropy between the distributions of v conditional on p, respectively on s — and also equals the expected Kullback-Leibler divergence of the distribution*

[15] If we start with a signal $\hat{s}$ of any other dimension, then the conditional mean $E(v|\hat{s})$ is a sufficient statistic, it is of dimension n, and it can be easily translated into a signal s as modeled above.

conditional on p from that conditional on s. Further, market inefficiency equals the economic value of information, $(u_i - u_u)\gamma = \eta$.

This proposition contains the attractive result that the multivariate notion of market efficiency based on entropy is linked to the utility gain from information in exactly the same way as with a single asset in (11). The idea that the economic and information-theoretic values of information are linked goes back at least to Marschak (1959), and we further establish a link to the degree of market inefficiency.[16]

Further, the fact that market efficiency can be summarized by a single number that is linked to the value of information means that having many assets hardly changes the model:

Proposition 11 (Security Selection) *As with a single asset ($n = 1$), η decreases with the number I of informed investors; the equilibrium asset management fee is given by* (14)*; and the equilibrium number of searching investors and informed asset managers are given by* (18)–(19).

We see that, despite the presence of many assets, the asset-market equilibrium is summarized by a single number, the inefficiency η. Hence, based on this single number, we can solve the model as before. Indeed, the asset management fee depends on investors' utility gain from information, which is linked to inefficiency as before. Likewise, investors' incentive to search for an informed manager depends on η in the same way as before and, similarly, asset managers' incentive to acquire information is as before. Further, as before, utility is linked to the (squared) Sharpe ratio, so our results above translate directly to statements about security-selection strategies.

[16]See also Cabrales et al. (2013) for a recent contribution on entropy as the economic value of information and for further references.

5.4 Managers with Different Signals

Our model also applies when different managers received different signals. Indeed, adopting the formulation of Hellwig (1980), in that manager j receives signal[17]

$$s^j = v + \varepsilon^j, \tag{40}$$

where ε^j are i.i.d. conditional on v.

When every agent invests with only one manager,[18] the asset-market equilibrium is characterized by

$$p = \hat{\theta}_0 + \hat{\theta}_v(v - \hat{\theta}_q(q - \bar{q})), \tag{41}$$

where $\hat{\theta}_0$, $\hat{\theta}_v$, and $\hat{\theta}_q$ are constant and computed by matching coefficients in the market-clearing condition.

Proposition 12 *A linear equilibrium exists in which the price takes the form* (41) *and inefficiency is given by* $\eta = \frac{1}{2}\log\left(\frac{\mathrm{var}(v|p)}{\mathrm{var}(v|p,s^j)}\right)$. *The general equilibrium is characterized by* (14), (18), (19), *and*

$$\eta = \gamma(u_i - u_u) = -\frac{1}{2}\log\left(1 - \frac{\sigma_\varepsilon^2\sigma_q^2}{I^2/\gamma^2 + \sigma_\varepsilon^2\sigma_q^2 + \sigma_\varepsilon^4\sigma_q^2/\sigma_v^2}\right). \tag{42}$$

We note that the equilibrium is qualitatively the same as in the base-case model, which can be seen by comparing equations (12) and the last line in (42). The qualitative dependence of η on I and the key parameters is the same in the two cases.

[17] We remind the reader that we consider a continuum of managers, and therefore signals s^j, which renders the model as tractable as the Grossman-Stiglitz one.

[18] Proposition 12 is stated under the assumption that agents may interact with only one manager. This assumption is, however, not necessary. In the appendix we show that, if investing with a second manager must be done at the fee negotiated by all the other agents, then the agent would be strictly losing by paying the meeting cost c and fee f to receive the — smaller — marginal benefits of investing with another manager.

5.5 The Cost of Passive Investing

In the benchmark model, investors had to choose between incurring a search cost to find an active manager and using passive investing for free. In the real world, however, passive investing also comes at a cost. Indeed, buying a diversified portfolio takes time and is associated with transaction costs. The costs of passive investing has come down over time due to the introduction and adoption of discount brokers, low-cost index funds, and exchange traded funds (ETFs), e.g., those run by Vanguard. It is interesting to consider how these costs of passive investing and their reduction affects the market for active asset management and the security markets.

We augment the benchmark with the assumption that investors (who are ex ante identical as in the benchmark model) must pay a cost c^u for passive investing (i.e., to put on the portfolio $x_u(p)$). For simplicity, we assume that investors are using both passive and active investing, both of which are superior to leaving money under the mattress, i.e., $u_i - c - f = u_u - c^u > 0$.

Solving this generalized model requires only to note that the cost c^u modifies the gains from trade between a matched investor-manager pair from $\frac{\eta}{\gamma}$ to $\frac{\eta}{\gamma}+c^u$. These gains from trade feature both in the ex-ante decisions of the investor and manager, and in the determination of the fee. Specifically, the Nash bargaining problem becomes to maximize

$$\left(\frac{\eta}{\gamma} - f + c^u\right) f \tag{43}$$

with solution

$$f = \frac{c^u}{2} + \frac{\eta}{2\gamma}. \tag{44}$$

The investor and manager indifference conditions (18)–(19) are modified to

$$\frac{\eta}{2\gamma} + \frac{c^u}{2} = c(M, A) \tag{45}$$

$$M = \frac{A}{k}\left(\frac{\eta}{2\gamma} + \frac{c^u}{2}\right). \tag{46}$$

Based on these revised indifference conditions, we can characterize how the general equilibrium for assets and asset management depends on the cost of passive investing.

Proposition 13 (Cost of passive investing) *Suppose that c satisfies $c_M \leq 0$ and $c_A \geq 0$. As the cost of passive investing c^u decreases, the largest equilibrium changes as follows. The numbers of active investors A, informed investors I, and of informed active managers M are lower, and the asset price is less efficient. The asset management fee f may increase or decrease.*

As seen in the proposition, we would expect that lower costs of passive investing due to index funds and ETFs should drive down the relative attractiveness of active investing and therefore reduce the amount of active investing, rendering the asset market less efficient. The supply of informed managers catering to active investors declines. The search costs, too, react to the changes in the numbers of investors in managers, to the effect that the relative gains from investing with an informed manager, as well as the fee, may either increase or decrease.

6 Empirical Implications

Searching for Asset Managers. Search frictions in the asset management fund industry are documented by Sirri and Tufano (1998) and Jain and Wu (2000), and, consistent with our model, proxies for lower search costs are associated with more investors. Hortaçsu and Syverson (2004) find that, even for relatively homogeneous S&P500 index funds, search costs help explain the large number of funds and the sizeable dispersion in fund fees. Such fragmentation and fee dispersion also arises in our model with heterogeneous agents. The estimated search costs for index funds could be viewed as a lower bound on the search costs in active management. In a cross-country study, Khorana et al. (2008) find that mutual fund *"fees are lower in wealthier countries with more educated populations,"* which may be related to lower search frictions for well educated investors.

Performance Differences across Asset Managers. The central prediction of asset market efficiency is that all managers underperform by an amount equal to their fees and the early empirical literature documented consistent evidence for the average US mutual fund (Fama (1970)). However, the research over the past decades shows that this early evidence for the average asset manager hides a significant cross-sectional variation across managers. Indeed, the literature documents a significant difference between the net-of-fee performance of the best and worst mutual managers of mutual funds (Kosowski et al. (2006), Kacperczyk et al. (2008), Fama and French (2010)), hedge funds (Kosowski et al. (2007), Fung et al. (2008), Jagannathan et al. (2010)), private equity, and venture capital funds (Kaplan and Schoar (2005)). For instance, Kosowski, Naik, and Teo (2007) report that *"top hedge fund performance cannot be explained by luck, and hedge fund performance persists at annual horizons... Our results are robust and relevant to investors as they are neither confined to small funds, nor driven by incubation bias, backfill bias, or serial correlation."*

The strong performance of the best managers presents a rejection of Fama's hypothesis that asset markets are fully efficient and all asset managers underperform by their fees. Further, the net-of-fee performance spread between the best and worst managers is a rejection of the hypothesis by Berk and Green (2004) that all managers deliver the same expected net-of-fee return. The existence of the performance spread is, however, consistent with our model's predictions. In our model, top asset managers should be difficult to locate and their outperformance must compensate investors for their search costs.

Performance Differences Linked to Our Search Mechanism. While the mere existence of a performance spread among the best and worst asset managers rejects existing theories and favor our theory, this "victory" can be seen as a somewhat weak test since other theories might also predict such a performance spread. To test the model at a deeper level, we ask whether performance differences appear to be driven by our search mechanism, that is, are consistent with the predictions of Proposition 7.

Consistent with search costs being higher for alternative investments (hedge funds and private equity) than mutual funds, we see larger performance spreads among alternative managers. However, comparisons across markets may be driven by multiple differences, so we need to dig deeper still as we do in Table 1.

First, mutual funds that have an institutional share class outperform other mutual funds (Evans and Fahlenbrach (2012)), consistent with the idea that the institutional investors are more likely to have performed due diligence (Proposition 7(ii)). Institutional investors outperform retail investors more broadly (Gerakos et al. (2014)).

Second, Dyck and Pomorski (2015) find that large institutions outperform small ones in their private equity investments, consistent with Proposition 7(i). Further, consistent with our model's implication that size only matters up to a certain point (at which all investors decide to search), Dyck and Pomorski (2015) find a non-linear effect of size that eventually diminishes.

Third, funds of hedge funds perform better on their local investments where they have a search advantage (Sialm et al. (2014)), also consistent with Proposition 7(i).

Fourth, Guercio and Reuter (2014) find that mutual funds sold directly to searching investors outperform those that are placed via brokers who earn commissions/loads (to noise allocators).

Fifth, consistent with Proposition 7(iii), Chevalier and Ellison (1999) find that *"managers who attended higher-SAT undergraduate institutions have systematically higher risk-adjusted excess returns"* and Chen et al. (2004) find that *"Controlling for fund size [...] the assets under management of the other funds in the family that the fund belongs to actually increase the fund's performance."*

Lastly, consistent with Proposition 2(iv), the outperformance of searching investors is larger in less efficient markets. Dyck et al. (2013) find that "active management in emerging market equity outperforms passive strategies by more than 180 bps per year, and that this outperformance generally remains significant when controlling for risk through a variety of mechanisms. In EAFE equities (developed markets of Europe, Australasia, and the Far East), active management also outperforms, but only by about 50 bps per year, consistent with these markets being relatively more competitive and efficient." Together, these findings provide significant and diverse evidence for the model's performance predictions.

Asset Pricing and Market Efficiency. While the efficient market hypothesis is a powerful theory, it can nevertheless be difficult to test because of the so-called "joint hypothesis" problem. However, the existence of deviations from the Law of One Price (securities with the same cash flows that trade at different prices) is a clear rejection of fully efficient asset markets. The theory of efficiently inefficient markets is *not* the entire complement to fully efficient markets, but, rather, it should be viewed as an equally well-defined null hypothesis. Efficiently inefficient markets means that the marginal investor should be indifferent between passive investing and searching for asset managers, where the latter should deliver an expected outperformance balanced by asset management fees and search costs, consistent with the findings of Gerakos et al. (2014) for professional asset managers. The average manager might not deliver this outperformance due to noise allocators, but investors should be able to collect sufficient information to achieve an outperformance that compensates their costs in an efficiently inefficient market.

In an efficiently inefficient market, anomalies are more likely to arise the more resources a manager needs to trade against them (higher k) and the more difficult it is for investors to build trust in such managers (higher c). For instance, while convertible bond arbitrage is a relatively straightforward trade for an asset manager (low k), it might have performed well for a long time because it is difficult for investors to assess (high c).

Asset management fees. We predict that asset-management fees should be larger for managers of more inefficient assets and in more inefficient asset-management markets. For

Table 1: **Evidence on our predictions.** This table includes references on the performance differences between investors who are more likely to be searching investors vs. noise allocators when allocating to asset managers. The quotes from each paper's main results show that asset managers found by searching investors outperform those of noise allocators, consistent with our model's predictions.

References	**More likely *searching investors***	**More likely *noise allocators***	**Quotes**
Evans and Fahlenbrach (2012)	Institutional investors	Retail investors	*"retail funds with an institutional twin outperform other retail funds by 1.5% per year"*
Dyck, Lins, and Pomorski (2013)	Institutional investors	Retail investors	*"the value of active management depends on the efficiency of the underlying market and the sophistication of the investor"*
Gerakos, Linnainmaa, Morse (2015)	Institutional investors	Retail investors	*"institutional funds earned annual market-adjusted returns of 108 basis points before fees and 61 basis points after fees"*
Dyck and Pomorski (2015)	Larger institutional investors	Smaller institutional investors	*"A one standard deviation increase in PE holdings is associated with 4% greater returns per year"*
Sialm, Sun, and Zheng (2014)	Fund of funds investing locally	Fund of funds investing far away	*"funds of hedge funds overweight their investments in hedge funds located in the same geographical areas and that funds of hedge funds with a stronger local bias exhibit superior performance"*
Del Guercio and Reuter (2014)	Investors searching for direct-sold mutual funds	Investors buying broker-sold funds	*"direct-sold actively managed funds outperform actively managed broker-sold funds"*

instance, if search costs for managers are large, this leads to less active investing and higher management fees. Note that the higher management fee in this example is not driven by higher information costs for managers, but, rather, by the equilibrium dynamics between the markets for the asset and asset management. This may help explain why hedge funds have historically charged higher fees than mutual funds. Also, markets for more complex assets that are costly to study should be more inefficient and have higher management fees. This can help explain why equity funds tend to have higher fees than bond funds and why global equity funds have higher fees than domestic ones.

The Industrial Organisation of Asset Management. In our model, the overall asset management industry faces decreasing returns to scale since a larger amount of capital with informed managers (I) leads to more efficient markets (lower η), reducing manager performance, consistent with the evidence of Pastor et al. (2015).[19] While in our model individual managers do not face decreasing returns to scale, they may do so in the real world as emphasized by Berk and Green (2004), e.g., due to transaction costs.

Our model has several implications for the size of the asset management industry. The asset management industry grows when investors' search cost diminish or when asset managers' information costs go down, leading to more efficient asset markets, consistent with the evidence of Pastor et al. (2015). Other important models that speak to the size of the asset management industry include Berk and Green (2004), Garcia and Vanden (2009), and Pastor and Stambaugh (2012).

When investors' search costs go down, our model predicts that the number of managers will fall, but the remaining managers will be larger (in fact so much larger that the total size of the asset management industry grows as mentioned above). Such consolidation of the asset management industry is discussed in the press, but we are not aware of a direct test

[19] While the overall asset management industry clearly has decreasing returns to scale for reasons described in our model, there might also be effects related to the size of each individual firm in the real world. Anecdotally, small asset managers face increasing returns to scale (due to fixed costs of trading infrastructure, worse commissions and other terms from brokers for small managers, etc.) while very large managers face decreasing returns to scale due to market impact.

of this model prediction.

7 Conclusion

We propose a model in which investors search for asset managers with useful information about securities, just as real-world investors examine an asset manager's investment process, the number of employees, their turnover, and their professional pedigree, whether the manager operates a trading desk 24/7, co-location on major trading venues, costly information sources, risk management, valuation methods, financial auditors, and so on. Our search model captures this time-consuming vetting process. At the same time, managers spend significant resources on making informed investments, captured by embedding these asset managers in the Grossman-Stiglitz information model. Our search-plus-information model turns out to be highly tractable, allowing a closed-form solution with a specific search function and, for a general class of search functions, yielding clear results seen from investors' and managers' indifference curves.

We find that asset managers can increase asset price efficiency by letting investors essentially share information costs, but their ability to do so is limited by the search frictions in the asset-management industry. Therefore, the efficiency of asset markets is fundamentally connected to the efficiency of the asset management market. Our model shows how lower search frictions in asset management leads to improved asset price efficiency, lower asset management fees, less outperformance by asset managers before and after fees, fewer and larger asset managers (i.e., a consolidation of the asset management industry), improved market liquidity, and potential welfare improvements.

To compensate investors for their search cost associated with finding an informed asset manager, informed managers must outperform passive investing after fees, a new prediction that helps explain the empirical evidence that the best mutual funds, hedge funds, and private equity firms do in fact deliver such outperformance. Further, we find that large sophisticated investors should search for informed active managers, while smaller investors are better served by passive investing as the search costs outweigh the potential gains from improved performance of a small portfolio. Therefore, the model implies that asset mangers with larger and more sophisticated investors should perform better on average, consistent with the evidence that institutional managers outperform retail managers and a number of other consistent facts. Hence, the model helps explain a number of empirical facts that were puzzling in light of existing models and it lays the ground for further analysis of asset markets and asset management.

A Real-World Search and Due Diligence of Asset Managers

Here we briefly summarize some of the main real-world issues related to finding and vetting an asset manager. While the search process involves a lot of details, the main point that we model theoretically is that the process is time consuming and costly. For instance, there exist more mutual funds than stocks in the U.S. Many of these mutual funds might be charging high fees while investing with little or no real information, just like the uninformed funds in our model (e.g., high-fee index funds, or so-called "closet indexers" who claim to be active, but in fact track the benchmark, or funds investing more in marketing than their investment process). Therefore, finding a suitable mutual fund is not easy for investors (just like finding a cheap stock is not easy for asset managers).

We first consider the search and due diligence process of institutional investors such as pension funds, insurance companies, endowments, foundations, funds of funds, family offices, and banks. Such institutional investors invite certain specific asset managers to visit their offices and also travel to meet asset managers at their premises. If the institutional investor is sufficiently interested in investing with the manager, the investor often asks the manager to fill out a so-called due diligence questionnaire (DDQ), which provides a starting point for the due diligence process. Here we provide a schematic overview of the process to illustrate the significant time and cost related to the search process of finding an asset manager and doing due diligence, but a detailed description of each of these items is beyond the scope of the paper.[20]

- **Finding the asset manager: the initial meeting.**
  - **Search.** Institutional investors often have employees in charge of external managers. These employees search for asset managers and often build up knowledge of a large network of asset managers whom they can contact. Similarly, asset managers employ business development staff who maintain relationships with investors they know and try to connect with other asset owners, although hedge funds are subject to non-solicitation regulation preventing them from randomly contacting potential investors and advertising. This two-way search process involves a significant amount of phone calls, emails, and repeated personal meetings, often starting with meetings between the staff members dedicated to this search

[20]Standard DDQs are available online, e.g. from the Managed Funds Association (http://www.managedfunds.org/wp-content/uploads/2011/06/Due-Dilligence-Questionnaire.pdf) or the Institutional Limited Partner Association (http://ilpa.org/wp-content/publicmedia/ILPA_Due_Diligence_Questionnaire_Tool.docx). See also "Best Practices in Alternative Investments: Due Diligence," Greenwich Roundtable, 2010 (www.greenwichroundtable.org/system/files/BP-2010.pdf), the CFA Institute's "Model RFP: A standardized process for selecting money managers" (http://www.cfainstitute.org/ethics/topics/Pages/model_rfp.aspx), and "Best Practices for the Hedge Fund Industry," Report of the Asset Managers' Committee to the President's working group on financial markets, 2009 (http://www.cftc.gov/ucm/groups/public/swaps/documents/file/bestpractices.pdf). We are grateful for helpful discussions with Stephen Mellas and Jim Riccobono at AQR Capital Management.

process and later with meetings between the asset manager's high-level portfolio managers and the asset owner's chief investment officer and board.

  - **Request for Proposal.** Another way for an institutional investor to find an asset manager is to issue a request for proposal (RFP), which is a document that invites asset managers to "bid" for an asset management mandate. The RFP may describe the mandate in question (e.g., $100 million of long-only U.S. large-cap equities) and all the information about the asset manager that is required.
  - **Capital introduction.** Investment banks sometimes have capital introduction ("cap intro") teams as part of their prime brokerage. A cap intro team introduces institutional investors to asset managers (e.g., hedge funds) that use the bank's prime brokerage.
  - **Consultants, investment advisors, and placement agents.** Institutional investors often use consultants and investment advisors to find and vet investment managers that meet their needs. On the flip side, asset managers (e.g., private equity funds) sometimes use placement agents to find investors.
  - **Databases.** Institutional investors also get ideas for which asset managers to meet by looking at databases that may contain performance numbers and overall characteristics of the covered asset managers.

- **Evaluating the asset management firm.**
  - **Assets, funds, and investors.** Institutional investors often consider an asset manager's overall assets under management, the distribution of assets across fund types, client types, and location.
  - **People.** Key personnel, overall headcount information, headcount by major departments, stability of senior people.
  - **Client servicing.** Services and information disclosed to investors, ongoing performance attribution, market updates, etc.
  - **History, culture, and ownership.** When was the asset management firm founded, how has it evolved, general investment culture, ownership of the asset management firm, and do the portfolio managers invest in their own funds.

- **Evaluating the specific fund.**
  - **Terms.** Fund structure (e.g., master-feeder), investment minimum, fees, high water marks, hurdle rate, other fees (e.g., operating expenses, audit fees, administrative fees, fund organizational expenses, legal fees, sales fees, salaries), transparency of positions and exposures.
  - **Redemption terms.** Any fees payable, lock-ups, gating provisions, can the investment manager suspend redemptions or pay redemption proceeds in-kind, and other restrictions.

- **Asset and investors.** Net asset value, number of investors, do any investors in the fund experience fee or redemption terms that differ materially from the standard ones?

- **Evaluating the investment process.**
  - **Track record.** Past performance numbers and possible performance attribution.
  - **Instruments.** The securities traded and geographical regions.
  - **Team.** Investment personnel, experience, education, turnover.
  - **Investment thesis and economic reasoning.** What is the underlying source of profit, i.e., why should the investment strategy be expected to be profitable? Who takes the other side of the trade and why? Has the strategy worked historically?
  - **Investment process.** The analysis of the investment thesis and process is naturally one of the most important parts of finding an asset managers. Investors analyze what drives the asset manager's decisions to buy and sell, the investment process, what data is used, how is information gathered and analyzed, what systems are used, etc.
  - **Portfolio characteristics.** Leverage, turnover, liquidity, typical number of positions and position limits.
  - **Examples of past trades.** What motivated these trades, how do they reflect the general investment process, how were positions adjusted as events evolved.
  - **Portfolio construction methodology.** How is the portfolio constructed, how are positions adjusted over time, how is risk measured, position limits, etc.
  - **Trading methodology.** Connections to broker/dealers, staffing of trading desk and is it operating 24/7, possibly co-location on major exchanges, use of internal or external broker algorithms, etc.
  - **Financing of trades.** Prime brokers relations, leverage.

- **Evaluating the risk management.**
  - **Risk management team.** Team members, independence, and authority.
  - **Risk measures.** Risk measures calculated, risk reports to investors, stress testing.
  - **Risk management.** How is risk managed, what actions are taken when risk limits are breached and who makes the decision.

- **Due diligence of operational issues and back office.**
  - **Operations overview.** Teams, functions, and segregation of duties.

  - **Lifecycle of a trade.** The different steps a trade makes as it flows through the asset manager's systems.
  - **Cash management.** Who can move cash, how, and controls around this process.
  - **Valuation.** Independent pricing sources, what level of PM input is there, what controls and policies ensure accurate pricing, who monitors this internally and externally.
  - **Reconciliation.** How frequency and granularly are cash and positions reconciled.
  - **Client service.** Reporting frequency, transparency levels, and other client services and reporting.
  - **Service providers.** The main service providers used abd any major changes (recent or planned).
  - **Systems.** What are the major homegrown or vendor systems with possible live system demos.
  - **Counterparties.** Who are the main ones, how are they selected, how is counterparty risk managed and by whom.
  - **Asset verification.** Some large investors (and/or their consultants) will ask to speak directly to the asset manager's administrator in order to independently verify that assets are valued correctly.

- **Due diligence of compliance, corporate governance, and regulatory issues.**
  - **Overview.** Teams, functions, independence.
  - **Regulators and regulatory reporting.** Who are the regulators for the fund, summary of recent visits/interactions, frequency of reporting.
  - **Corporate governance.** Summary of policies and oversight.
  - **Employee training.** Code of ethics and training.
  - **Personal trading.** Policy, frequency, recent violations and the associated penalties for breach.
  - **Litigation.** What litigation the firm has been involved with.

- **Due diligence of business continuity plan (BCP) and disaster recovery plan.**
  - **Plan overview.** Policy, staffing, and backup facilities.
  - **Testing.** Frequency of tests and intensity.
  - **Cybersecurity.** How IT systems and networks are defended and tested.

The search process for finding an asset manager is very different for retail investors. Clearly, there is no standard structure for the search process for retail investors, but here are some considerations:

- **Retail investors searching for an asset manager.**
  - **Online search.** Some retail investors can search for useful information about investing online and they can make their investment online. However, finding the right websites may require a significant search effort and, once located, finding and understanding the right information within the website can be difficult as discussed further below.
  - **Walking into a local branch of a financial institution.** Retail investors may prefer to invest in person, e.g., by walking into the local branch of a financial institution such as a bank, insurance provider, or investment firm. Visiting multiple financial institutions can be time consuming and confusing for retail investors.
  - **Brokers and intermediaries.** Bergstresser et al. (2009) report that a large fraction of mutual funds are sold via brokers and study the characteristics of these fund flows.
  - **Choosing from pension system menu.** Lastly, retail investors get exposure to asset management through their pension systems. In defined contributions pension schemes, retail investors must search through a menu of options for their preferred fund.
- **Searching for the relevant information.**
  - **Fees.** Choi et al. (2010) find experimental evidence that "search costs for fees matter." In particular, their study "asked 730 experimental subjects to allocate \$10,000 among four real S&P 500 index funds. All subjects received the funds prospectuses. To make choices incentive-compatible, subjects expected payments depended on the actual returns of their portfolios over a specified time period after the experimental session. ... In one treatment condition, we gave subjects a one-page 'cheat sheet' that summarized the funds front-end loads and expense ratios. ... We find that eliminating search costs for fees improved portfolio allocations."
  - **Fund objective and skill.** Choi et al. (2010) also find evidence that investors face search costs associated with respect to the funds' objectives such as the meaning of an index fund. "In a second treatment condition, we distributed one page of answers to frequently asked questions (FAQs) about S&P 500 index funds. ... When we explained what S&P 500 index funds are in the FAQ treatment, portfolio fees dropped modestly, but the statistical significance of this drop is marginal."
  - **Price and net asset value.** In some countries, retail investors buy and sell a mutual fund shares as listed shares on an exchange. In this case, a central piece of information is the relation between the share price and mutual fund's net asset value, but investors must search for these pieces of information on different websites and are often they not synchronous.
- **Understanding the relevant information.**

– **Financial literacy.** In their study on the choice of index funds, Choi et al. (2010) find that "fees paid decrease with financial literacy." Simply understanding the relevant information and, in particular, the (lack of) importance of past returns is an important part of the issue.

– **Opportunity costs.** Even for financially literate investors, the non-trivial amount of time it takes to search for a good asset manager may be viewed as a significant opportunity cost given that people have other productive uses of their time and value leisure time.

B Proofs

Proof of Proposition 1. This result is effectively provided, and proved, in Grossman and Stiglitz (1980), but we include a sketch here in the interest of being self-contained. An agent having conditional expectation of the final value μ and variance V optimally demands a number of shares equal to

$$x = \frac{\mu - p}{\gamma V}. \tag{B.1}$$

To compute the relevant expectations and variance, we conjecture the form (6) for the price and introduce a slightly simpler "auxilary" price, $\hat{p} = v - \bar{v} + \varepsilon - \theta_q(q - \bar{q})$, with the same information content as p:

$$\mathrm{E}[v|p] = \mathrm{E}[v|\hat{p}] = \bar{v} + \beta_{v,\hat{p}}\hat{p} = \bar{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2}\hat{p} \tag{B.2}$$

$$\mathrm{E}[v|s] = \mathrm{E}[v|v+\varepsilon] = \bar{v} + \beta_{v,s}(s - \bar{v}) = \bar{v} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}(s - \bar{v}) \tag{B.3}$$

$$\mathrm{var}(v|p) = \mathrm{var}(v|\hat{p}) = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2} = \frac{\sigma_v^2\left(\sigma_\varepsilon^2 + \theta_q^2\sigma_q^2\right)}{\sigma_v^2 + \sigma_\varepsilon^2 + \theta_q^2\sigma_q^2} \tag{B.4}$$

$$\mathrm{var}(v|s) = \sigma_v^2 - \frac{\sigma_v^4}{\sigma_v^2 + \sigma_\varepsilon^2} = \frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}. \tag{B.5}$$

We can now insert these demands into the market-clearing condition (5), which is a linear equation in the random variables q and s. Given that this equation must hold for all values of q and s, the aggregate coefficients on these variables must zero, and similarly, the constant term must be zero. Solving these three equations leads to the coefficients in (7), (8), and (9). Hence, by construction, a linear equilibrium exists.

To compute the relative utility, we start by noting that, with $a \in \{u, i\}$,

$$e^{-\gamma u_a} = \mathrm{E}\left[e^{-\frac{1}{2}\frac{(\mu_a - p)^2}{V_a}}\right], \tag{B.6}$$

where μ_a and V_a are the conditional mean and variance of v for an investor of type a. To complete the proof, one uses the fact that, for any normally distributed random variable $z \sim \mathcal{N}(\mu_z, V_z)$, it holds that (e.g., based on the moment-generating function of the noncentral chi-square)

$$\mathrm{E}\left[e^{-\frac{1}{2}z^2}\right] = (1 + V_z)^{-\frac{1}{2}}\, e^{-\frac{1}{2}\frac{\mu_z^2}{1+V_z}},$$

and performs the necessary calculations giving

$$u_u = \frac{1}{\gamma} \log\left(\frac{\sigma_{v-p}}{\sigma_{v|p}}\right) + \frac{1}{2\gamma}\frac{(\bar{v}-\theta_0)^2}{\sigma_{v-p}^2} \tag{B.7}$$

$$u_i = \frac{1}{\gamma} \log\left(\frac{\sigma_{v-p}}{\sigma_{v|s}}\right) + \frac{1}{2\gamma}\frac{(\bar{v}-\theta_0)^2}{\sigma_{v-p}^2}. \tag{B.8}$$

(We note that the last term, $\frac{1}{2\gamma}\frac{(\bar{v}-\theta_0)^2}{\sigma_{v-p}^2}$, represents the utility attainable by an agent who cannot condition on the price.)

By combining (B.7), (B.8), and the definition of η, we see that (11) holds. To see (12), we use (B.4), (B.5), and the expression (9) for θ_q. ■

Before continuing with the proofs of the next propositions, we state an auxiliary result regarding the number of managers. First, we define the unique value of M that solves managers' indifference condition (19) for any I by

$$\mathcal{M}^I(I) = \max\left\{\frac{\eta(I)I}{2\gamma k + \eta(I)\frac{N}{\bar{M}}}, \bar{M}\right\}, \tag{B.9}$$

where we have used that $I = A + N\frac{M}{\bar{M}}$. Given this definition, the number of managers depends on I as follows.

Lemma 1 *The function of I given by $I\eta(I)$ increases up to a point $\bar{I}$ and then decreases, converging to zero. Consequently, $\mathcal{M}^I(I)$ increases with I for I low enough, and decreases towards zero as I tends to infinity.*

Proof of Lemma 1. The function $x\eta(x)$ of interest is a constant multiple of

$$h(x) := x \log\left(\frac{a+x^2}{b+x^2}\right), \tag{B.10}$$

with $a > b > 0$. Its derivative equals

$$\begin{aligned} h'(x) &= \log\left(\frac{a+x^2}{b+x^2}\right) + x\frac{b+x^2}{a+x^2}\frac{2x(b+x^2)-2x(a+x^2)}{(b+x^2)^2} \\ &= \log\left(\frac{a+x^2}{b+x^2}\right) - \frac{2(a-b)x^2}{(a+x^2)(b+x^2)}. \end{aligned}$$

For $x = 0$, the first term is clearly higher: $h'(0) > 0$. For $x \to \infty$, the second is larger, so that $\lim h'(x) < 0$. Finally, letting $y = x^2$ and differentiating $h'(y)$ with respect to y one sees that $h''(y) = 0$ when y satisfies the quadratic

$$y^2 - (a+b)y - 3ab = 0, \tag{B.11}$$

which clearly has a root of each sign. Thus, since $y = x^2$ is always positive, $h''(x)$ changes sign only once. Given that $h'(x)$ starts positive and ends negative and its derivative changes sign only once, we see that h' itself must change sign exactly once. This result means that h is hump-shaped. Finally, we can apply L'Hôpital's rule to $h(x) = \log\left(\frac{a+x^2}{b+x^2}\right)/(1/x)$ to conclude that $\lim_{x\to\infty} h(x) = 0$.

To make a statement about the number of informed managers M, we use (B.9) and the first result. ■

Proof of Proposition 2. Part (i) is a restatement of the fact that investors matched with good managers rationally choose to pay the fee and invest with the manager rather than invest as uninformed. Part (ii) is a juxtaposition of the facts that uninformed managers do not provide any investment value and that their fee is strictly positive. Part (iii) is the indifference condition for the active investors. For part (iv), we note that the outperformance $u_i - f - u_u = c$ is clearly larger if the equilibrium c is larger. Finally, part (v) follows from expressing the aggregate outperformance as

$$\left(A + N\frac{M}{\bar{M}}\right)(u_i - f - u_u) + N\left(1 - \frac{M}{\bar{M}}\right)(-f) = Af - N\left(1 - 2\frac{M}{\bar{M}}\right)f \tag{B.12}$$

using that $u_i - u_u = \frac{\eta}{\gamma} = 2f$. This outperformance is positive if and only if $N\left(1 - 2\frac{M}{\bar{M}}\right) \leq A$. ■

Proof of Proposition 3. (i) Consider the largest-I equilibrium under the search cost c_1, denoted using the subscript 1. We show that, under c_2, an equilibrium exists with larger I. To see this, note that since (18) holds with equality for c_1, we have $\eta(I_1) \geq 2\gamma c_2(M_1, A_1)$. Consider now the set

$$\left\{I \,|\, I \geq I_1, I - \mathcal{M}^I(I)\frac{N}{\bar{M}} \leq \bar{A}\right\}, \tag{B.13}$$

where $I - \mathcal{M}^I(I)\frac{N}{\bar{M}}$ is the number of searching investors A corresponding to I. This set is not empty because it includes I_1. Either $\eta(I) > 2\gamma c_2\left(\mathcal{M}^I(I), I - \mathcal{M}^I(I)\frac{N}{\bar{M}}\right)$ over the entire set, in which case $A = \bar{A}$ corresponds to an equilibrium for c_2, or $\eta(I) = 2\gamma c_2\left(\mathcal{M}^I(I), I - \mathcal{M}^I(I)\frac{N}{\bar{M}}\right)$ for a value $I_2 \geq I_1$, which is the desired conclusion.

The asset-market efficiency and fee are determined monotonically by the level of I. The number M of managers can either increase or decrease given the result on the shape of $\mathcal{M}^I$.

Finally, if $\mathcal{M}^I(I_2) \leq \mathcal{M}^I(I_1)$, then $A_2 \geq A_1$ from $A = I - M\frac{N}{\bar{M}}$. If $\mathcal{M}^I(I_2) \geq \mathcal{M}^I(I_1)$, then the same conclusion follows from (19).

(ii) Since the functions c_j are continuous on $[0, \bar{A}] \times [M_0, \bar{M}]$ for any $M_0 > 0$, they converge to zero uniformly on this compact set. Pick M_0 low enough so that $\mathcal{M}^I(I) > M_0$ for any $I \in [\bar{A}, \bar{A} + N]$.

Since η is bounded away from zero on the set of interest, for high enough j there is an equilibrium with $A = \bar{A}$. By letting $\bar{A}_j \to \infty$, the equilibrium value A_j goes to ∞. Hence, the market converges toward full efficiency in the limit. ■

Proof of Proposition 4. We note that equation (19) also defines a function $\mathcal{M}(A)$ associating a value A with a unique value M (because the left-hand side decreases in M and the right-hand side increases in M). Similarly, $c_A \geq 0$ ensures that (18) defines a function $\mathcal{A}(M)$ associating each value M with a unique value of A. Further, adding the condition $c_M \leq 0$ implies that $\mathcal{I}(M) \equiv \mathcal{A}(M) + M\frac{N}{M}$ increases with M.

Another helpful observation is that, from (18) and (19), A and M are positively related across equilibria, just as I and M are. The highest-I equilibrium is therefore also the highest-A equilibrium.

One can describe the effect of k using the language of graphs. (A more rigorous argument can be made following a similar logic to that in the proof of Proposition 3.) At the highest I, the increasing function $\mathcal{I}^{-1}$ crosses $\mathcal{M}$ from below; since a lower value of k translates into an upward shift of the function $\mathcal{M}$, there exists at least one equilibrium at the new k with a higher value of I than before. Since $\mathcal{I}$ does not vary with k and it is increasing, M also increases. The inefficiency η decreases as I increases.

The level of A, on the other hand, can either increase or decrease. To see the latter fact, imagine a function c that increases abruptly in A around the original equilibrium, while being flat with respect to M. Since η decreases, A has to decrease from (18). Formally, make use of

$$\frac{d\eta}{dk} = c_M \frac{dM}{dk} + c_A \frac{dA}{dk}. \tag{B.14}$$

■

Proof of Proposition 5. Letting x denote either σ_v^2 or σ_q^2, we note that the partial derivatives are positive, $\frac{\partial \eta}{\partial x} > 0$ (i.e., keeping I constant). To derive the equilibrium effects of a change in risk, we rewrite (18)–(19) abstractly as

$$0 = -\frac{1}{2}\eta + \gamma c(M, A) \equiv g^I(I, M) = g^I(\mathcal{I}(M), M) \tag{B.15}$$

$$0 = -\frac{1}{2}\eta + \gamma k \frac{M}{A} \equiv g^M(I, M) = g^M(I, \mathcal{M}(I)), \tag{B.16}$$

and note that I being maximal implies that the difference $\mathcal{I}^{-1}(I) - \mathcal{M}^I(I)$ increases in a neighborhood of the equilibrium I, or ${\mathcal{M}^I}'(I) < (\mathcal{I}^{-1})'(I)$. Using subscripts to indicate partial derivatives, this translates into[21]

$$-\frac{g_I^M}{g_M^M} < -\frac{g_I^I}{g_M^I}, \tag{B.17}$$

[21] Note that $g^I(\mathcal{I}(M), M)$ can be written as $g^I(I, \mathcal{I}^{-1}(I))$ for $I = \mathcal{I}(M)$.

which is equivalent to

$$g_M^I g_I^M < g_I^I g_M^M \tag{B.18}$$

because $g_M^I < 0$ and $g_M^M > 0$. The dependence of I and M on x is given as a solution to

$$\begin{pmatrix} g_I^M & g_M^M \\ g_A^I & g_M^A \end{pmatrix} \begin{pmatrix} I_x \\ M_x \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\frac{\partial \eta}{\partial x} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{B.19}$$

therefore by

$$\begin{pmatrix} I_x \\ M_x \end{pmatrix} = \frac{1}{g_M^I g_I^M - g_I^I g_M^M} \begin{pmatrix} g_M^I - g_M^M \\ g_I^M - g_I^I \end{pmatrix} \begin{pmatrix} \frac{1}{2}\frac{\partial \eta}{\partial x} \end{pmatrix}. \tag{B.20}$$

We note that $g_M^I - g_M^M < 0$ and $g_I^M - g_I^I < 0$, while the determinant $g_M^I g_I^M - g_I^I g_M^M$ is negative from (B.18). Thus, both I and M increase as σ_v^2 or σ_q^2 increases.

By dividing equation (18) by (19), A is seen to increase with M.

The effect on the efficiency of the asset market, on the other hand, is not determined. To see this clearly, differentiate (18) to get

$$\frac{1}{2}\frac{d\eta}{dx} = \gamma \left(c_M M_x + c_A A_x \right) \tag{B.21}$$

and remember that $c_M \leq 0$ and $c_A \geq 0$. Since $M_x > 0$ and $A_x > 0$, by setting one of the partial derivatives c_M and c_A to zero and keeping the other non-zero, the sign of $\frac{d\eta}{dx}$ can be made either positive or negative. Consequently, the efficiency may increase as well as decrease, a conclusion that translates to the fee f.

Exactly the same argument works when increasing $(\sigma_v, \sigma_\varepsilon)$ or $(\sigma_v, \sigma_\varepsilon, \sigma_q)$ proportionally. ■

Proof of Proposition 6. This proposition follows from the considerations in the body of the paper along with the observation that the derivation of the fee and indifference condition continue to hold, where the risk aversion and search cost are made investor specific. In particular, the fee f_a for investor a is given by equation (14) with γ replaced by γ_a and part (i) of the proposition comes from (16) with c replaced by c_a and likewise for f_a and γ_a. ■

Proof of Proposition 7. We compute expected return on the wealth invested with a manager, working under the assumptions that all managers choose positions targeting investors with relative risk aversion $\bar{\gamma}_R$. Given the total wealth under management $\bar{W}$, the manager invests as an agent with absolute risk aversion $\bar{\gamma} = \bar{\gamma}^R/\bar{W}$. It is clear that all investors with an informed manager achieve the same gross excess return. Its average is computed as the total dollar profit per capital invested $\bar{W}$, using the fact that the aggregate position is $(\bar{\gamma}\mathrm{Var}(v|s))^{-1}(\mathrm{E}[v|s]-p)$, that is,

$$\bar{R}_i \equiv \mathrm{E}\left[\frac{1}{\bar{W}}(\bar{\gamma}\mathrm{Var}(v|s))^{-1}(\mathrm{E}[v|s]-p)(v-p)\right] = \frac{1}{\bar{\gamma}^R}\mathrm{E}\left[SR_i^2\right]. \tag{B.22}$$

Similarly, the gross return to an investor with an uninformed manager is

$$\bar{R}_u \equiv \mathrm{E}\left[\frac{1}{\bar{W}}(\bar{\gamma}\mathrm{Var}(v|p))^{-1}(\mathrm{E}[v|p]-p)(v-p)\right] = \frac{1}{\bar{\gamma}^R}\mathrm{E}\left[SR_u^2\right]. \tag{B.23}$$

There are two reasons why $\mathrm{E}[SR_i^2] > \mathrm{E}[SR_u^2]$: better information, and lower risk (which translates into higher leverage, in absolute value). We note that the second effect is not necessary for the result. As for the first effect, namely the fact that

$$\mathrm{E}\left[(\mathrm{E}[v-p|s,p])^2\right] > \mathrm{E}\left[(\mathrm{E}[v-p|p])^2\right], \tag{B.24}$$

it follows immediately from Jensen's inequality (conditional on p).

Consider now the expected return of an investor in a fund conditional on the investor's wealth:

$$\mathrm{E}[R|W_a] = Pr(i|W_a)\bar{R}_i + (1-Pr(i|W_a))\bar{R}_u \tag{B.25}$$

where $Pr(i|W_a) = \frac{\bar{A}Pr\left(\gamma_a^R c_a < \frac{1}{2}\eta W_a | W_a\right) + \frac{M}{\bar{M}}N}{\bar{A}Pr\left(\gamma_a^R c_a < \frac{1}{2}\eta W_a | W_a\right) + N}$. Since $Pr\left(\gamma_a^R c_a < x\right)$ increases with x and $\bar{R}_i > \bar{R}_u$, we see that $\mathrm{E}[R|W_a]$ increases with W_a.

Precisely the same argument works for c_a, albeit with reversed signs.

The results also hold after fees. Since investors with informed managers are, on average, more sophisticated (low c_a), they pay lower fees; and since they tend to be less risk averse (low γ_a^R), they pay these lower total fees for a larger investment.

For part (ii), we let $R^{(m)}$ and $\bar{W}^{(m)}$ denote the return, respectively the average wealth of the investors, of manager m. These two quantities are independent conditional on the manager's type (informed or uninformed). Since there are two manager types, $\tau = i$ and $\tau = u$, the covariance $\mathrm{Cov}(R^{(m)}, W^{(m)})$ is positive if and only if the conditional expectations $\mathrm{E}[R^{(m)}|\tau]$ and $\mathrm{E}[\bar{W}^{(m)}|\tau]$ are ranked the same as a function of the type τ of the manager.

In the present case, it is easy to see that the average investor of an informed manager has higher wealth. Specifically,

$$\mathrm{E}[W_a \,|\, \tau = i] = \frac{A}{A + \frac{M}{\bar{M}}N}\mathrm{E}\left[W_a \,|\, \frac{\gamma_a^R c_a}{W_a} < \frac{\eta}{2}\right] + \frac{\frac{M}{\bar{M}}N}{A + \frac{M}{\bar{M}}N}\mathrm{E}[W_a] > \mathrm{E}[W_a], \tag{B.26}$$

since $\mathrm{E}[W_a \,|\, \frac{\gamma_a^R c_a}{W_a} < \frac{\eta}{2}] > \mathrm{E}[W_a]$. We already saw that $\bar{R}_i > \bar{R}_u$.

The same argument works for any decreasing function of c_a, thus sophistication.

Part (iii) follows easily along the same lines. ■

Proof of Proposition 8. We have

$$
\begin{aligned}
\eta &= \log\left(\mathrm{E}\left[e^{-\frac{1}{2}(v-p)\frac{\mathrm{E}[v-p|p]}{\mathrm{var}(v|p)}}\right]\right) - \log\left(\mathrm{E}\left[e^{-\frac{1}{2}(v-p)\frac{\mathrm{E}[v-p|s,p]}{\mathrm{var}(v|s)}}\right]\right) \\
&\approx \frac{1}{2}\left(\mathrm{E}\left[(v-p)\frac{\mathrm{E}\left[v-p|s,p\right]}{\mathrm{var}(v|s)}\right] - \mathrm{E}\left[(v-p)\frac{\mathrm{E}\left[v-p|p\right]}{\mathrm{var}(v|p)}\right]\right) \qquad \text{(B.27)} \\
&= \frac{1}{2}\left(\mathrm{E}[SR_i^2] - \mathrm{E}[SR_u^2]\right),
\end{aligned}
$$

where the second line follows from linear approximations to the exponential and logarithmic functions, and the third owes to the fact that the conditional variances are constant. ■

Proof of Proposition 9. We can write the (il)liquidity as a function of I given by

$$
\lambda(I) = \frac{\gamma\frac{\sigma_v^2\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2} + (\bar{A} - I + N)\frac{\mathrm{var}(v|s)}{\mathrm{var}(v|p)}\frac{\sigma_v^2\theta_q}{\sigma_v^2+\sigma_\varepsilon^2+\theta_q^2\sigma_q^2}}{I + (\bar{A} - I + N)\frac{\mathrm{var}(v|s)}{\mathrm{var}(v|p)}}. \qquad \text{(B.28)}
$$

Note first that, for $N = 0$, the numerator of λ is minimized by $I = \bar{A}$. The denominator is increasing in I because $\mathrm{var}(v|s) < \mathrm{var}(v|p)$ and $\mathrm{var}(v|p)$ decreases with I. Consequently, for $N = 0$, λ is minimal at $I = \bar{A}$; furthermore $\lambda'(\bar{A}) < 0$. Given that (B.28) is a smooth function, we also infer that λ decreases for I in a neighborhood of $\bar{A}$ — and therefore also if N lies in a suitable neighborhood of zero: $-\lambda' > \varepsilon > 0$ for (I, N) in a neighborhood of $(\bar{A}, 0)$. Thus, if N and c are low enough that I is sufficiently close to $\bar{A}$, then λ decreases as c decreases further.

Once c is low enough that $A = \bar{A}$, λ is trivially constant. ■

Proof of Propositions 10–11. Under the conjecture of a price that is linear in s and q, demands are

$$
\begin{aligned}
x_u &= (\gamma\Sigma_{v|p})^{-1}\left(\mathrm{E}[v|p] - p\right) \qquad \text{(B.29)} \\
x_i &= (\gamma\Sigma_{v|s})^{-1}\left(\mathrm{E}[v|s] - p\right) \qquad \text{(B.30)}
\end{aligned}
$$

with

$$
\begin{aligned}
\Sigma_{v|s} &= \Sigma_v - \Sigma_v\left(\Sigma_v + \Sigma_\varepsilon\right)^{-1}\Sigma_v \qquad \text{(B.31)} \\
&= \Sigma_v\left(\Sigma_v + \Sigma_\varepsilon\right)^{-1}\Sigma_\varepsilon \qquad \text{(B.32)} \\
\Sigma_{v|p} &= \Sigma_v - \Sigma_v\left(\Sigma_v + \Sigma_\varepsilon + \theta_q^\top\Sigma_q\theta_q\right)^{-1}\Sigma_v. \qquad \text{(B.33)}
\end{aligned}
$$

To calculate the utilities (conditional on p), we use the formula

$$
\mathrm{E}\left[e^{x^\top Ax + b^\top x}\right] = \det(I_n - 2\Omega A)^{-\frac{1}{2}}e^{\frac{1}{2}b^\top(I_n-2\Omega A)^{-1}\Omega b} \qquad \text{(B.34)}
$$

for $x \sim \mathcal{N}(0, \Omega)$, with I_n the n-dimensional identity matrix.

Specifically, we compute

$$\mathrm{E}\left[e^{-\frac{1}{2}(\mathrm{E}[v|s]-p)^\top \Sigma_{v|s}^{-1}(\mathrm{E}[v|s]-p)}|p\right] \tag{B.35}$$

by letting $x = \mathrm{E}[v|s] - \mathrm{E}[v|p]$, $A = -\frac{1}{2}\Sigma_{v|s}^{-1}$, $b^\top = (\mathrm{E}[v|p] - p)^\top \Sigma_{v|s}^{-1}$ to evaluate

$$\begin{aligned} &\mathrm{E}\left[e^{x^\top Ax + b^\top x - \frac{1}{2}(\mathrm{E}[v|p]-p)^\top \Sigma_{v|s}^{-1}(\mathrm{E}[v|p]-p)}\right] \\ &= \det(I_n + \Omega\Sigma_{v|s}^{-1})^{-\frac{1}{2}} e^{\frac{1}{2}(\mathrm{E}[v|p]-p)^\top \Sigma_{v|s}^{-1}(I_n+\Omega\Sigma_{v|s}^{-1})^{-1}\Omega\Sigma_{v|s}^{-1}(\mathrm{E}[v|p]-p) - \frac{1}{2}(\mathrm{E}[v|p]-p)^\top \Sigma_{v|s}^{-1}(\mathrm{E}[v|p]-p)} \end{aligned} \tag{B.36}$$

with

$$\begin{aligned} \Omega &= \mathrm{Var}\left(\mathrm{E}[v|s]|p\right) \\ &= \Sigma_{v|p} - \Sigma_{v|s}. \end{aligned} \tag{B.37}$$

We now simplify the last exponent in (B.36):

$$\begin{aligned} &\frac{1}{2}\left(\mathrm{E}[v|p] - p\right)^\top \Sigma_{v|s}^{-1}(I_n + \Omega\Sigma_{v|s}^{-1})^{-1}\Omega\Sigma_{v|s}^{-1}\left(\mathrm{E}[v|p] - p\right) - \frac{1}{2}\left(\mathrm{E}[v|p] - p\right)^\top \Sigma_{v|s}^{-1}\left(\mathrm{E}[v|p] - p\right) \\ &= -\frac{1}{2}\left(\mathrm{E}[v|p] - p\right)^\top \Sigma_{v|s}^{-1}(I_n + \Omega\Sigma_{v|s}^{-1})^{-1}\left(\mathrm{E}[v|p] - p\right) \\ &= -\frac{1}{2}\left(\mathrm{E}[v|p] - p\right)^\top \Sigma_{v|s}^{-1}(\Sigma_{v|p}\Sigma_{v|s}^{-1})^{-1}\left(\mathrm{E}[v|p] - p\right) \\ &= -\frac{1}{2}\left(\mathrm{E}[v|p] - p\right)^\top \Sigma_{v|p}^{-1}\left(\mathrm{E}[v|p] - p\right). \end{aligned} \tag{B.38}$$

This is the same exponent as in the expression for the uninformed investor's utility (conditional on p). Therefore, the first term in (B.36), the determinant, is the only term that distinguishes the utility of the informed from that of the uninformed. For the informed investor, we have

$$\begin{aligned} \det(I_n + \Omega\Sigma_{v|s}^{-1})^{-\frac{1}{2}} &= \det(\Sigma_{v|p}\Sigma_{v|s}^{-1})^{-\frac{1}{2}} \\ &= \frac{\det(\Sigma_{v|s})^{\frac{1}{2}}}{\det(\Sigma_{v|p})^{\frac{1}{2}}}. \end{aligned} \tag{B.39}$$

The certainty-equivalent difference follows as

$$\eta = \frac{1}{2}\log\left(\frac{\det(\mathrm{Var}(v|p))}{\det(\mathrm{Var}(v|s))}\right), \tag{B.40}$$

which has an informational interpretation through the notion of entropy.

One can go further by using the fact that the Kullback-Leibler divergence of a k-dimensional multi-variate normal distribution with mean μ_1 and variance Σ_1 from one with

μ_0 and variance Σ_0 is

$$D_{KL} = \frac{1}{2}\left(tr\left(\Sigma_1^{-1}\Sigma_0\right) + (\mu_1 - \mu_0)^\top \Sigma_1^{-1}(\mu_1 - \mu_0) - k + \log\left(\frac{\det(\Sigma_1)}{\det(\Sigma_0)}\right)\right). \tag{B.41}$$

In our case, $\Sigma_0 = \Sigma_{v|s}$, $\Sigma_1 = \Sigma_{v|p}$, $\mu_0 = \mathrm{E}[v|s]$, and $\mu_1 = \mathrm{E}[v|p]$. Taking expectations, conditional on p, of the second term, we get

$$\begin{aligned}
\mathrm{E}\left[(\mu_1 - \mu_0)^\top \Sigma_1^{-1}(\mu_1 - \mu_0)|p\right] &= \mathrm{E}\left[tr\left((\mu_1 - \mu_0)(\mu_1 - \mu_0)^\top \Sigma_1^{-1}\right)|p\right] \\
&= tr\left(\mathrm{E}\left[(\mu_0 - \mu_1)(\mu_0 - \mu_1)^\top \Sigma_1^{-1}|p\right]\right) \\
&= tr\left(\mathrm{E}\left[(\mu_0 - \mu_1)(\mu_0 - \mu_1)^\top |p\right]\Sigma_1^{-1}\right) \\
&= tr\left(\mathrm{Var}(E[v|s]|p)\Sigma_{v|p}^{-1}\right) \\
&= tr\left(\mathrm{Var}(E[v|s])\Sigma_{v|p}^{-1}\right) \\
&= tr\left(\left(\Sigma_{v|p} - \Sigma_{v|s}\right)\Sigma_{v|p}^{-1}\right) \\
&= k - tr\left(\Sigma_{v|s}\Sigma_{v|p}^{-1}\right).
\end{aligned} \tag{B.42}$$

It follows that $\mathrm{E}\left[D_{KL}\right] = \eta$.

Finally, to relate $\det(\mathrm{Var}(v|p))$ to the number of informed investors I, we need θ_q, which we estimate from the market-clearing condition. Equating the coefficients for q, respectively s, we obtain

$$I_n = (\bar{A} + N - I)\left(\gamma\Sigma_{v|p}\right)^{-1}\left(\beta_{v,p} - I_n\right)\theta_s^\top\theta_q^\top - I\left(\gamma\Sigma_{v|s}\right)^{-1}\theta_s^\top\theta_q^\top \tag{B.43}$$

$$0 = (\bar{A} + N - I)\left(\gamma\Sigma_{v|p}\right)^{-1}\left(\beta_{v,p} - I_n\right)\theta_s^\top + I\left(\gamma\Sigma_{v|s}\right)^{-1}\beta_{v|s} - I\left(\gamma\Sigma_{v|s}\right)^{-1}\theta_s^\top. \tag{B.44}$$

Using the second equation in the first, we get

$$\theta_q^\top = \frac{\gamma}{I}\Sigma_{v|s}^{-1}\Sigma_v\Sigma_s^{-1} \tag{B.45}$$

$$= \frac{\gamma}{I}\Sigma_\varepsilon^{-1}. \tag{B.46}$$

We therefore see that the form of the equilibrium quantities is the same as with one asset; in particular, given (B.33), η as defined in terms of utilities decreases with I. ■

Proof of Proposition 12. We omit the details of the derivation, which is standard. The proof uses the well-known fact (B.1) to calculate the demands:

$$x_i^j = \frac{\mathrm{E}[v|v - \hat{\theta}_q(q - \bar{q}), s^j] - p}{\gamma \operatorname{var}(v|v - \hat{\theta}_q(q - \bar{q}), s^j)} \tag{B.47}$$

$$x_u = \frac{\mathrm{E}[v|v - \hat{\theta}_q(q - \bar{q})] - p}{\gamma \operatorname{var}(v|v - \hat{\theta}_q(q - \bar{q}))}. \tag{B.48}$$

We note that, in computing the optimal demands, the following quantities are helpful:

$$\operatorname{var}(v|p)^{-1} = \sigma_v^{-2} + \hat{\theta}_q^{-2}\sigma_q^{-2} \tag{B.49}$$

$$\operatorname{var}(v|p, s^j)^{-1} = \sigma_v^{-2} + \sigma_\varepsilon^{-2} + \hat{\theta}_q^{-2}\sigma_q^{-2}. \tag{B.50}$$

Furthermore, given the relation between η and the ratio of these two variances, using $\hat{\theta}_q = \frac{\gamma}{A}\sigma_\varepsilon^2$, we derive

$$\eta = -\frac{1}{2}\log\left(1 - \frac{\gamma^2\sigma_\varepsilon^2\sigma_q^2}{A^2 + \gamma^2\sigma_\varepsilon^2\sigma_q^2\left(\sigma_\varepsilon^2\sigma_v^{-2} + 1\right)}\right). \tag{B.51}$$

Remark: No agent would choose to search and invest with a second manager if the cost and fee that she would have to pay were the same. Intuitively, this result is due to the diminishing marginal value of information. Precisely, we have

$$\operatorname{var}(v|p, s^{j_1}, s^{j_2})^{-1} = \sigma_v^{-2} + 2\sigma_\varepsilon^{-2} + \hat{\theta}_q^{-2}\sigma_q^{-2} \tag{B.52}$$

and the utility gain

$$\gamma\left(u_{2i} - u_i\right) = \frac{1}{2}\log\left(\frac{\operatorname{var}(v|p, s^j)}{\operatorname{var}(v|p, s^{j_1}, s^{j_2})}\right) < \frac{1}{2}\log\left(\frac{\operatorname{var}(v|p)}{\operatorname{var}(v|p, s^j)}\right) = \gamma\left(u_i - u_u\right).$$

■

Proof of Proposition 13. The effect of the cost c^u is to increase the gains from trade between an investor and the manager — from η to $\eta + \frac{c^u}{\gamma}$. Following the same line of reasoning as in the proof of Proposition 5, we find

$$\begin{pmatrix} A_{c^u} \\ M_{c^u} \end{pmatrix} = \frac{1}{g_M^A g_A^M - g_A^A g_M^M}\begin{pmatrix} g_M^A - g_M^M \\ g_A^M - g_A^A \end{pmatrix}\left(\frac{1}{2}\frac{\partial(\eta + c^u)}{\partial c^u}\right), \tag{B.53}$$

which is positive given the proof of Proposition 5 and $\frac{\partial(\eta + c^u)}{\partial c^u} = 1 > 0$.

The other results follow from the facts that η decreases with A and that $fA = kM$. The effect on the fee f is ambiguous because $f = \frac{1}{2}\left(\frac{\eta}{\gamma} + c^u\right) = c$ may either increase or decrease, as one can see by considering examples for c such as $c(M, A) = \frac{\bar{c}}{M}$ and $c(M, A) = \frac{\bar{c}A^2}{M}$ for some positive constant $\bar{c}$. ■

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