Modelling yields at the lower bound through regime shifts[1]

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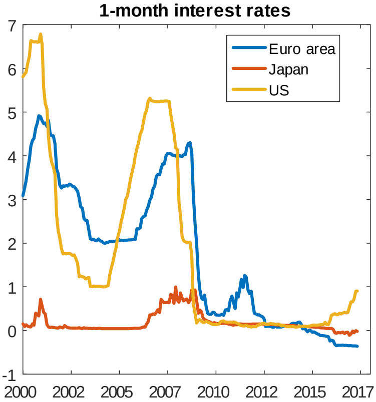
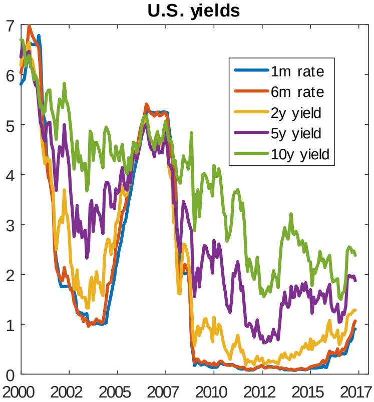
ABFER 2017

23 May 2017

[1] The views expressed do not necessarily reflect those of the BIS or the ECB.

Short-term interest rates and U.S. yields





Motivation

- Lessons from recent effective lower bound (ELB) experiences
  - ELB is not zero, due to cash storage costs; its exact level is uncertain
  - ELB spells can be extremely persistent; and expected to be persistent
  - US experience suggests very slow pace of normalisation after exiting ELB
- Shadow rate models (e.g. Bauer&Rudebusch 2015; Wu&Xia 2016) empirically successful (and parsimonious), but:
  - they impose a hard constraint, arguably too strong an assumption
  - the state vector dynamics are the same at the ELB as in normal times
  - this suggests fast pace of normalisation after exiting ELB

Modelling yields at the ELB

- We study an alternative model of yields at the ELB. Two regimes: Normal (N) and Lower bound (L), with stochastic switches
  - allows for different dynamics conditional on regime
- Regime-switching probabilities are state dependent: the probability of switching to L is high when the policy rate is close to 0; the prob. of switching to N increases as the short rate rises
- Benefit: explicit account of state nonlinearity at the ELB; allows ELB episodes to be very persistent; bond prices reflect these features – also after exiting
- Cost: more parameters $\longrightarrow$ use solely observable state variables

Results (so far)

- Application to US term structure using yield factors
- Good fit; clear identification of regimes
- The RS model rules out a deeply negative policy rate (but allows it to dip below the estimated LB)
- The model implies a slow pace of policy rate normalisation in coming years
- Compared to an affine model: higher term premia in recent years / lower average expected policy rates
- Regime shift risk is priced by investors, but magnitude is small

Regime-switching model

- State vector
$$x_{t+1} = \mu^j + \Phi^j x_t + \Sigma^j \varepsilon_{t+1}$$
with $j = N, L$ and $x_t = \begin{bmatrix} c_t & s_t & r_t \end{bmatrix}'$; i.e. curvature, slope, r.
- By assumption, under L the policy rate:
  - is expected to remain constant;
  - does not affect the other factors

$$\begin{aligned} r_{t+1} &= \mu_r^L + \sigma^L \varepsilon_{r,t+1}, \\ \Phi^L &= \begin{bmatrix} \phi_{cc}^L & \phi_{cs}^L & 0 \\ \phi_{sc}^L & \phi_{ss}^L & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Regime-switching model

- RS probabilities are state-dependent:
  - general intuition: the lower r, the more likely a switch to L; the higher r, the more likely a switch to N.
- Specifically:

$$\pi_t^{\mathbb{P},NL} = \int^{\theta_r} \frac{1}{\sigma_r^N \sqrt{(2\pi)^2}} \exp\left(-\frac{1}{2}\left(\frac{r-\mu_{t+1}^{N,r}}{\sigma_r^N}\right)^2\right) dr.$$

  and $\pi_t^{\mathbb{P},LN} = 1 - \pi_t^{\mathbb{P},NL}$.
- Assume constant $\mathbb{Q}$-RS probabilities, $\pi^{\mathbb{Q},NL}$ and $\pi^{\mathbb{Q},LN}$.

Adding yields

- Pricing according to Dai, Singleton and Yang (2007) and Bansal and Zhou (2002)
- Recall state vector

$$x_{t+1} = \mu^j + \Phi^j x_t + \Sigma^j \varepsilon_{t+1}$$

- Risk-aversion induces

$$x_{t+1} = \mu^{\mathbb{Q}j} + \Phi^{\mathbb{Q}j} x_t + \Sigma^j \varepsilon_{t+1}$$

and, using a conditionally log-normal approximation

$$y_{t,n} = \frac{A_n^j}{n} + \frac{B_n^j}{n} x_t$$

where

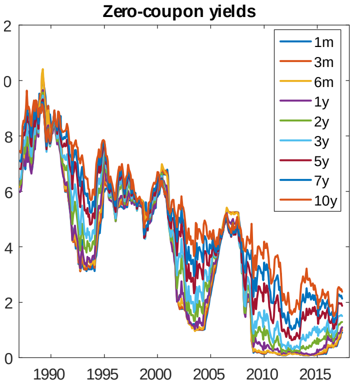
$$\begin{aligned} A_n^j &= \sum_{k=1}^{S} \pi^{\mathbb{Q}jk} \left(\delta_0^j + A_{n-1}^k + B_{n-1}^k \mu^{\mathbb{Q}j} - \frac{1}{2} B_{n-1}^k \Sigma^j \Sigma^j \left(B_{n-1}^k \right)' \right) \\ B_n^j &= \sum_{k=1}^{S} \pi^{\mathbb{Q}jk} \left(\delta_x' + B_{n-1}^k \Phi^{\mathbb{Q}j} \right) \end{aligned}$$

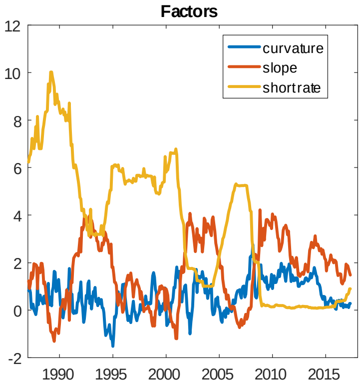
Pricing consistency

- Recall $x_t = \begin{bmatrix} c_t & s_t & r_t \end{bmatrix}'$. We set $c_t = r_t + y_{t,120} - 2y_{t,36}$ and $s_t = y_{t,120} - r_t$.
- Hence $y_{t,120} = s_t + r_t$ and $y_{t,36} = \frac{1}{2}(s_t - c_t) + r_t$.
- Need to ensure consistency with $y_{t,120} = \frac{1}{120}\left(A^j_{120} + B^j_{120}x_t\right)$, so that
  - $A^j_{120} = 0$ and $B^j_{120} = \begin{bmatrix} 0 & 120 & 120 \end{bmatrix}$
- and $y_{t,36} = \frac{1}{36}\left(A^j_{36} + B^j_{36}x_t\right)$ so that
  - $A^j_{36} = 0$ and $B^j_{36} = \begin{bmatrix} -18 & 18 & 36 \end{bmatrix}$
- This induces nonlinear constraints on two rows of $\mu^{\mathbb{Q}j}$ and $\Phi^{\mathbb{Q}j}$.
- We impose these constraints in ML estimation.

Data

- Monthly US yield data (end-month), January 1987 – April 2017.
- Zero-coupon yields from Fed Board (Gürkaynak, Sack, Wright, 2006)
- 1m, 3y, 10y yields used for factors; not included among yields





Estimation

- We have a large number of parameters, so we need to impose some restrictions.
- We estimate the N parameters on a sub-sample when the economy clearly was in the N regime $(1987-2007)$.
- We estimate the VAR L parameters under $\mathbb{P}$ on the Dec. $2008-$ Oct. 2015 sample.
- We set the short rate threshold θ_r used in $\pi_t^{\mathbb{P},NL}$ to the 10th percentile of the distribution of r.
- Remaining parameters are estimated using maximum likelihood.

Results

Parameter estimates

- Mean levels of state variables

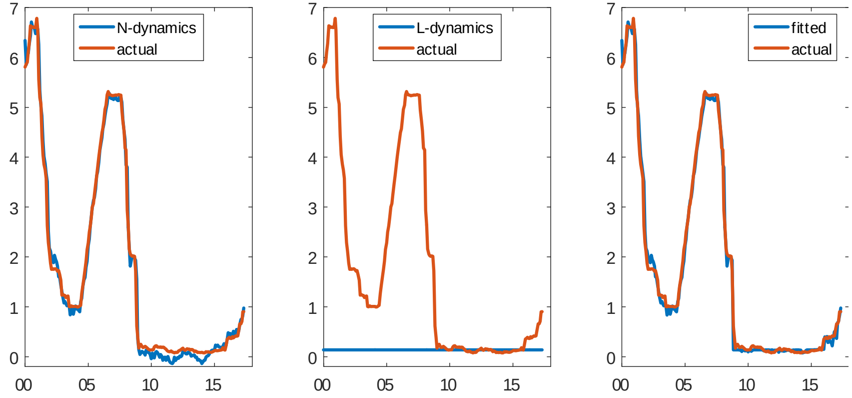
Conditional means

	N	L
c	0.473	0.962
s	1.581	2.361
r	3.286	0.136

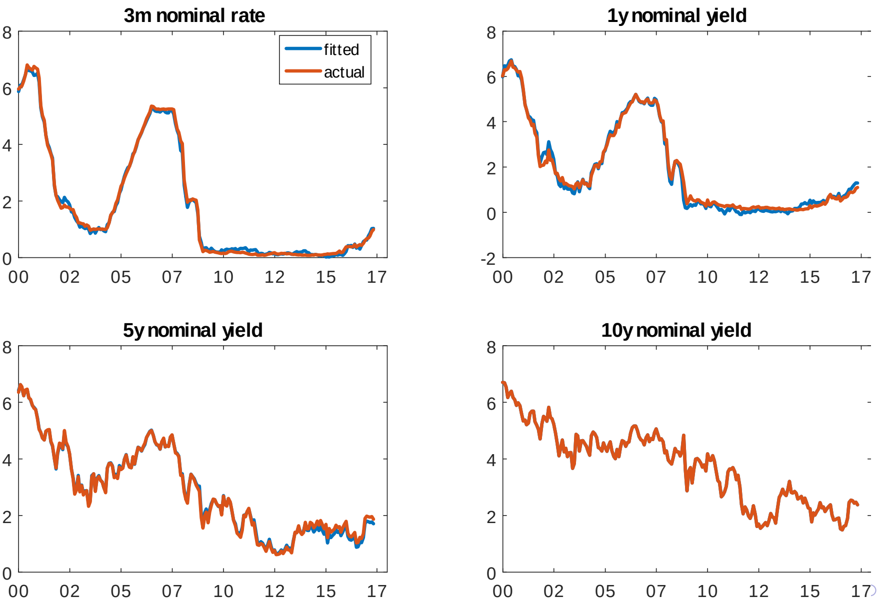
- The lower bound is estimated at 13.6 basis points.
- The steady state short rate in the normal regime is 3.29%.
- Standard deviation of yield measurement errors is $\sigma^m = 0.124$.

Short-term interest rate (zoom in on period 2000 -)

- The RS model effectively combines the N and L dynamics to ensure consistency with actual data.



Yield fit


3m nominal rate
fitted
actual
1y nominal yield
5y nominal yield
10y nominal yield
8
6
4
2
0
-2
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02
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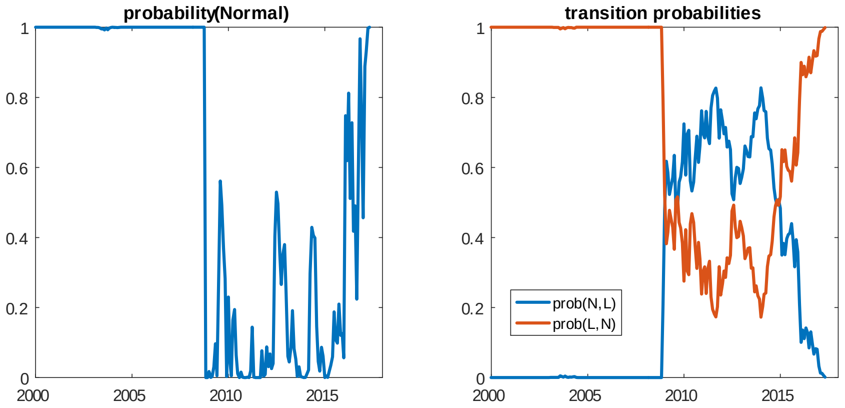
15
17

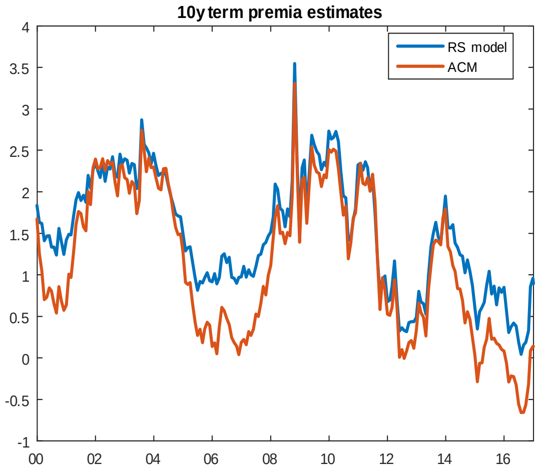
Filtered probabilities of N/L regimes



Risk premia

Term premium

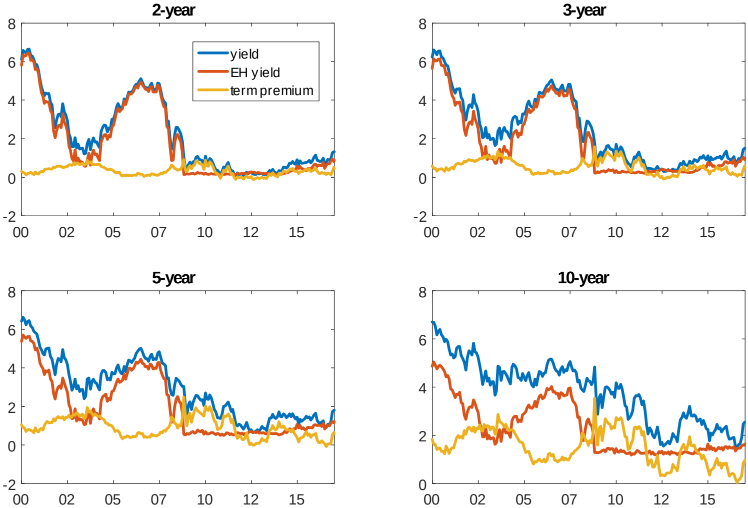
- RS model implies higher premia than 1-regime affine model recently.



ACM premium source: Adrian, Crump, Moench (2013)

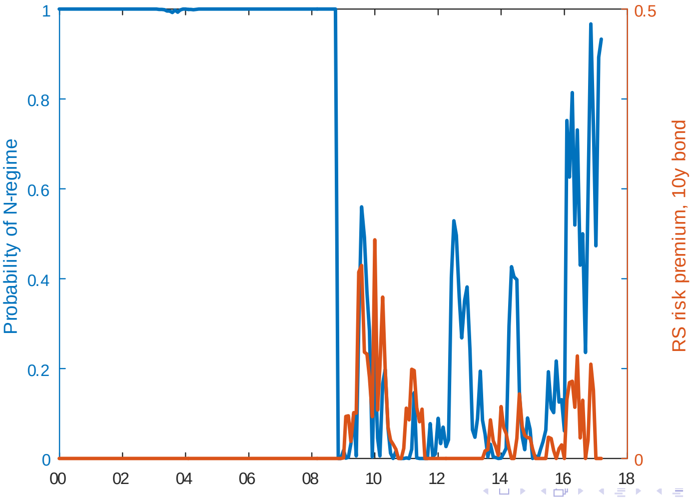
Expected short rate and term premia

- Interest rate expectations matter more for yields during normal times; the term premium dominates at the LB.



Is regime shift risk priced?

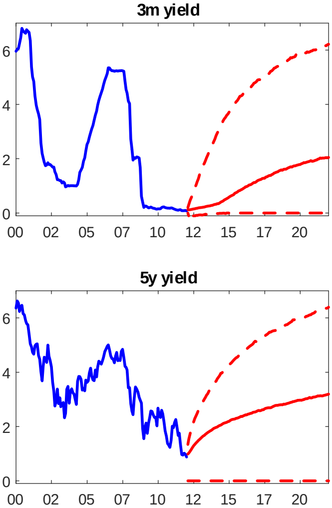
- Regime shift premia essentially zero during *N*-regime; small during *L*-regime.



Forecasts

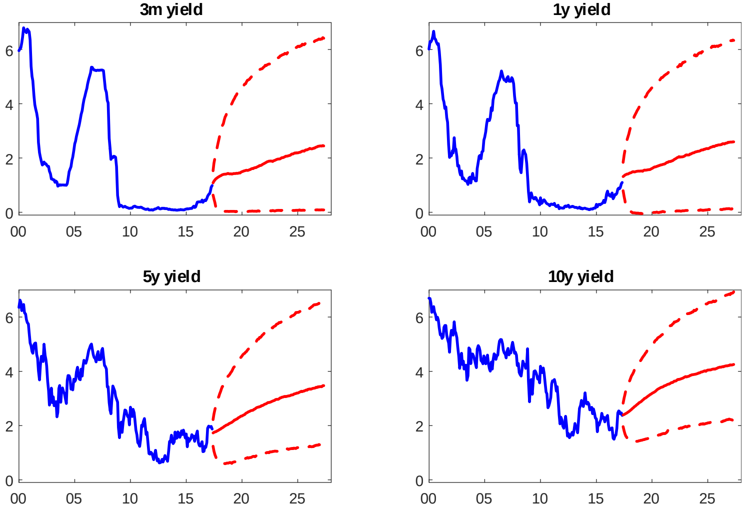
Yield forecasts at end-2011

- Wide confidence bands, but rate distributions do not include deeply negative values.



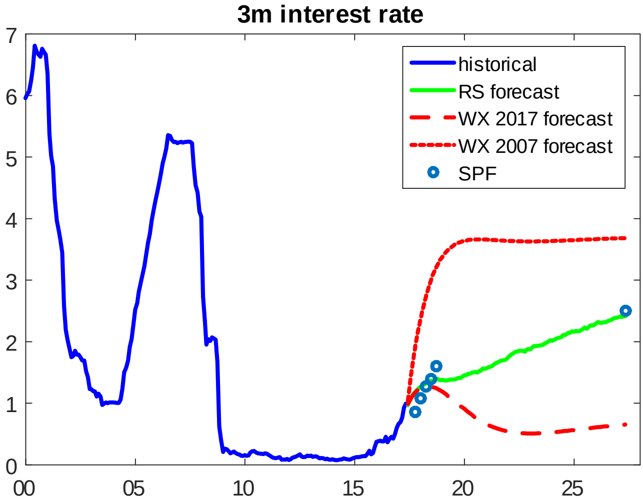
Yield forecasts at end-April 2017

- Gradual increase in interest rates and yields.



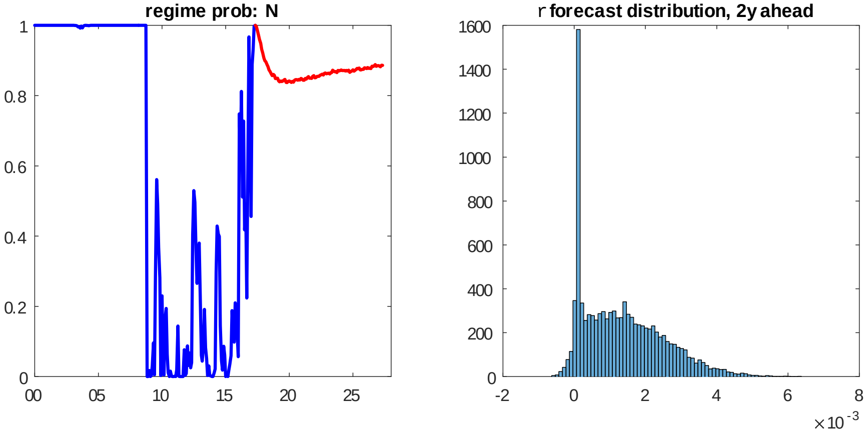
Short rate forecasts at end-April 2017

- Forecast from regime switching model more in line with SPF than shadow rate model (WX) forecasts.



Regime prob. forecast and short rate forecast distribution

- Highly skewed interest rate forecasts increases the future L-regime probability, influencing longer term bond pricing today



Concluding remarks

- We propose a dynamic term structure model with regime switches to account for lower bound spells
- Application to US term structure using yield factors: good fit; clear identification of regimes
- The RS model rules out a deeply negative policy rate (but allows it to dip below the estimated LB)
- Compared to an affine model: higher term premia in recent years / lower average expected policy rates
- Regime shift risk is priced by investors, but magnitude is small
- The model implies a slow pace of policy rate normalisation in coming years, in line with SPF forecasts