# Habits and Leverage

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# Motivation

- Much discussion in the academic literature and in policy circles about leverage and its impact on the real economy and on financial markets
- Various related themes, such as:
  - Excess credit supply may lead to financial crisis
  - The excessive growth of household debt and the causal relation between households' deleveraging and their low future consumption growth
  - Leverage cycle: Leverage is high when prices are high and volatility is low
  - Active deleveraging of financial institutions generate "fire sales" of risky financial assets, which further crash asset prices
  - The leverage ratio of financial institutions is a risk factor
  - Balance sheet recessions
  - ....

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  - Heterogeneous time varying risk-bearing capacity $\Longrightarrow$ leverage dynamics

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- Our model predicts:
  1. Aggregate debt $\uparrow$ in good times when prices $\uparrow$ and volatility $\downarrow$
  2. Poorer agents borrow more than richer agents
  3. Leveraged agents enjoy a "consumption boom" in good times, followed by a consumption slump
  4. Crisis time $\Longrightarrow$ leveraged agents delever by "fire-selling" stocks, but their debt/wealth ratio $\uparrow$ due to strong discount effects.
  5. Intermediaries leverage is a priced risk factor.
  6. Wealth dispersion $\uparrow$ in good times

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- Model aggregates to standard representative agent models with external habit

$\Longrightarrow$ It can be calibrated to yield reasonable asset pricing quantities.

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- Endowments $w_i$ are also heterogeneous, with $\int w_i di = 1$

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  - Endowments satisfy

$$w_i > \frac{a_i(\overline{Y} - \lambda) + \lambda - 1}{\overline{Y}}$$

# Optimal Risk Sharing

- No consumption externalities $\Longrightarrow$ solve planner's problem
- **Consumption shares:** $$s_{it} = \frac{C_{it}}{D_t} = a_i + (w_i - a_i)\frac{\overline{Y}}{Y_t}$$

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  - Cross-section: risk aversion $\downarrow$ if $w_i \uparrow$ or $a_i \downarrow$
  - Time-series: (1) all agents' risk aversion $\uparrow$ if $Y_t \uparrow$

    (2) risk aversion of $i \uparrow$ more if $w_i$ is low or $a_i$ is high

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- Less risk averse agents provide insurance to more risk averse agents

# Competitive Equilibrium

- Given price processes $\{P_t, r_t\}$, agents solve

$$\max_{\{C_{it}, N_{it}, N_{it}^0\}} E_0 \left[ \int_0^\infty e^{-\rho t} \log\left(C_{it} - X_{it}\right) dt \right] \qquad \text{subject to}$$

$$dW_{it} = N_{it}(dP_t + D_t dt) + N_{it}^0 B_t r_t dt - C_{it} dt \quad \text{with} \quad W_{i,0} = w_i P_0$$

- A **competitive equilibrium** is a set of stochastic processes for prices $\{P_t, r_t\}$ and allocations $\{C_{it}, N_{it}, N_{it}^0\}$ such that agents maximize their utilities, and good and financial markets clear $\int C_{it} di = D_t$, $\int N_{it} di = 1$, $\int N_{it}^0 = 0$.

# Representative Agent and State Price Density

- Our model aggregates to Menzly, Santos, and Veronesi (2004):
- As in Campbell and Cochrane (1999), define

$$\textit{Surplus consumption ratio} = S_t = \frac{D_t - \int X_{it} di}{D_t} = \frac{1}{Y_t} \tag{1}$$

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- **Proposition**. The equilibrium state price density

$$M_t = e^{-\rho t} D_t^{-1} S_t^{-1} \tag{2}$$

  – which follows

$$dM_t/M_t \ = \ -r_t dt - \sigma_{M,t} dZ_t \quad \text{with} \quad \sigma_{M,t} = (1+v)\sigma_D(S_t)$$

- We use $S_t$ as state variable for notational convenience.

# Competitive Equilibrium – 2

- **Proposition.** The competitive equilibrium has:

(Stock price) $$P_t \;=\; \left(\frac{\rho + k\overline{Y}S_t}{\rho\left(\rho + k\right)}\right) D_t$$

(Risk-free rate) $$r_t \;=\; \rho + \mu_D - (1+v)\sigma_D(S_t)^2 + k\left(1 - \overline{Y}S_t\right)$$

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- Stock and bond holdings depend on $w_i - a_i$ and the function $H(S_t)$.
- Stock price and risk-free rate are independent of distribution of $w_i$ and $a_i$.

$\Longrightarrow$ Prices and quantities have no causal relation with each other.

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  (v) suffer consumption decline after consumption boom

  $*$ Spatial interpretation: e.g. counties with high $w_i$ or low $a_i$

  $*$ Good times $\Longrightarrow$ debt $\uparrow$ and consumption $\uparrow$ $\Longrightarrow$ but lower future growth.

  $*$ Crucial role of identification strategies to provide causal link between leverage and future consumption

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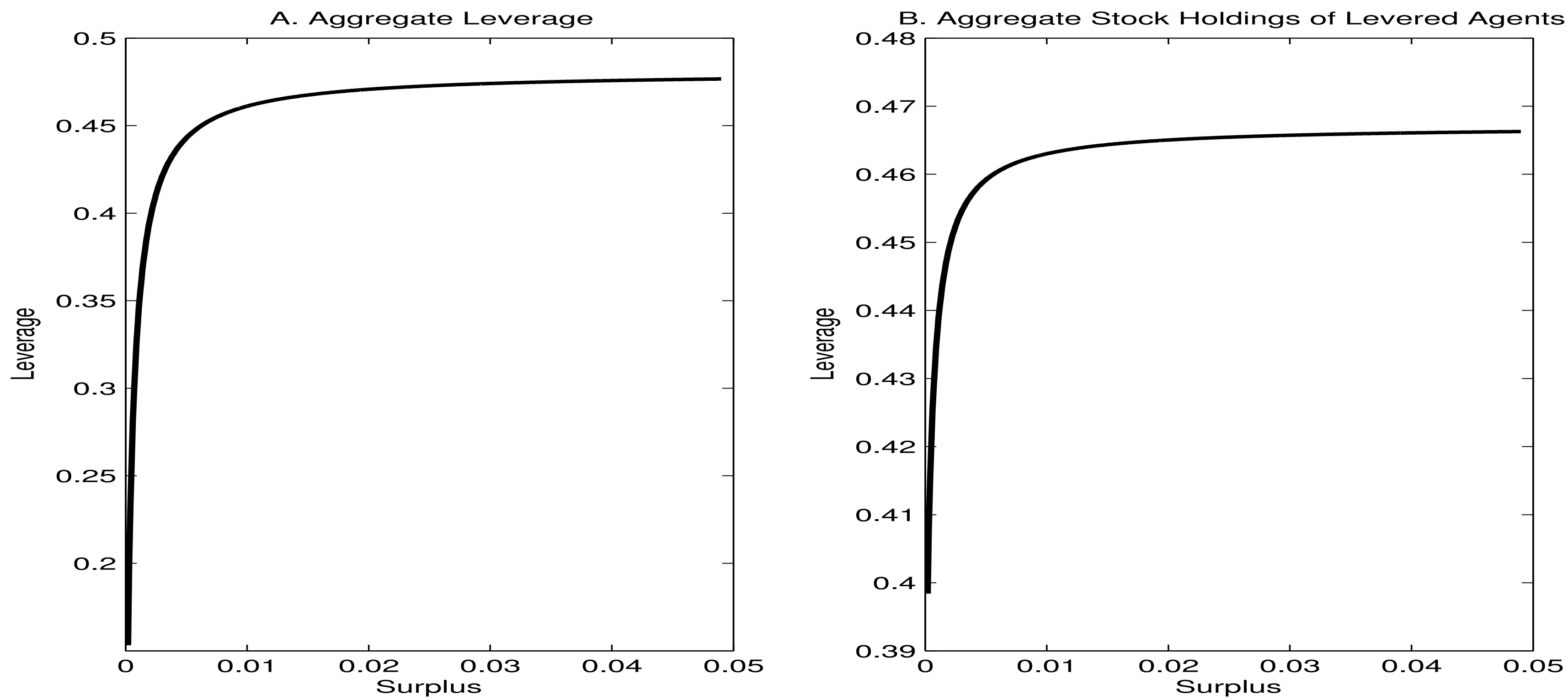

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- Let $\ell_t = Q(S_t)$, and hence $S_t = q(\ell_t) = Q^{-1}(\ell_t)$

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- The risk premium for any asset with return $dR_{it} = (dP_{it} + D_{it})/P_{it}$ is

$$E_t[dR_{it} - r_t dt] = \underbrace{Cov_t\left(\frac{dD_t}{D_t}, dR_{it}\right)}_{\text{Consumption CAPM}} + \underbrace{\frac{q'(\ell_t)}{q(\ell_t)} Cov_t\left(d\ell_t, dR_{it}\right)}_{\text{Leverage risk premium}}$$

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- Two potential measures of leverage:

Debt/Output Ratio: $\ell_t = Q_{it}^{D/O}(S_t) = -\frac{N_{it}^0 B_t}{D_t} = v\left(w_i - a_i\right) H\left(S_t\right)$

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$$\text{Debt/Equity Ratio:} \quad \ell_t = Q_{it}^{D/W}(S_t) = -\frac{N_{it}^0 B_t}{W_{it}} = \frac{\sigma_{Wi}(S_t)}{\sigma_P(S)} - 1$$

- **Result:** The price of leverage risk is

(a) $\lambda_t^{D/O} = \frac{q^{D/O\,\prime}(\ell_t)}{q^{D/O}(\ell_t)} > 0$ if $\ell_t =$ Debt/Output Ratio ("book leverage").

(b) $\lambda_t^{D/W} = \frac{q^{D/W\,\prime}(\ell_t)}{q^{D/W}(\ell_t)} < 0$ if $\ell_t =$ Debt/Equity Ratio ("market leverage").

- In bad times:
  - agents deleverage $\Longrightarrow$ debt/output $\downarrow$ $\Longrightarrow$ book leverage risk price $> 0$.
  - high discounts $\Longrightarrow$ debt/equity $\uparrow$ $\Longrightarrow$ market leverage risk price $< 0$.

# Quantitative Predictions

- Previous results independent of the functional form of $\sigma_D(Y_t)$.
- Assume now a specific functional form to make model comparable to MSV and obtain reasonable asset pricing implications:

$$\sigma_D(Y_t) = \sigma^{max}(1 - \lambda Y_t^{-1})$$

- $\Longrightarrow$ Economic uncertainty increases in bad times, but bounded between $[0, \sigma^{max}]$
- $\Longrightarrow$ Obtain same process for $Y_t$ as in MSV $\Longrightarrow$ Use their same parameters.
  - Additional parameter $\sigma^{max}$ chosen to fit average consumption volatility
- All asset pricing results are similar (or stronger) than MSV.

**Table 1. Parameters and Moments**

Panel A. Parameters (MSV)

| $\rho$ | $k$ | $\overline{Y}$ | $\lambda$ | $\overline{v}$ | $\mu$ | $\sigma^{max}$ |
|---|---|---|---|---|---|---|
| 0.0416 | 0.1567 | 34 | 20 | 1.1194 | 0.0218 | 0.0641 |

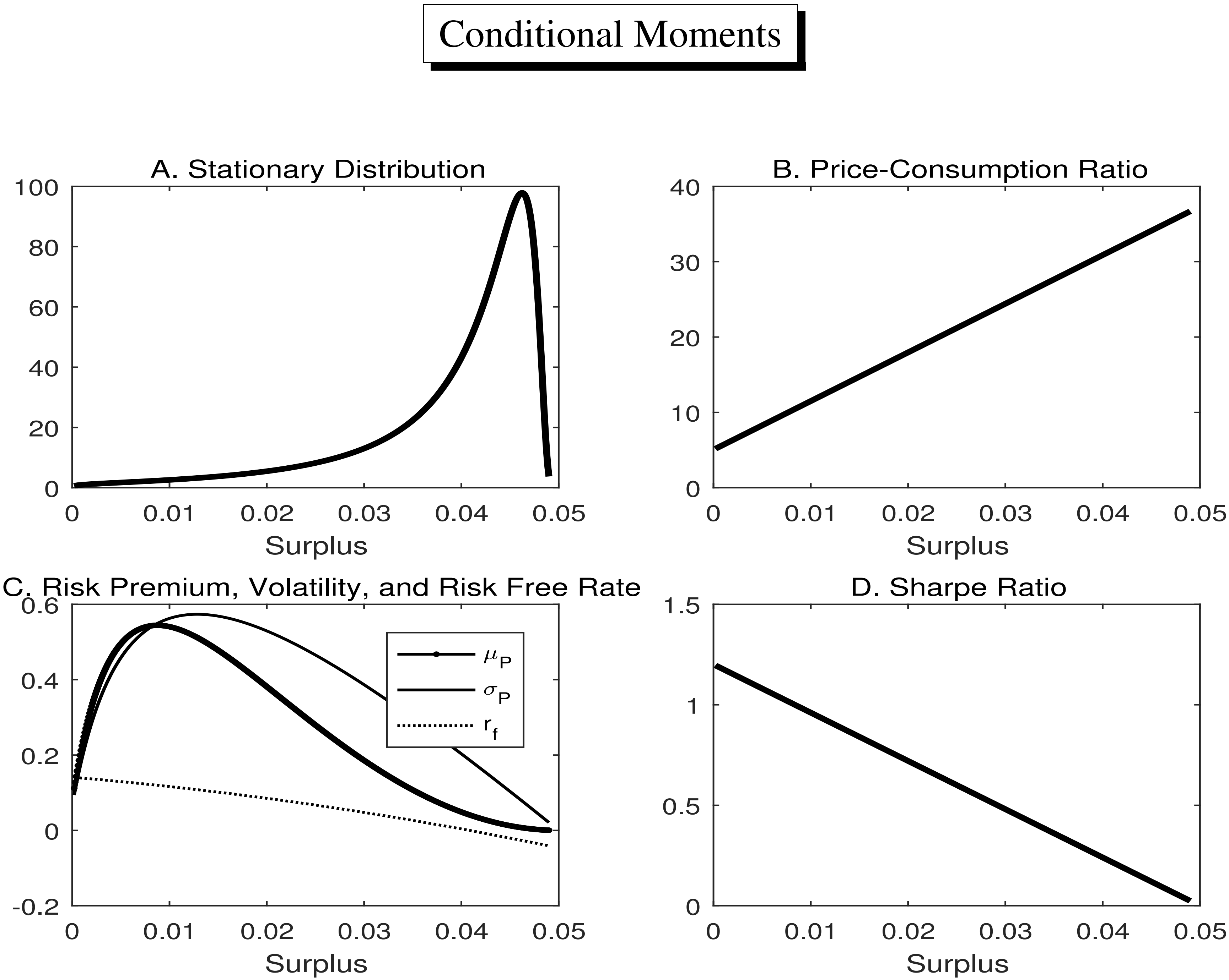
Panel B. Moments (1952 – 2014)

| | $E[R]$ | $Std(R)$ | $E[r_f]$ | $Std(r_f)$ | $E[P/D]$ | $Std[P/D]$ | $SR$ | $E[\sigma_t]$ | Std($\sigma_t$) |
|---|---|---|---|---|---|---|---|---|---|
| Data | 7.13% | 16.55% | 1.00% | 1.00% | 38 | 15 | 43% | 1.41% | 0.52% |
| Model | 8.19% | 25.08% | 0.54% | 3.77 % | 30.30 | 5.80 | 32.64% | 1.43% | 1.18% |

Panel C. P/D Predictability $R^2$

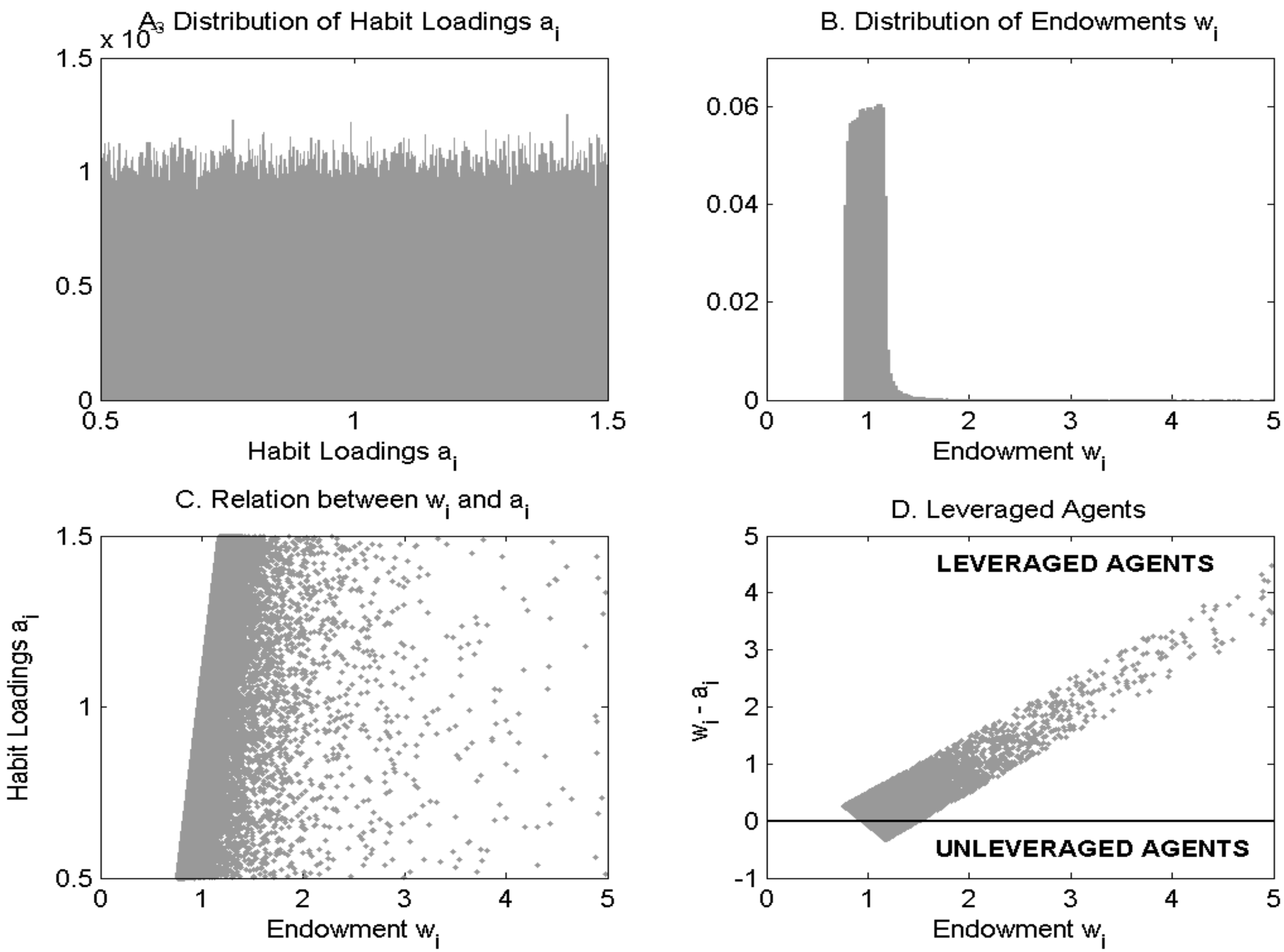
| | 1 year | 2 year | 3 year | 4 year | 5 year |
|---|---|---|---|---|---|
| Data | 5.12% | 8.25% | 9.22% | 9.59% | 12.45% |
| Model | 14.18% | 23.67% | 30.01% | 33.81% | 35.92 |

- Model matches asset pricing moments well.



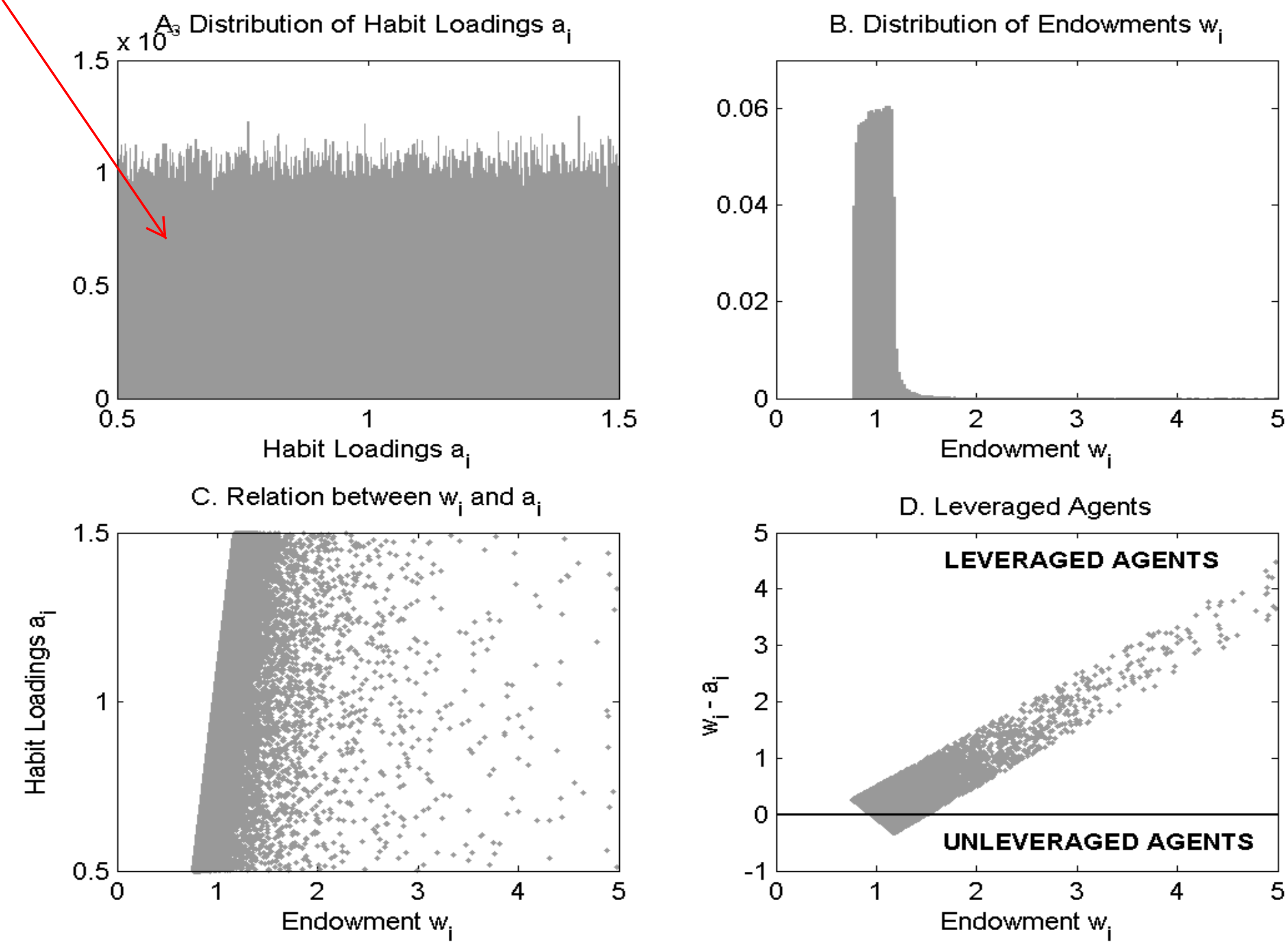
Conditional Moments
A. Stationary Distribution
100
80
60
40
20
0
0
0.01
0.02
0.03
0.04
0.05
Surplus
B. Price-Consumption Ratio
40
30
20
10
0
0
0.01
0.02
0.03
0.04
0.05
Surplus
C. Risk Premium, Volatility, and Risk Free Rate
0.6
0.4
0.2
0
-0.2
0
0.01
0.02
0.03
0.04
0.05
Surplus
$\mu_P$
$\sigma_P$
$r_f$
D. Sharpe Ratio
1.5
1
0.5
0
0
0.01
0.02
0.03
0.04
0.05
Surplus

# The Cross-Section of Agents' Behavior: Who Levers?



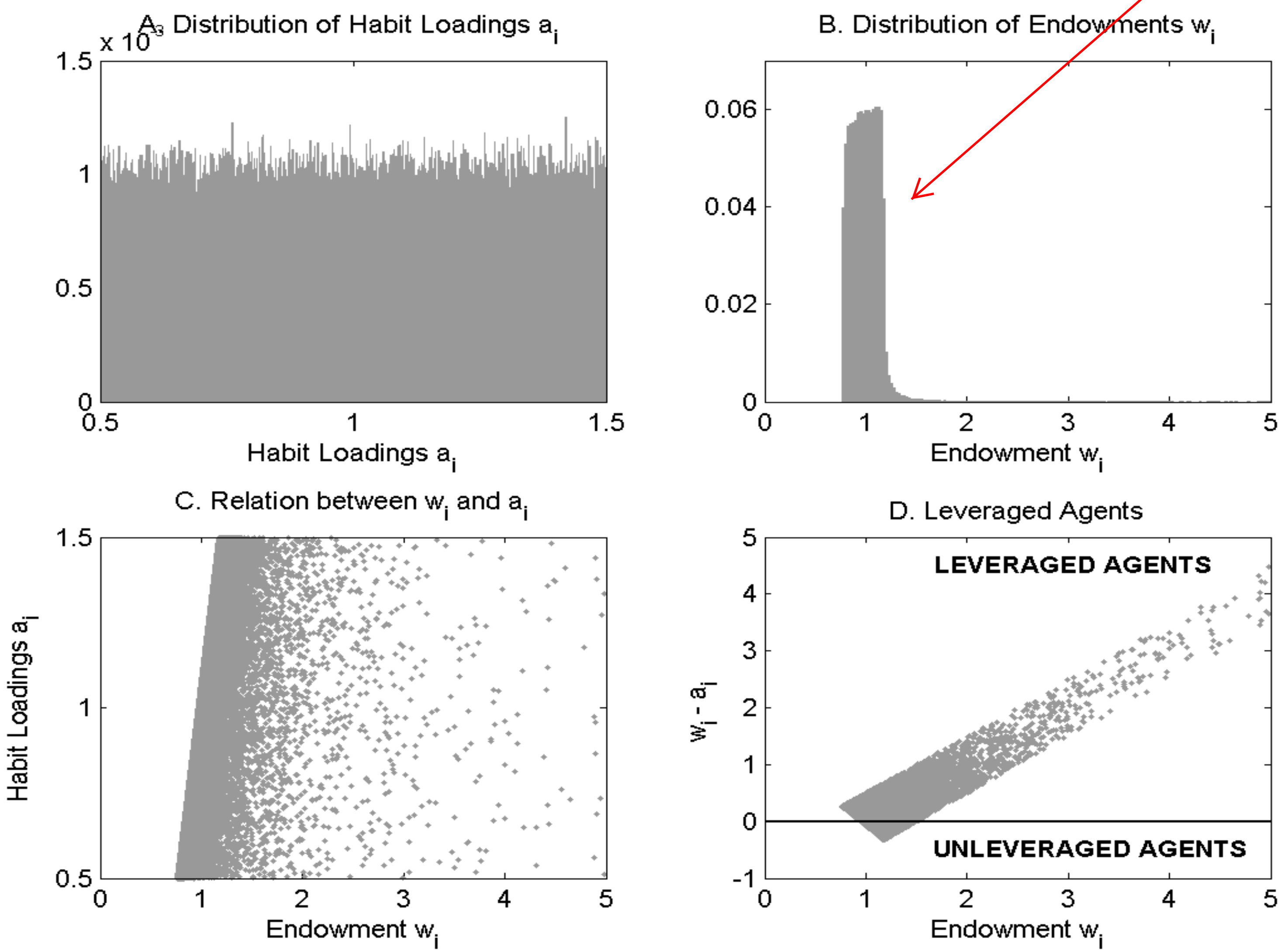
# The Cross-Section of Agents' Behavior: Who Levers?

Uniform distribution of habit ai



# The Cross-Section of Agents' Behavior: Who Levers?

Positively skewed distribution of wi



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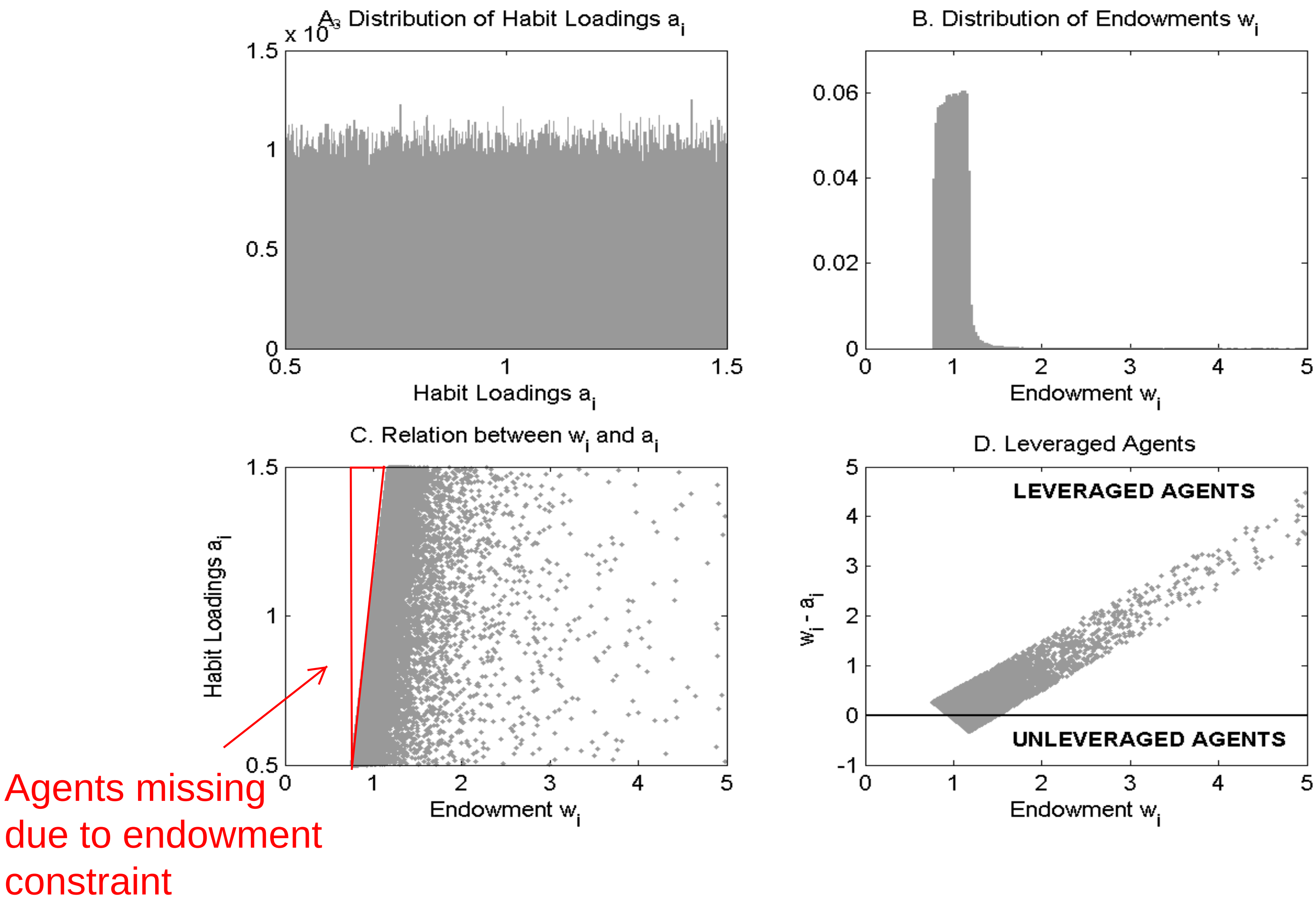

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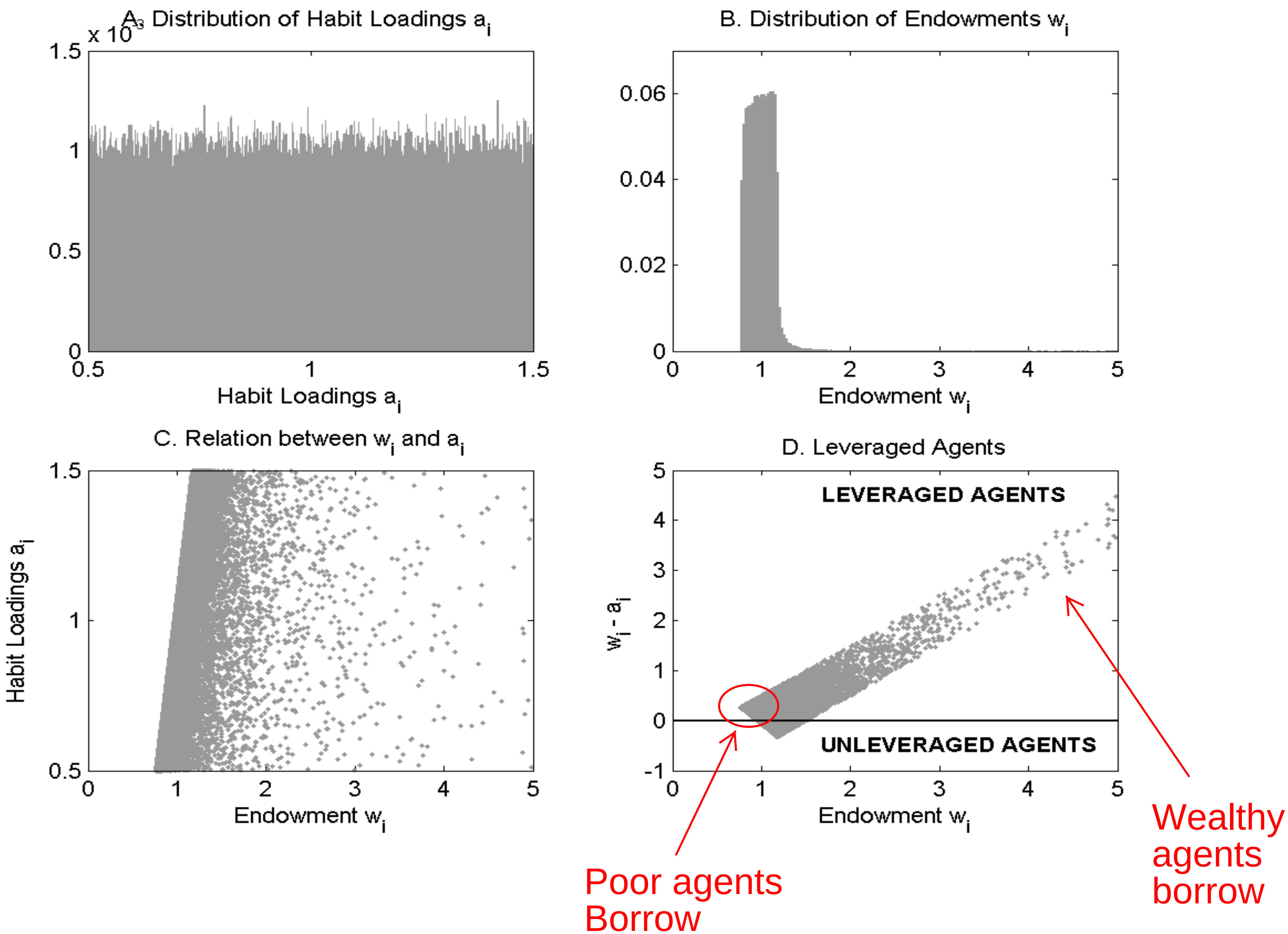
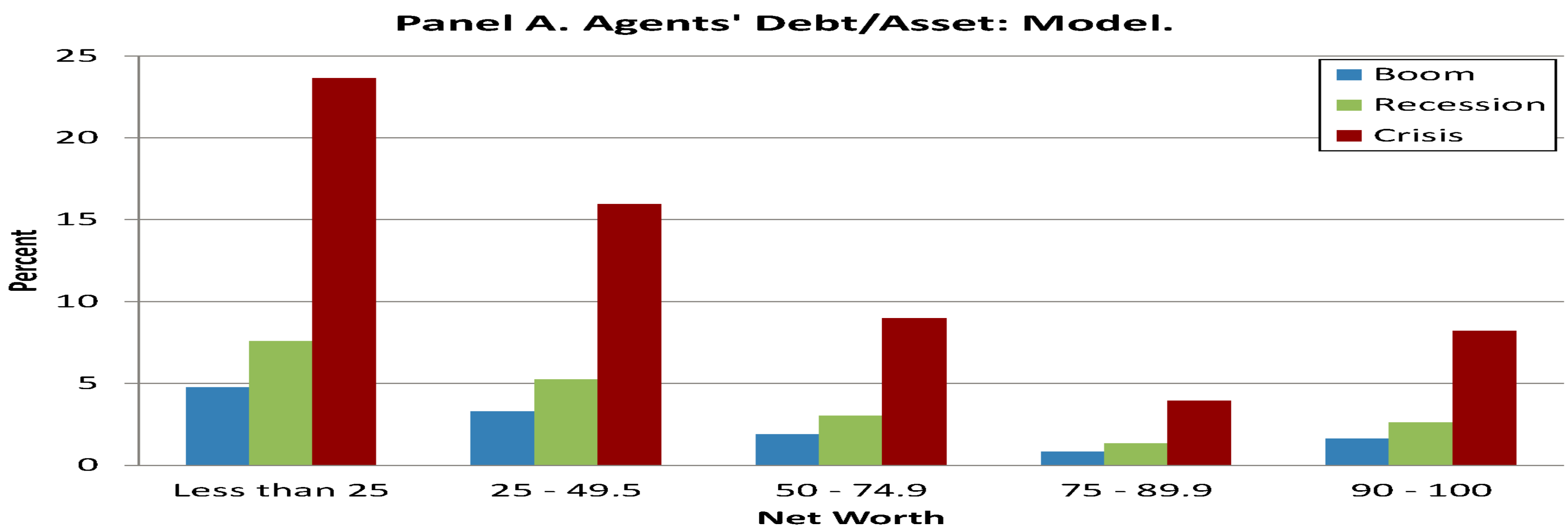


# Leverage in Good and Bad Times



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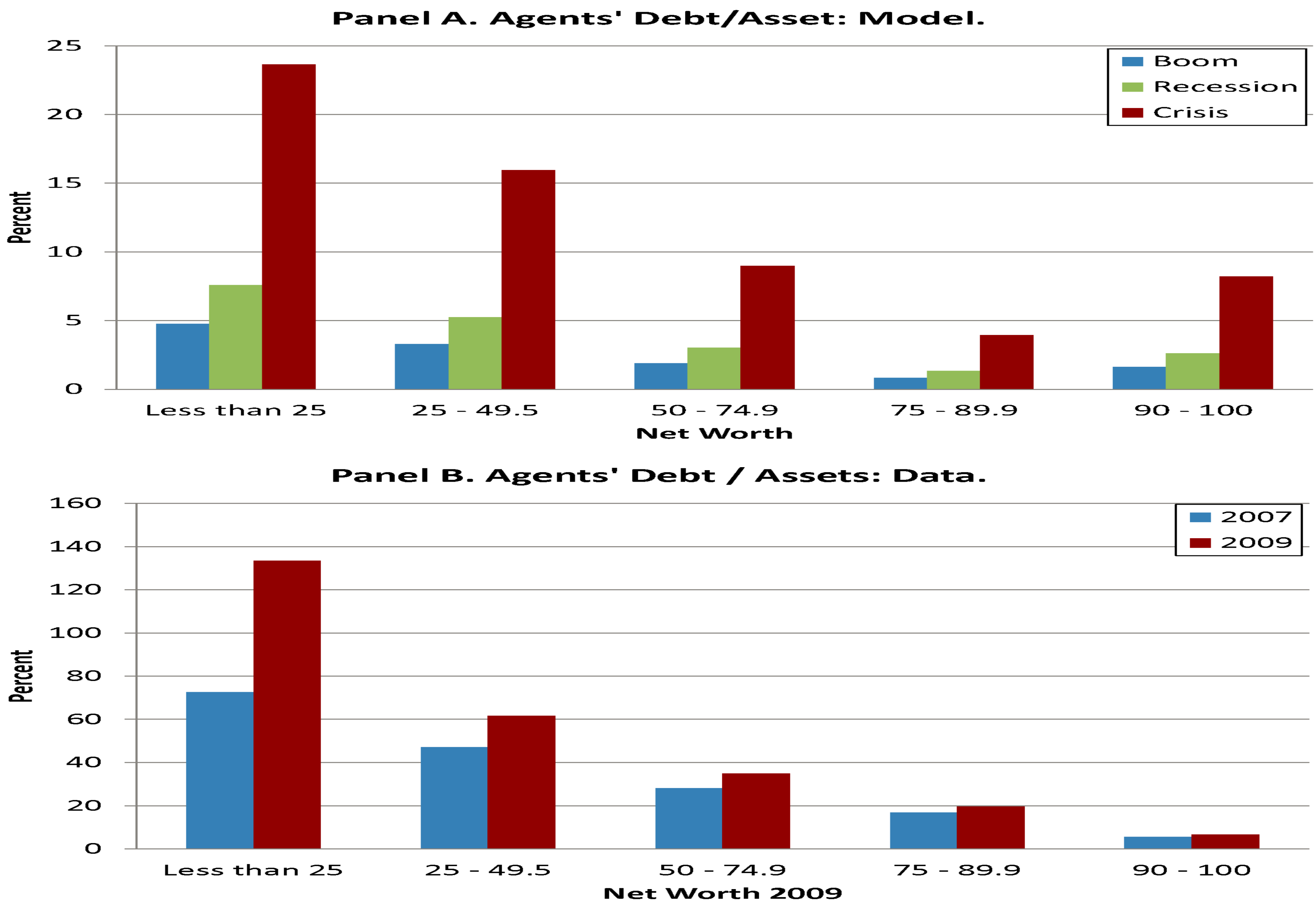
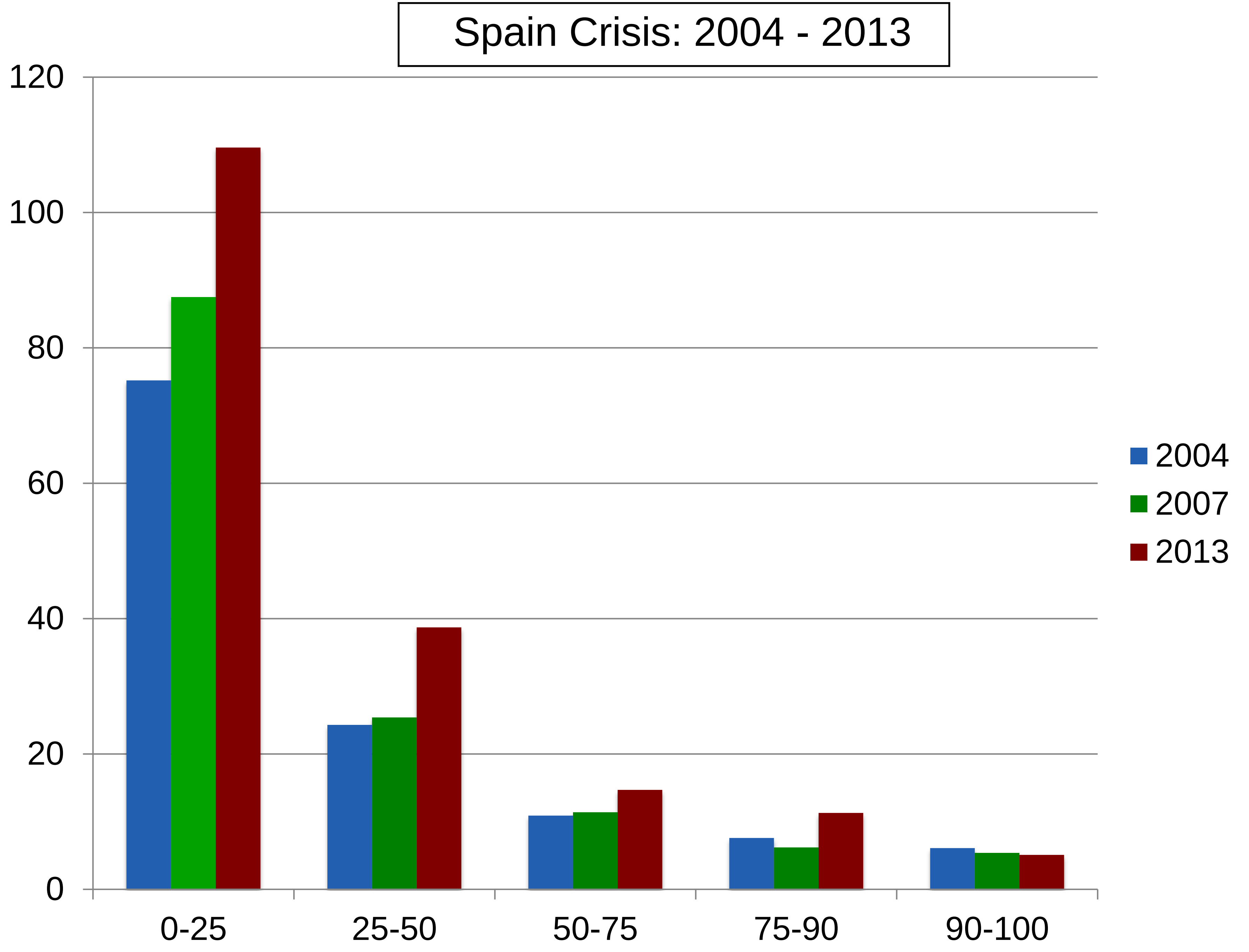
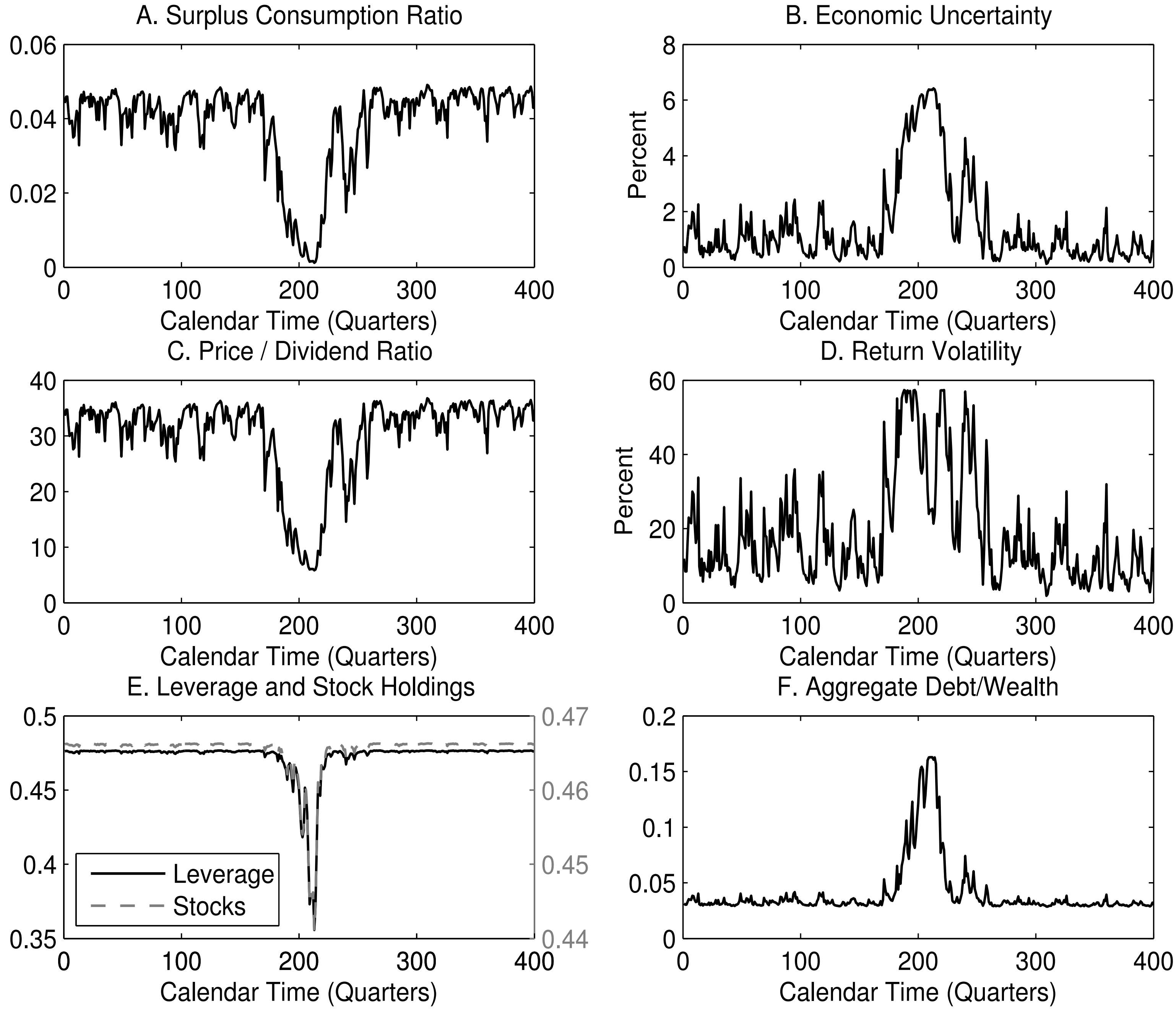


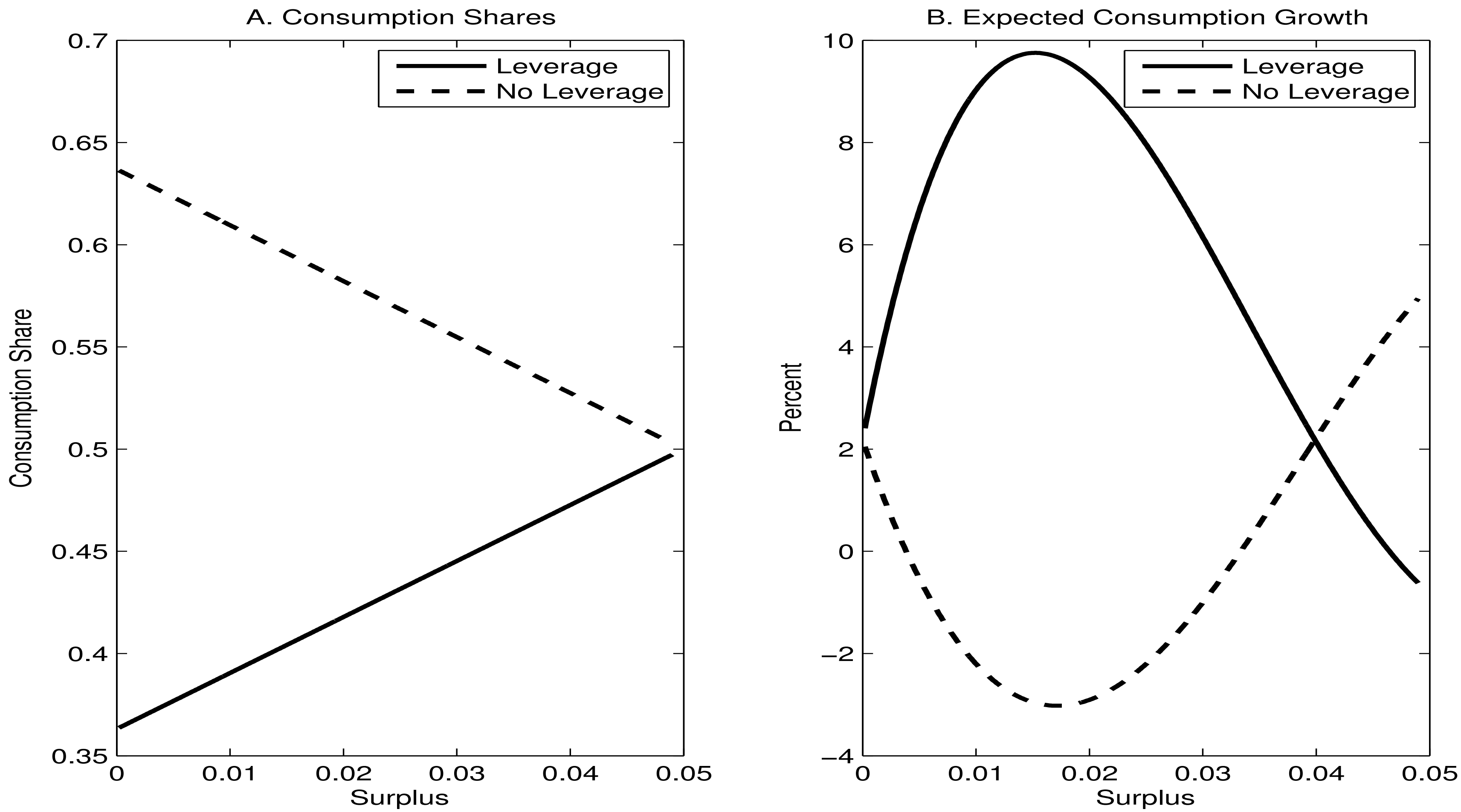

Spain Crisis: 2004 - 2013
120
100
80
60
40
20
0
0-25
25-50
50-75
75-90
90-100
2004
2007
2013

# “Fire Sales” in a Simulation Run

## "Fire Sales" in a Simulation Run



# Consumption of Levered Agents




- Consumption boom of levered agents during good times
- But expected negative consumption growth going forward

# Wealth and Wealth Dispersion

- **Proposition.** Agent $i$'s wealth/output ratio:

$$\frac{W_{it}}{D_t} = \frac{1}{\rho}\left[\frac{\rho}{\rho+k} a_i\left(1-\overline{Y}S_t\right)+w_i\overline{Y}S_t\right]$$

- and wealth share:

$$\frac{W_{it}}{\int W_{jt}dj} = a_i + (w_i - a_i)\frac{(\rho+k)\overline{Y}S_t}{\rho+k\overline{Y}S_t}$$

  – Higher $w_i$ or lower $a_i \Longrightarrow$ higher wealth in good times

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$$Var^{CS}\left(\frac{W_{it}}{\int W_{jt}dj}\right) = Var^{CS}(a_i)\left(1-\frac{(\rho+k)\overline{Y}S_t}{\rho+k\overline{Y}S_t}\right)^2 + Var^{CS}(w_i)\left(\frac{(\rho+k)\overline{Y}S_t}{\rho+k\overline{Y}S_t}\right)^2$$

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  - Preference heterogeneity $\Longrightarrow$ U-shaped wealth dispersion
    - Less risk averse richer in good times but poorer in bad times

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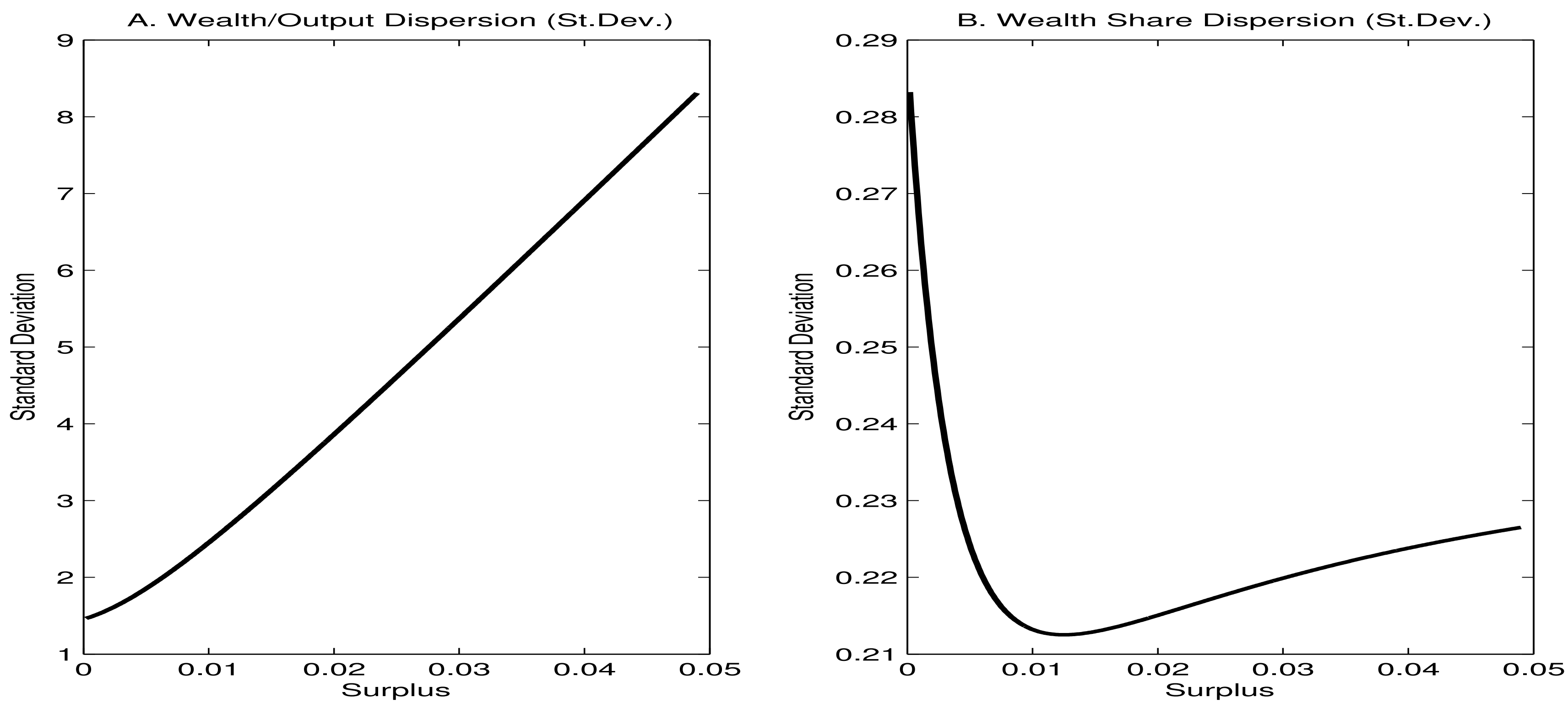
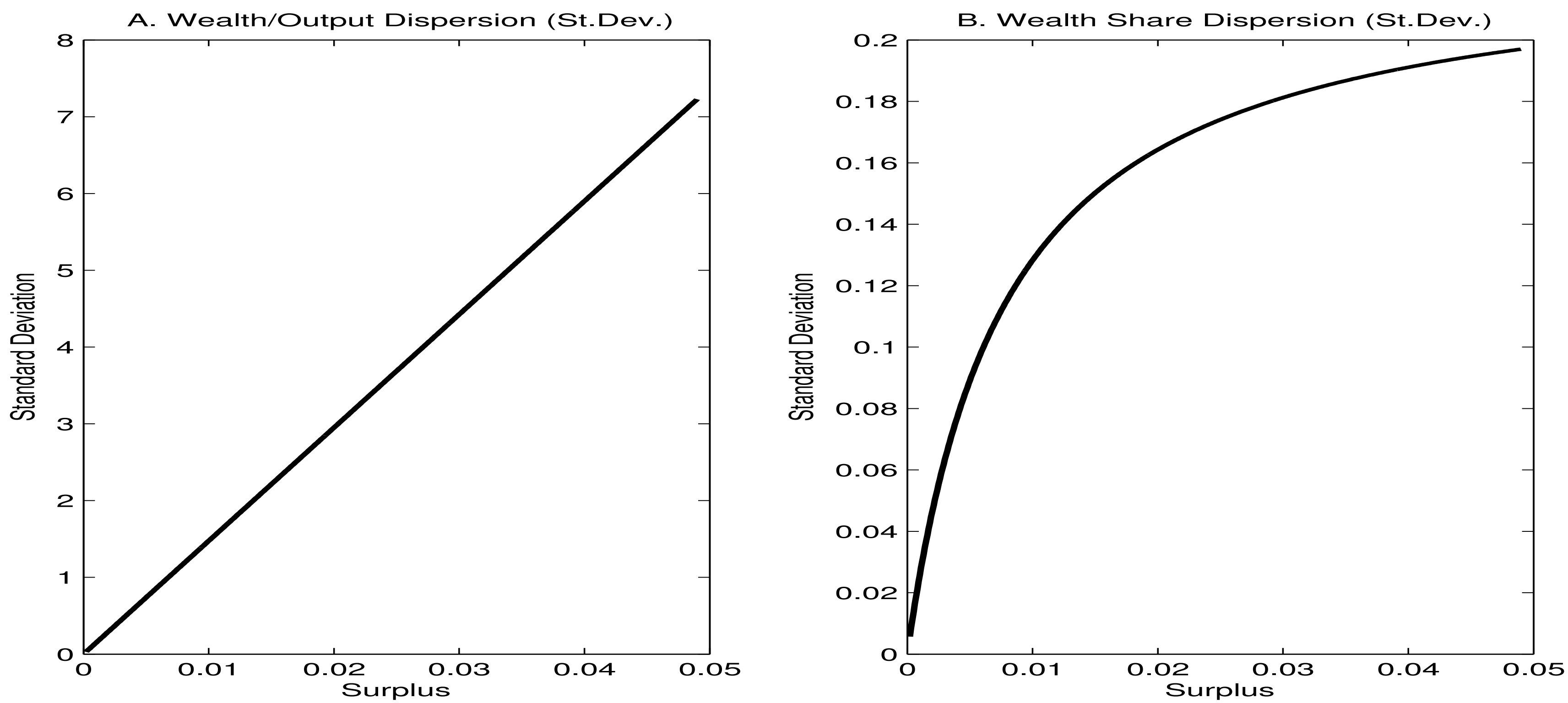
# Wealth Dispersion




- Level effect: Wealth/output dispersion increases in good times
- Relative effect: Wealth-share dispersion decreases on some range
  - Poor but very leveraged agents become better off as times get better

# Wealth Dispersion with only Heterogeneous Endowments




- Relative wealth dispersion now increases in good times
  - Only agents with high endowment (i.e. $w_i > a$) borrow $\Longrightarrow$ they become even wealthier in good times

# Conclusions

- A frictionless dynamic general equilibrium model with heterogeneous agents and external habits seem consistent with many stylized facts.
- Risk sharing motives generate endogenous leverage dynamics
- Our model predicts:
  1. Aggregate debt $\Uparrow$ in good times when prices $\Uparrow$ and volatility $\Downarrow$
  2. Poorer agents borrow more than richer agents
  3. Leveraged agents enjoy a "consumption boom" in good times, followed by a consumption slump
  4. Crisis time $\Longrightarrow$ leveraged agents delever by "fire-selling" stocks, but their debt/wealth ratio $\Uparrow$ due to strong discount effects.
  5. Intermediaries leverage is a priced risk factor.
  6. Wealth dispersion $\Uparrow$ in good times
- Leverage dynamics is due to the differential impact of aggregate shocks on agents' risk aversion.

# The Cross-Section of Consumption and Wealth