Resolving Estimation Ambiguity*

Paul H. Décaire†, Denis Sosyura‡, and Michael D. Wittry§

March 27, 2024

Abstract

Economic models develop conceptual frameworks for fundamental decisions but rarely prescribe a specific estimation approach. Using novel data on the inputs and assumptions in professional stock valuations, we study how financial analysts address estimation uncertainty when calculating a firm's cost of capital. Analysts use the same return-generating model (CAPM) but diverge in their estimation choices for key inputs, such as equity betas. Such estimation choices are driven by idiosyncratic analyst-specific criteria, persist throughout their career and across brokerages, and generate large cross-analyst variation in discount rates for the same stock. The dispersion in discount rates is associated with higher market measures of investor disagreement, such as trading volume. Overall, we provide micro evidence on how financial experts resolve estimation uncertainty.

JEL classification: G30, G31, G41, D25, D82, D83, O13, Q15, R14

Keywords: Ambiguity, Uncertainty, CAPM, Subjectivity, Trading volume, Disagreement

*For their helpful comments and suggestions, we thank Mike Barnett, Hank Bessembinder, Rawley Heimer, and brownbag participants at Arizona State University. For their excellent research assistantship, we thank our research team: Eisa Alloughani, Ezra Daniel, Sebastian Gortarez Ortiz, Jennifer Gunawan, Tej Kulkarni, Michael Lasserre, Biwon Lee, Tara Mellul, Aditeya Pattiwar, Andre Radic, Reed Simons, Roy Shubham, Cole Streitmatter, Mingyang Zhang, and Yuxin Zheng. We thank Marius Guenzel for generously sharing analysts' personal characteristic data with us and Luke Taylor for his help and support. Paul Décaire and Denis Sosyura gratefully acknowledge financial support from the Center for Responsible Investing at Arizona State University.

†W.P. Carey School of Business, Arizona State University, email: paul.decaire@asu.edu.

‡W.P. Carey School of Business, Arizona State University, email: dsosyura@asu.edu.

§Fisher College of Business, Ohio State University, email: wittry.2@osu.edu.

1 Introduction

Economic theory provides models that offer insights for fundamental decisions. However, it rarely prescribes a specific approach to estimate them. This methodological leeway gives rise to estimation ambiguity. Agents know that there exists a combination of methods that will yield a parameter's true value estimate, but they rarely know the correct weights of each approach. Thus, even if all agents agree on the underlying economic model and have access to the same data, variation in their methodological choices due to estimation ambiguity can produce different modeling outcomes.

When agents estimate the same model but face a variety of feasible methods and plausible assumptions, how do they arrive at their decisions and how do their estimation choices affect real aggregate outcomes? Multiple theories generate insights about how agents should address estimation ambiguity. However, obtaining direct empirical evidence to pin down the most likely theories has remained elusive because agents' inputs, assumptions, and methodological selection processes are difficult to discern.

We examine these questions by studying the inner workings of estimating a firm's required rate of return—a key valuation driver and a foundation for investment decisions. We collect over 40,000 stock valuation models of financial analysts from top brokerage firms worldwide. We observe the discount rate model selected, its inputs and values, as well as a discussion of the information sources and methodological choices. Our panel data structure allows us to track each analyst over time and across the different settings they analyze, enabling us to investigate whether and why an agent's methodological choices might vary or exhibit persistence. This setting also generates useful variation in the degree of uncertainty across model inputs. In particular, the cost of equity estimation combines inputs that are observable in financial markets, such as the risk-free rate, and those that require estimation, such as equity betas. As a result, we can exploit the variation in estimation uncertainty across different inputs for the same firm in the same year, while holding constant the selected estimation model.

Securities valuation offers a convenient laboratory to study estimation ambiguity. Most finance professionals rely on the same model of the return-generating process—namely, the

CAPM, to estimate a firm's cost of capital, as shown in surveys (Graham and Harvey, 2001) and field data (Kruger et al., 2015; Dessaint et al., 2021), but theory provides no guidance on estimating the model's inputs. Since the CAPM inputs are based on public data, a natural question is whether finance professionals obtain comparable results while estimating the same model.

We find that financial analysts arrive at significantly different discount rates for the same firm at the same time. For a given firm, the average difference in the estimated cost of equity for two different analysts publishing equity reports at the same time is 180 basis points or 18 percent of the mean cost of equity in our sample. Similarly, the average difference in the weighted average cost of capital (WACC) for two analysts covering the same firm at the same time is 140 basis points or 16 percent of the mean WACC. Given the outsized quantitative importance of the discount rate in financial models, these magnitudes are somewhat unexpected because most prior work rarely considers analyst dispersion in the context of discount rates.

The discount rate dispersion is associated with a large variation in a firm's private valuations. For the median firm, a one standard deviation increase in the estimated cost of equity is associated with a 22% drop in intrinsic value, comparable to the difference between a buy and hold recommendation, when using a simple dividend discount model. This effect is quantitatively important because higher discount rates are not offset by other modeling choices, such as higher explicit or terminal growth rates. For example, in 45% of the valuation models, an above-consensus estimate of the discount rate is associated with a below-consensus estimate of the terminal growth rate, suggesting that they diverge in the opposite direction. As a result, the dispersion in the estimated discount rate appears to arise independently from cash flow disagreement.

Next, we study why professional agents with comparable tools arrive at different results for the same estimation task to understand the underlying factors of this particular channel of disagreement. We find that the discount rate variation is driven not by the return generating model, but by the methodological choices in model estimation. Décaire and Graham (2023) show that when estimating discount rates nearly all analysts (97%) use the CAPM, but our results suggest they diverge sharply on its inputs.

We investigate the sources of variation in model inputs and find that they are driven by variables that require estimation and have more estimation uncertainty. The largest sources of variation come from the differences in the estimation of betas and the equity risk premium (ERP). Using a Campbell-Shiller decomposition, we show that estimation ambiguity is a key driver of heterogeneity in the discount rate by decomposing it into the estimates of the risk-free rate and beta×ERP. The evidence shows that nearly 80% of the variation in the cost of equity is driven by the estimates of beta×ERP, and only a small minority of the variation (21%) is attributable to the choice of the risk-free rate.

The variation in the estimated betas and ERP is surprisingly large. For example, the average difference in the estimated betas for two different analysts publishing equity reports at the same time is 0.219 or 20% of the unconditional sample mean for CAPM beta. The variation in the estimated ERP is similarly large. During the median year in our sample (2000-2023), the 25th percentile of the ERP estimate is 5 percent per year, and the 75th percentile is 6.4 percent, indicating an interquartile range of 1.4 percentage points or 25% of the mean. Similarly, during the median sample year, the average difference in the ERP for two analysts covering the same firm at the same time is 130 basis points or 23% of the mean (5.7 percent per year).

To understand the drivers of divergence in model inputs, we conduct a contextual analysis of valuation reports and extract the description of estimation procedures and data sources for the main inputs. We manually read a sample of reports to construct a training algorithm and develop a machine-learning procedure to obtain key insights from analysts' qualitative discussions. We augment this approach with an advanced analysis that incorporates artificial intelligence and big-data contextual structures.

We find that a key driver of the differences in beta estimates for the same stock is the trailing estimation horizon, the number of years of monthly data used to estimate the beta. The estimation horizon varies from one to ten years, and the most common horizons use monthly stock returns over the trailing five years (40%), two years (32%), or three years (16%). The choice of the estimation horizon is particularly important for valuation outcomes during periods of strong market returns and economic growth, as the spread between the maximum and minimum beta estimated with different rolling windows is pro-cyclical (e.g., see Figure 1).

In contrast to the large dispersion in beta and ERP estimates, analysts tend to converge on the inputs linked to observable values, such as the risk-free rate. In the U.S., the majority (98%) of valuations rely on the 10-year Treasury as a proxy for the risk-free rate, while a minority of reports use the 30-year Treasury (1%). Overseas, valuation reports show more variation in the choice of the risk-free asset, ranging from government bonds issued by the foreign firm's country, another country in the firm's region (e.g., Germany in the Eurozone), or the United States. These choices generate modest cross-sectional variation in the risk-free rate and rationalize the evidence from the Campbell-Shiller decomposition.

Our findings suggest that analysts' estimation outcomes reflect persistent methodological choices rather than ad-hoc tweaking of financial models to yield a particular result. For example, valuation models with higher betas do not systematically use smaller ERP, as would be expected if analysts attempted to reverse engineer estimation outcomes by adjusting estimation procedures.

A natural question is why critical valuation inputs vary widely across financial experts who work for similar financial firms and have comparable professional backgrounds. Does the variation in the modeling choices reflect the rules of the brokerage house, the institutional features of the firm, the common practice of its industry, or the modeling preferences of an analyst?

To address this question, we estimate a variance decomposition (ANOVA) for the dispersion in the CAPM betas observed for a given firm-year into several components attributable to institutional norms, personal preferences, or the focal firm. By including firm fixed effects, we absorb time-invariant differences in betas across firms driven by their different exposure to systematic factors, and exploit within-firm variation across personal preferences and institutional norms respectively captured with analysts and brokerage houses fixed effects. We find that personal preferences explain 28% of the within-firm variation in beta estimates, several times more than those related to institutional norms (1%).

Consistent with a key role of an analyst in selecting estimation parameters, we find that over 80% of the analysts use the same estimation horizon across all reports in their sample. These methodological choices persist when the same analyst covers different stocks or moves across brokerage houses. In contrast, we do not find similar consistencies in estimation choices for analysts within the same brokerage house or for stocks within the same industry. This evidence suggests that the choice of model parameters rests with the individual analyst, and this choice is persistent throughout an analyst's career. This persistency squares with the analysis in Bewley (2002) applied to the role of ambiguity in parameter selection. In the absence of a clearly dominant alternative, individuals exposed to ambiguity will maintain the status quo.

Next, we study how analyst characteristics are related to their choice of model estimation parameters. Using a wide array of analyst characteristics, such as education, seniority, location, and demographics, we find that they have modest effects in explaining modeling choices. Overall, our evidence suggests that estimation choices are persistent and driven by idiosyncratic individual-specific criteria, such that the same analyst applies a consistent set of estimation parameters over time and across firms, but the choice of model is unrelated to a set of common observables.

Our findings help distinguish among the most common theories on resolving estimation ambiguity proposed in the literature. First, robust methodological choices, such as the max-min criteria discussed in Gilboa and Schmeidler (1989) and Hansen and Sargen (2001), suggest selecting a method that would be optimal under a justifiable worst-case scenario.[1] For example, an analyst guided by this approach could select a trailing horizon for beta estimation that would yield the most liberal (conservative) beta such that a firm's cash flows are never overestimated (underestimated). A second group of theories suggests agents address estimation ambiguity by aggregating a model's outcomes over its likely specifications following a Bayesian selection criterion (Giacomini et al., 2019, 2022). Lastly, behavioral theories offer a promising avenue to interpret our results. Ambiguity-averse agents might favor familiar strategies (Heath and Tversky, 1991; Fox and Tversky, 1995), such as those covered during their academic training or the ones they were exposed to during their formative professional years, instead of considering all possible methods. A more extreme version of this criterion relates to Raiffa (1961)'s critique, which suggests that one can resolve estimation

[1]Existing empirical evidence must discipline the set of justifiable scenarios that can be reasonably considered.

ambiguity using a random draw over the possible (justifiable) methods under consideration.

Our results are broadly consistent with a behavioral explanation. First, the data suggests that the adoption of a max-min criterion is unlikely in our setting as such a strategy would result in using different beta estimation horizons across firms and, over time, when selecting the most conservative or liberal estimates for each valuation exercise (e.g., see Figure 8). In fact, in only 21% of cases would an analyst adopting such decision criteria end up using the same beta horizon for two consecutive years for the average firm. Rather, we observe that analysts' methodological preferences are extremely persistent and seem to be driven by idiosyncratic individual-specific criteria. Simple heuristics and anchoring (Tversky and Kahneman, 1974) driven by early career influences or randomly selecting a method and sticking with it is more consistent with the patterns we document in the data than the sophisticated approaches discussed in the literature.[2] Indeed, that 80% of analysts consistently employ the same methodology, coupled with the fact that only 3-4% engage in any form of aggregation, indicates that the alternative criteria explored in the literature are, at best, adopted by a minority of professionals.

In our final analyses, we study how the disagreement in the estimated discount rates is associated with market outcomes. In the cross-section, while the average disagreement in estimated cash flows declines with a greater number of valuations that diversify away extreme estimates, the differences in the discount rate need not cancel out and may be augmented with a greater number of valuations due to the convexity in the discount rate's effect on the share price—Jensen's inequality. Consistent with this mechanism, we find that stocks with a greater disagreement in the discount rate have significantly higher trading volume. A one standard deviation in the discount rate disagreement (153 bps) is associated with a 4 percent increase in the abnormal trading volume.

Overall, our findings suggest that a large share of the dispersion in stock valuation is attributable to modeling choices in estimating a firm's discount rate. Most of the variation results from estimating the same return-generating model but with different trailing horizons (beta estimates) and under different assumptions about the equity risk premium. These

[2]At the moment, our evidence cannot distinguish between a role for familiarity or a simple random draw. We are currently collecting data to address this.

methodological choices reside with the financial analyst and provide micro foundation for prior evidence on the importance of individuals' factors in asset valuation.

The central contribution of this paper is to provide granular evidence on the inner workings of estimating financial models by sophisticated intermediaries. Our paper contributes to prior research on (1) estimating the cost of capital, (2) disagreement in stock valuations, (3) ambiguity in professional decisions.

We expand our understanding of how key market agents determine and apply the cost of capital in their professional decisions. Since the discount rate is rarely observable to the econometrician, most research has inferred the discount rate from surveys (Graham and Harvey, 2001), observed investment actions (Kruger et al., 2015; Dessaint et al., 2021; Décaire, 2023), or select disclosures (Gormsen and Huber, 2023). While these approaches yield useful WACC measures, they remain silent about how agents arrive at their estimates and update them in response to market conditions. Our paper complements this work by providing evidence on how equity analysts estimate each component of the discount rate.

The discount rates in analyst reports are important for both corporate managers and investors. Décaire and Graham (2023) find that analyst discount rates are unbiased predictors of future stock returns, indicating that they provide useful forward-looking information. Décaire and Bessembinder (2021) show that the discount rate uncertainty has significant real effects on project valuation and corporate investment. We supplement these findings by identifying the drivers of variation in analyst discount rates.

Our paper is also a part of the broader literature that studies the origins of disagreement in private valuations. Theory makes an important distinction between market participants' disagreement in information inputs and valuation methods (see Hong and Stein, 2007 for a review), but this distinction has been difficult to test. Andrei et al. (2019) and Meeuwis et al. (2022) show that investors react differently to common information shocks and suggest that they rely on different mental models. Using investor activity on a social media platform, Cookson and Niessner (2020) find that investor disagreement is evenly split between a self-declared investment approach and different information sets. Using a survey, Giglio et al. (2021) find that investors disagree about both cash flows and expected returns and conclude that it is crucial to jointly model their disagreement about both parameter groups and their comovement. Rather than relying on indirect measures of disagreement in valuation, such as surveys, social media, or trades, our paper provides direct evidence on the inner workings of agents' financial models. We show why financial experts arrive at different outcomes even when estimating the same model with public data. While most prior literature on analyst disagreement (see Sadka and Scherbina, 2007 for a review) has focused on the dispersion in cash flow forecasts, we show that the dispersion in the estimated discount rates is an equally important determinant of private valuations.

We also contribute to the discussion linking ambiguity in valuation with the role of subjectivity in finance (Keynes, 1921; Shiller, 2014), by showing how individuals' idiosyncratic preferences about the methods used to estimate financial models can be associated with market-wide disagreement.

Finally, our findings relate to the literature investigating the effect of ambiguity on estimation outcomes of practitioners. Kahneman et al. (2021) discusses how judges can give different judgments on similar cases due to different mental models. Multiple recent papers investigate whether estimation ambiguity affects estimate dispersion in science, such as Huntington-Klein et al. (2021) and Menkveld et al. (2023), showing that even well-intentioned researchers can arrive at different outcomes in the absence of clear guidance. In parallel work, Mitton (2022) posits that academic researchers might exploit estimation ambiguity to obtain significant results in their work. Using "what-if" scenarios and simulations, he shows one can obtain drastically different outcomes by selecting different ways to process the data. Our work extends these papers' insights in three ways. First, we shed light on the likely mechanism used by practitioners to resolve estimation ambiguity, offering avenues to refine theories studying how agents address ambiguity in the real world. Second, we investigate the factors that can explain why individuals select a particular approach when resolving ambiguity. Lastly, we present evidence about how estimation ambiguity likely impacts aggregate outcomes, by studying its relation to the financial market.

2 Conceptual Framework

We start this section by introducing the conceptual model used by equity analysts to estimate firms' discount rate and cost of equity. We then use those models to precisely illustrate the sources of estimation ambiguity that we investigate in the paper.

We can express discount rates, the WACC, as:

$$WACC_{i,t} = W^E_{i,t} * (rf_t + \beta_{i,t} * ERP_t) + W^D_{i,t} * (1 - \tau_{i,t}) * r^D_{i,t} \tag{1}$$

Where W^E and W^D are the weights of equity and debt in the firm's capital structure, respectively. rf is the risk-free rate, ERP denotes the equity risk premium, β corresponds to the CAPM beta, τ is the firm's marginal tax rate, and r^D is the cost of debt. While this model is conceptually clear, and it provides guidelines about how these variables interact together in the overall computation of a firm's cost of capital, substantial ambiguity remains in the measurement and estimation of some of its parameters, making room for individuals' subjective interpretation. In that sense, studying how financial professionals determine their CAPM beta is well suited for our analysis: there is a clear theoretical framework underpinning its estimation, but limited guidance about its exact estimation.

Estimation ambiguity materializes in many aspects of the CAPM beta measurement. Focusing on the basic formula:

$$r_{i,t} - rf_t = \alpha + \beta * (r^M_t - rf_t) \tag{2}$$

Where $r_{i,t}$ is the firm's stock market return at a predetermined frequency, and r^M_t is the market return. While the underlying theory is simple, analysts have limited guidance in setting up the regression design when it comes to choosing (i) the number of days, months, or years over which the analysis should be performed (i.e., beta's horizon), (ii) the market benchmark to be used to proxy for the market returns r^M_t, and (iii) the frequency of the returns (i.e., daily, weekly, or monthly). Different assumptions about these elements of the CAPM regression can yield material differences in the estimate, and thus, generate different costs of capital for the firm.

We focus our analysis on the effect of beta's horizon, as this dimension of estimation ambiguity can be both directly investigated in the numerical data and equity analysts generally discuss this modeling choice in equity reports. This added level of visibility allows us to benchmark our regression results with a textual analysis from discussions in equity reports to verify our conclusions. This permits us to confirm the conclusions of our empirical analysis with precise discussions about how professionals determine their cost of capital. In the context of our analysis, estimation ambiguity arises because analysts know that distinct horizons, or combination of horizons, can best represent the firm's true beta, but they do not know the weight to put on each of those cases (Figure 2).

Lastly, we note that not all financial professionals personally estimate their CAPM betas. Mainstream data providers such as Bloomberg, Refinitiv, and Yahoo also provide their own estimates. While not explicitly acknowledged by some of the analysts, taking the CAPM beta from those databases implies that professionals implicitly adopt the provider's distinct methodology. For example, Bloomberg by default uses a 2-year horizon at the weekly frequency, while Refinitiv favors using a 5-year horizon at the monthly frequency.

In contrast, determining other components of the WACC, such as the risk-free rate, is a more direct process, which offers a natural benchmark to evaluate the nature of estimation ambiguity. For example, Figure 7 shows that analysts nearly unanimously (91%) use the 10-year domestic treasury yield to estimate the risk-free rate in their models. Ultimately, our setting allows us to hold the model used by our financial agents fixed and determine if inputs suffering from estimation ambiguity exhibit greater heterogeneity.

3 Methodology and Data

3.1 Data Source

The bulk of our data is sourced from equity research reports (i.e., the original documents) published by sell-side analysts. We start with an initial batch of 157,549 equity reports with mentions of the keywords "DCF", "discounted cash flow", and "weighted average cost of capital", or "WACC" from 42 major equity research firms. We restrict the time window to reports published in the first quarter of the calendar year (January 1^{st} to April 1^{st}) from

2000 to 2023. This ensures that our data are systematically measured at a similar time point in the year. In cases where analysts publish more than one report on the same firm during the first quarter, we systematically keep the earliest publication in that calendar year to avoid duplicates for a given analyst-firm-year pair. This procedure results in 45,992 equity reports that include at least one of our variables of interest, and for which each firm-year pair is covered by at least 2 analysts.

We collect numerical values for each of the inputs in four steps. First, documents are pre-processed using a Python program to identify sections of text, tables, and figures containing relevant information for the study (e.g., discount rates, risk-free rate, or equity betas). Second, we convert each of these forms of publication into text snippets. Third, for each variable, we use artificial intelligence to extract the numerical value from the snippets. Fourth, we export the text snippets and the numerical values extracted by artificial intelligence to Excel – and our research team manually verifies every single number. This last step of the collection effort (manual verification) is crucial to ensure the integrity of the data used in the analysis. While artificial intelligence is an efficient tool for text extraction, error rates in the processing of complex sentences can be well above acceptable levels when AI is left unsupervised (Gilardi et al., 2023).

The disclosure of this information is done on a purely voluntary basis. However, prior literature has found that the intensity of information disclosure of DCF modeling assumptions is positively associated with report accuracy (Asquith et al., 2005; Hashim and Strong, 2018), and more detailed information disclosure leads to larger market reactions following changes in recommendations (Huang et al., 2023). Moreover, by detailing their valuation theses, informed analysts have the opportunity to differentiate their work from their uninformed rivals, gaining credibility in the process. More closely related to the study of analysts' discount rates and their inputs, Décaire and Graham (2023) shows that the average discount rate reported by analysts is an unbiased predictor of firms' one-year realized returns. They interpret this result as suggestive evidence that data is informative and captures inherent features associated with stocks' returns.

3.2 Firms and Coverage

The equity reports are produced by 42 of the largest equity research departments operating throughout the world. Because we aim to understand disagreement in the valuations of the same security during the same time period, we restrict our sample to firm-year observations with equity reports produced by at least two analysts. Panel A of Table 1 highlights the specific coverage for each of the DCF inputs in terms of number of firms and firm-year observations that meet this requirement. For example, in the discount rate sample, the 45,992 reports cover 4,261 firms located in 63 countries during the 2000-2023 period. Panel A also shows that as the DCF inputs increase in specificity, e.g., moving from discount rates to the components of a standard weighted average cost of capital (WACC) calculation, our sample sizes reduce as fewer equity reports include every such input.

Table 1 Panel A also reports on a limited set of firm characteristics for our discount rate sample. For example, the average firm is covered by 10 analysts that we can identify over its life and is included in the sample for 4.0 years. As with standard commercial databases that report on analyst expectations (e.g., EPS forecasts via I/B/E/S), our sample skews older and larger than the full universe of publicly traded firms that could be downloaded from Compustat or CRSP. However, Décaire and Graham (2023) show that the characteristics of the firms in their sample (which closely matches out sample) are similar to the firms from I/B/E/S data in terms of size and investment intensity.

In terms of geographic coverage, 34% in North America, 33% of firms have their headquarters located in Europe,18% in Asia, 11% in Oceania, 3% in South America, and 1% in Africa. The reports are produced by equity analysts located in Europe in 45% of the cases, 22% Asia, 18% in North America, 11% in Oceania, 2% in South America, and 2% in Africa.

Two dozen NAICS industry sectors (2-digit) are represented in our sample, with the eight largest broad sectors accounting for 74% of the total coverage: 34% for manufacturing (NAICS 31-32-33), 16% for information (NAICS 51), 9% for professional services (NAICS 54), 6% for retail trades (NAICS 44-45), 6% for mining and oil & gas (NAICS 12), 4% for transportation (NAICS 48-49), 5% for utilities (NAICS 22), and 4% for finance and insurance (NAICS 52). Overall, these statistics suggest that our sample is comprehensive, representative, and comparable to its commercial counterparts.

4 Results

4.1 Required Rate of Return Heterogeneity and its Sources

We start by investigating whether the required rate of return estimates produced by equity analysts differ materially when evaluating the same firm at the same point in time (e.g., in the first quarter of the same year). Table 1 reports that the absolute value of the mean pairwise difference between two analysts' estimated weighted average cost of capital ($WACC_{|A-B|,i,t}$) for a given firm is 1.4%. This number is highly statistically significant. Moreover, this effect is quantitatively important because higher discount rates are not offset by other modeling choices, such as terminal growth rates (TGR). For example, Table 2 suggests that 55% of the time, analysts deviate from the consensus estimate for discount rates and terminal growth rates in the same direction, minimizing the impact of either the growth rate estimate or discount rate estimate on overall valuation. However, this leaves nearly *half* of the cases in our sample (45%) that deviate in the opposite direction. Overall, we interpret the lack of discernible patterns in 2 as reasonable evidence that analysts do not systematically fudge their valuation models to reach a predetermined price target.

Figure 3 displays the entire distribution of the absolute value of pairwise differences in estimated WACCs. The red bar shows that less than 5% of analysts agree exactly about the required rate of return for a given firm in a given year. Furthermore, consistent with the large standard deviation, Figure 3 shows substantial variation in the estimated differences, including over 10% of the sample reflecting a spread between analysts' estimated costs of capital of at least 3 percentage points.

All in all, Figure 3 implies that two financial professionals covering the same firm at the same time, presumably with similar information sets, using the same text-book modeling approach to required rate of return (e.g., WACC), arrive at significantly different estimates for the discount rate in their valuations models, which can have large impacts on prices. Such differences are consistent with estimation uncertainty. That is, there are numerous ways to estimate the same model, all of which are reasonable, but theory provides little guidance.

For example, the beta horizon, return frequency, choice of risk-free rate horizon, choice of risk-free rate source (U.S., national, etc.) are all important input decisions analysts face when attempting to estimate the model for a firm's cost of capital.

To better understand the sources of the heterogeneity in estimated costs of capital, we next split the estimated WACCs into their respective parts, e.g., the risk-free rate and CAPM beta multiplied by the equity risk premium. We again construct the absolute value of the pairwise difference between two analysts covering the firm. Table 1 reports the means (medians) for $rf_{|A-B|,i,t}$ and $(CAPM\ Beta \times ERP)_{|A-B|,i,t}$ are 1.0% (0.8%) and 1.6% (1.1%), respectively. That is, the absolute magnitude of the mean spread in CAPM Beta multiplied by the equity risk premium is 60% larger than that of the risk-free rate, a difference that is significantly different zero at the 1% level.

Figure 4 displays the distributions of the absolute values of the pairwise differences in the risk-free rate and estimated $(CAPM\ Beta \times ERP)$. In contrast to the $WACC_{A-B,i,t}$ distribution in Figure 3, Panel A of Figure 4 shows that analysts agree exactly on the risk-free rate in nearly 10% of the cases. Moreover, nearly 40% of all pairwise differences in the risk-free rate estimates are smaller than 0.5%

Panel B suggests that the distribution of heterogeneity in CAPM Betas and the equity risk premium differs sharply from that of the risk-free rate. In particular, only 2% of the analysts in our sample exactly agree on both the CAPM Beta and the equity risk premium when covering the same firm in the same year. Moreover, the overall distribution $(CAPM\ Beta \times ERP)_{|A-B|,i,t}$ is flatter than that of the risk-free. Specifically, only around 20% of the observations are between 0.0% and 0.5% (vs. nearly 35% of such cases for the risk-free rate), while the decay is much less pronounced.

The upshot from Figures 3 and 4 is that while there is substantial heterogeneity in estimated required rates of return by analysts covering the same firm at the same time, there does seem to be components of the model that create more agreement amongst the analysts. In particular, analysts tend to use risk-free rate parameters that are significantly closer together in magnitude. These results are consistent with the argument that analysts face more acute ambiguity over parameters that must be estimated rather than those that can be observed easily with market data.

4.2 Estimated Cost of Equity Capital

The conceptual framework in Section 2 suggests that estimation ambiguity introduces more uncertainty for model inputs that require estimation (such as betas) than for the inputs that are observable in public data (such as the risk-free rate). The distributions of the pairwise differences for ($CAPM\ Beta \times ERP$) and rf presented in Section 4.1 provide suggestive support for this argument. However, in this section, we provide direct evidence on this prediction.

Our analysis exploits variation in estimation uncertainty across different model parameters for the same focal firm, while holding constant the estimation model. Our first test decomposes the discount rate heterogeneity between two analysts evaluating the same firm i at the same point in time t. Using the definition of WACC, we can express this decomposition as follows:

$$\underbrace{r^E_{A,i,t} - r^E_{B,i,t}}_{r^E_{A-B,i,t}} = \underbrace{rf_{A,i,t} - rf_{B,i,t}}_{rf_{A-B,i,t}} + \underbrace{\beta_{A,i,t} * ERP_{A,i,t} - \beta_{B,i,t} * ERP_{B,i,t}}_{[\beta * ERP]_{A-B,i,t}} \tag{3}$$

Where $r^E_{\cdot,i,t}$ denotes the analyst's estimate of the cost of equity, and the terms below the underbrace correspond to the notation shorthands to facilitate exposition. This decomposition allows us to determine whether changes in the cost of equity are driven by the differences in the estimated risk-free rate or the compensation for market risk. The order of subtraction, $r^E_{A,i,t} - r^E_{B,i,t}$ or $r^E_{B,i,t} - r^E_{A,i,t}$, is inconsequential for the decomposition results.

To capture the cost of equity differential attributable to each model input, we estimate the following equation:

$$var(r^E_{A-B,i,t}) = cov(rf_{A-B,i,t}, r^E_{A-B,i,t}) + cov([\beta * ERP]_{A-B,i,t}, r^E_{A-B,i,t}) \tag{4}$$

$$1 \approx \frac{cov(rf_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})} + \frac{cov([\beta * ERP]_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})} \tag{5}$$

By construction, the combined variation attributable to the risk-free rate and the compensation for market risk ($\beta \times ERP$) explains 100% of the heterogeneity in the estimated cost of equity. The two right-hand side terms in Equation (5) are estimated using a univariate linear regression of their respective right-hand side terms in Equation (3).

Table 3 shows the estimation results. Panel A corresponds to the full sample, while Panels B and C refer to the subsamples of domestic and international firms. Across all panels, columns 1-3 focus on the share of the heterogeneity in the cost of equity attributable to the choice of the risk-free rate. Columns 4-6 perform an equivalent decomposition for the compensation for market risk ($\beta \times ERP$).

The results in Panel A of Table 3 show that the dominant majority (79%) of the within-firm variation in the cost of equity is attributable to analyst dispersion in the compensation for risk ($\beta \times ERP$). The remaining one-fifth (21%) of the variation is attributable to the differences in the estimated risk free rate. This relationship remains robust as we gradually saturate specifications with fixed effects that absorb temporal variation in the cost of equity (year fixed effects) and the cross-firm variation (firm fixed effects). With the inclusion of both groups of fixed effects, the results are robust to exploiting only within-firm variation in analyst estimates derived in the same calendar year.

Panels B and C split the sample between U.S. and foreign firms. These splits are motivated by the variation in the choice set for the risk-free proxy between the domestic and international settings. While the selection of the risk-free rate in the U.S. is usually confined to U.S. Treasuries of different maturities, the risk-free asset for foreign firms is far less obvious. Common risk-free proxies in analyst reports for foreign firms include sovereign bonds issued by the government of the U.S, the firm's own country, or another country in the firm's region (e.g., Germany in the Eurozone). These choices generate additional cross-sectional variation in the risk-free rate.

The results in Panels B and C are consistent with the textual analysis of analyst reports. The choice of the risk-free proxy explains twice as much variation in the cost of equity for international firms (22.5%) than it does for U.S. firms (11.3%). Moreover, for U.S. firms, the risk-free rate dispersion is a statistically insignificant factor in explaining the discrepancies in analysts' estimated cost of equity capital.

In summary, most of the cross-analyst variation in a firm's cost of capital is attributable to parameters that require estimation—namely, beta and the market risk premium. The choice of the risk-free rate explains a modest share of the variation. The quantitative importance of the risk-free parameter increases in an international setting characterized by an expanded choice set of the risk-free proxies.

4.3 Textual Analysis of Analyst Methodologies

Given the large discrepancies in the final required rates of return, as well as the individual components of the WACC, our next step is to take a closer look at the text of the equity reports to better understand the subjective choices they make within the context of their models. In particular, we try to understand the exact implementation and parameterization of the CAPM betas amd the risk-free rates that analysts use.

4.3.1 CAPM Beta

First, we analyze analysts' discussion on the exact implementation of the CAPM Beta that they use. For this section we restrict the sample to include only the reports that discuss the specifics of their CAPM Beta. Table 1 Panel D reports the specifics of this sample, which includes coverage of almost 800 firms by over 500 analysts for a total of 1,023 observations

We focus our investigation on two components of the model analysts use to estimate a firm's CAPM beta: return frequency and return horizon. Both of these inputs display substantially more heterogeneity than the parameters in analysts' risk-free rates. For example, while Figure 5 shows that the most common approaches are to use weekly returns and a horizon of 60 months in beta estimations, almost 50% of observations do not use these two implementations. Specifically, Panel A highlights that around 40% of analysts use monthly returns, and Panel B suggests around 30% use a 24 month horizon, which are the next most common return frequency and horizon, respectively.

It is worth noting that Panel B of Figure 5 indicates that analysts appear to focus on annual anchors when determining their favored horizon Tversky and Kahneman (1974). That is, none of the equity reports in our sample choose a CAPM Beta horizon of 18 or 30 months, for example. In principle, theory does not require financial professionals to determine the horizon at the annual frequency. Rather, this appears to be the result of implicit heuristics that permeate professionals, as well as academics, and simply works to reduce the dimensionality of the problem at hand into a smaller set of more manageable options.

Despite the total heterogeneity in CAPM Beta horizons employed across analysts, there is strikingly little variation in these same horizons within individual analysts in the time-series or even in the cross-section of firms covered by the same analyst in the same year. Put differently, individual analysts seem to routinely use the exact same beta horizon strategy each time they estimate a covered firm's required rate of return. For example, Figure 6 shows the frequency in which individual analysts in our sample use one, two, or three separate CAPM beta horizons.

Panel A includes all analysts that we observe at least two reports for. In 76% of the cases, analysts use a single horizon for every report we observe. An additional 5% of analysts use the same averaging strategy each time, for example, equally weighting the 24 and 60 month horizon.s Importantly, these statistics are not a function of how many reports we observe for each indvidual, or the distribution of these reports across firms or time. For example, Panel B restricts the sample to only analysts in wich we observe at least 4 separate equity reports, and Panel C displays the same distributions, but limits to analysts that cover multiple firms over multiple years. In all three panels, around 80% of individual agents employ a single horizon strategy with respect to estimate the CAPM beta. This is consistent with cautious behavior in the face of estimation uncertainty.

4.3.2 Risk-Free Rate

Next, we focus on the risk-free rate. Theory postulates that this parameter should capture the rate an investor could earn from lending and pay for borrowing in the risk-free asset (e.g., see Sharpe, 1964; Lintner, 1965). Theory is largely silent, however, on how its practical implementation.

This means Sharpe's (1964) and Lintner's (1965) framework offers no direction on the most basic detail in implementing the risk-free rate in the CAPM formula: the empirical benchmark on which to base the rate itself. Thus, financial professions are faced with a litany of potential (low-default risk) rates they could possibly use in their estimation, from U.S. Treasury rates, to LIBOR, to country- or region-specific treasury rates. At the same time, unlike CAPM betas, risk-free rates do not need to be estimated. Rather, they can simply be observed in market data. This means that the ambiguity analysts face could be less acute for the risk-free rate.

We use a sample of equity reports discussing the specifics of the risk-free rate used in the implementation of the CAPM forumla. Table 1 Panel C reports the details of this sample of reports. Overall, we see the specifics of the risk-free rate parameter for almost 1,700 firms and just over 3,200 firm-year observations.

Figure 7 displays the distributions of both different benchmark rates, as well as the horizon of the rate that analyst employ. Panel A provides the breakdown of risk-free benchmark rates by category: domestic, regional, and U.S. benchmarket, where the domestic rate is the treasury rate for the country of the firm's headquarters, the regional rate is the treasury rate for one of the countries located in the same continent as the firm's headquarters, and the U.S. benchmark rate is a U.S. treasury security.

Panel A suggests that nearly 70% of the nearly 3,300 observations in this sample use the domestic treasury. This number increases to over 75% that implement a regional rate. There are, perhaps unsurprisingly, large differences between US headquartered firms and those headquartered elsewhere. For example, analysts nearly unanimously use a U.S. Treasury benchmark rate for their risk-free rate. The same is not true for international firms. It is, of course, important to note that there is significantly more heterogeneity in the *choice* of risk-free rate proxies for non-US firms than there is for US firms. Consistent with this, Figure 7 Panel A shows that analysts sometimes use a domestic or regional rate, but also at times, choose to use the U.S. benchmark rate as the risk-free parameter in their models.

According to the discussions in equity reports, this heterogeneity in risk-free rate arises due to challenges determining whether the treasuries of smaller countries properly capture the properties that models want when determining a risk-free rate proxy. For example, theoretically, a security should be liquid enough such that all investors can properly lend and borrow to maximize their personal return with respect to their risk aversion through the combination of the risk-free asset and the tangency portfolio. It is arguable that some country-level treasuries do not provide such liquidity and analysts optimally choose a different risk-free rate. However, once again, theory provides no guidance for the rate an analyst should then use instead, leading to significant ambiguity regarding this choice.

Even when analysts agree on the broad benchmark (e.g., those following U.S. firms using a U.S. Treasury), there is an additional parameter set for the analysts to consider: horizon of the security. Panel B of Figure 7 shows the distribution of the risk-free rate horizons chosen by analysts. Even for internationally headquarter firms, there is significantly less heterogeneity in the horizon of the rate used. For example, 2,951 of the 3,284 (90%) of all the equity reports in this sample report using a horizon of 10 years. This includes 80% of international firms. These results suggest a higher degree of estimation ambiguity over CAPM beta estimation than the risk-free.

Overall, the results in Section 4.3 suggests three things about the estimation of firms' required rates of return in our sample. First, while analysts largely agree on the choice of model to perform the analysis, they disagree over how to estimate its inputs. Second, the degree of heterogeneity is much more substantial for parameters analysts need to estimate (e.g., CAPM Beta) relative to those in which they can observe a reasonable benchmark (e.g., risk-free rate). Finally, while different agents seem to disagree about how to exactly estimate the models, individual agents tend to pick one strategy and apply it broadly over each estimation. Given this, better understanding exactly how each analyst chooses such a strategy is extremely important.

The existing literature discussing how ambiguity-averse agents address ambiguous situations have mostly focused on robust strategies (Tsoy, 2023), where agents end up evaluating the situation using worst-case justifiable models (Gilboa and Schmeidler, 1989). However, such an approach to deal with ambiguous situations implies that agents have the ability to systematically identify a worst-case model. In our setting, this may equate to an analyst selecting a trailing horizon for the beta estimation that would yield the most liberal (conservative) beta such that the analyst never overestimates (underestimates) a firm's cash flow. Such an exercise seems to be non-trivial. For example, Figure 8 shows the cross-sectional distribution of firms for which each beta horizon is the most conservative (Panel A) and the most liberal (Panel A) for each month of our sample. Though the 24-month and 72-month horizons make up the highest percentage of most conservative strategies, they also make up the highest percentage of most liberal strategies. Figure 8 suggests that agents could not simply apply a single horizon as a worst-case justifiable model across the firms they cover.

Moreover, even for an agent covering a single firm through time, it isn't clear that a single horizon could be applied in a worst-cast justifiable way. Table 5 shows the annual autocorrelations for both the most conservative (Panel A) and the most liberal (Panel B) strategies. In particular, Table 5 reports that no single CAPM beta horizon would provide a worst-case strategy year-over-year, as the highest autocorrelation among the various horizons is 0.249 for the 72-month liberal strategy. In fact, the majority of horizons have a negative autocorrelation, suggesting that the most conservative or liberal strategy one year, is significantly less likely to be the same strategy the following year. Our evidence suggesting that analysts apply a single beta horizon across firms and years is entirely inconsistent with such a solution to estimation ambiguity.

Alternatively, decision theory offers strategies to optimally aggregate the recommendation of various models' outcomes when agents face ambiguous situation. While a rudimentary version of such approach is used by some of the professional in our setting (3-4%), most favor estimating the model in a single way.

Finally, Heath and Tversky (1991) and Fox and Tversky (1995) provide a fruitful avenue to think about why and how professionals address ambiguity in our setting, by considering its source. When ambiguity arises from the lack of familiarity or expertise about a particular situation, restricting the set of scenarios considered to those where agents are more familiar can be a first step to help agents mitigate the effect of ambiguiy. This might help rationalize why 3 particular horizons, 2, 3, and 5-year, are used as anchors in 85% of cases. Those horizons appear to be reasonable heuristics, as they are sufficiently long to be estimated with relative precision, while being short enough to ensure that stale information is unlikely to entirely contaminate the estimates. Combined with the fact that the choice of which benchmark to use when estimating the model appears to be driven by individual idiosyncrasies, this suggests that professionals' approach to resolving ambiguity is akin to a coin toss.[3] To the extent that these decisions reverberate into broader market disagreement, as we document in the latter part of the paper, this conclusion appears concerning.

[3] Raiffa (1961) critic, such that when agents face ambiguity and existing strategies are hardly applicable (e.g., Gilboa and Schmeidler (1989) or Giacomini et al. (2019)), the decision to select a particular method

4.4 What Impacts Analysts' Choice of CAPM Beta?

The prior sections establish that financial professionals arrive at different required rates of return for the same firm in the same year, even when the analysts apply the exact same theoretical framework in the estimation of these discount rates. Moreover, Section 4.2 suggests that these differences are driven mostly by variables that require estimation (e.g., the CAPM beta), rather than parameters analysts can simply observe with standard data services (e.g., risk-free rate proxies like U.S. Treasury securities). However, understanding what explains such variation in choices remains particularly elusive in the literature. For example, are idiosyncratic modeling decisions driven by people working in different brokerage houses that have default different standards, are they due to personal traits or characteristics that correlate with important baseline financial metrics such as risk aversion, or do any observed differences simply reflect personal preferences or arbitrary choices over the various parameter and methodology heuristic set?

4.4.1 Brokerage House and Individual Analyst Effects

We start the analysis of this at a very high level, as we first want to understand what is most important in explaining heterogeneity in the estimated CAPM betas analysts report in their equity valuations. Specifically, we perform a variance decomposition (ANOVA) on the level of CAPM betas using three cross-sectional dependent variables: analyst, brokerage house, and firm indicators. The firm indicators account for differences in betas across firms due to their differential exposure to systematic factors.

Table 6 Panel A reports the results of this decomposition. The entire model fit is strong, with an R^2 of 78% and an adjusted R^2 of 57%. Moreover, as expected, the firm indicators account for the majority (78%) of the variation explained by the model. Perhaps unsurprisingly, the brokerage house indicators explain almost none of the variation in analysts' CAPM beta. For example, even though the percentage of model sum of squares is 2%, the adjusted partial R^2 associated with the brokerage house indicators is just 0.01.

On the other hand, the individual analyst indicators account for nearly 20% of the variation explained by the model and have an adjusted partial R^2 of 28%. Together, the results in Panel A suggest that the largest portion of variation in the estimated CAPM betas we observe in equity research reports is attributable to the individual agent publishing the report.

The results in Table 6 Panel A must be interpreted with a degree of caution. The order of inclusion of the cross-sectional indicator variables in our models can change the exact estimation results. We do two things to ensure the impact of this ordering is minimal. First, we estimate the sum of squares sequentially (Smith and Cribbie, 2014). Second, we include the brokerage house indicators as the second variable in our model after the firm indicators. This means, if anything, we are *overestimating* the explanatory power of the brokerage indicators relative to the analyst indicators.

A second potential concern with the results in Panel A is the vast number of degrees of freedom for the firm and analyst indicators relative to the brokerage indicators. That is, the number of firms and analysts included in the model is several orders of magnitude larger than the number of brokerage houses. This works to mechanically increase the portion of model-explained variation that we attribute to firms and analysts relative to brokerage houses.

Though the adjusted partial R^2s in Column (3) ease this concern, to further probe the sensitivity of the baseline ANOVA decomposition, we perform two additional ANOVA decompositions in Panels B and C that restrict the sample. Panel B of Table 6 presents the results of the same decomposition where we limit the sample to only analysts in which we observe at least 5 reports, while Panel C presents the results where we limit the sample to only analysts in which we observe at least 5 reports, and firms that have at least 5 total reports as well. The results in Panels B and C provide very similar qualitatively results. For example, the adjusted partial R^2 numbers for the firm indicators remain between 30-40% across each specification, while those for the analyst indicators remain between 20-30%. Furthermore, the adjusted partial R^2 for brokerage houses never exceeds 1%, implying that the exact brokerage house that employs the analyst is not an important determinant in the estimate of CAPM beta. This is potentially unexpected, as it would have matched the priors of many academics if estimation procedures such as CAPM were set at the brokerage house level, such that all analyst at a given contributor estimated betas the way their predecessor did, and their predecessor estimated betas the way theirs did and so on and so forth. However, Table 6 strongly rejects such a practical implementation of estimation procedures at the brokerage house level.

4.4.2 Individual Analyst Personal Characteristics

The next step is to better understand what, if any, individual analyst characteristics drive the variation in observed CAPM betas. To do this, we fuzzy match by the names of the lead analysts identified in each equity report published in our sample with a large social networking database that identifies several important personal characteristics. For example, this merge allows us to cleanly observe the age, gender, race, and education level for a subsample of the analysts in our sample. Moreover, this data facilitates a location mapping for the analysts by the country code of the phone number listed in the equity report.

To extract the portion of observed CAPM betas that is attributable to the individual analysts, we estimate the following fixed effect regression model:

$$CAPM\ Beta_{a,i,t} = \alpha + \beta_a + \gamma_i + \delta_b \tag{6}$$

where a references an individual analyst, i references a firm, and b references a brokerage hosue. After estimating this regression, we save the estimated coefficients $\hat{\beta}_a$, a strategy that mimics Bertrand and Schoar (2003). These coefficients represent an individual's unobserved impact on the CAPM Beta estimation, conditional on firm and brokerage house fixed effects.

We use these estimated coefficients as the dependent variable in our next level of variance decompositions. That is, we again perform an ANOVA decomposition, in which we estimate the model with a cross-section of the following personal characteristics: a gender indicator, race indicators, an indicator for a graduate degree, and analyst country indicators. Because we estimated the anlayst fixed effect coefficients across firms and time, our data is collapsed to the individual level.

Table 7 Panel A displays the results of this decomposition. After merging personal characteristics and collapse the data, we are left with only 154 analyst observations. However, this sample of analysts appears to be representative of the full sample of analysts, insofar as the estimated WACCs of this subsample are statistically identical to those of the analyst in our full sample.

Panel A suggests the overall fit of the model is quite poor. That is, the R^2 is rather low at only 9%. Moreover, the adjusted R^2 of the model is 0%, suggesting that beyond the mechanical effect of the categorical variables, none of the variation in the analyst-level fixed effects is explained by the model. Furthermore, each individual variable adds nothing in terms of adjusted partial R^2, which range between -2% and 2%.

Table 7 Panel B adds an indicator variable for young, which is equal to one if an analyst's age is below the sample median at the time his or her report is published. We observe the birth year of individual analysts less frequently than other personal characteristics, which drops our sample to only 96 analysts. However, the results in Panel B are nearly identical to those in Panel A, both qualitatively and quantitatively. The overall model fit erodes a bit in terms of adjusted R^2, but the partial adjusted R^2s for the individual components remains close to zero for each predictor.

All in all, Table 7 suggests that personal characteristics play nearly no role in the level of analysts' estimated CAPM beta. That is, there are no statistical or economically significant differences in CAPM betas between analysts with and without a masters degree, for example. Though the results on the estimated level of CAPM are strongly suggestive that analysts from different backgrounds and with different characteristics do not systematically take different approaches in their estimation procedure, we go one level deeper and analyze the impact of these same personal characteristics on analysts' choices for the CAPM beta horizon.

Figure 9 replicates the CAPM horizon distribution from Figure 5 Panel B, but separates by analyst gender, race, education and region, as well as by firm industry. For example, Panel A of Figure 9 differentially compares the distribution of chosen CAPM beta horizons for females (blue bars) males (red bars). The figures also include 90% standard error bars. There are, of course, some observed differences between the distributions for males and females. For example, more than 40% of females use the 60 month horizon, while the same number for males is less than 30%, a difference that is statistically significant for that particular distribution. However, the overall distribution for *all* the beta horizons for males and females look strikingly similar.

This pattern holds across the other personal characteristic splits in Panels B through E. For example, Panel B suggests there are no significant differences across any horizon between those analysts that pursued graduate education and those that did not. Moreover, even the analysts' country (Panel D) show very small differences in the distribution of the chosen horizons. Thus, in sum, Figure 9 implies that personal characteristics and measurable individual differences across analysts play little role in the estimation procedure for the financial professionals in our sample.

We are careful to note that there are a few significant differences across personal characteristics when analyzing CAPM beta horizons one by one. For example, as we mentioned above, the percentage of females using the 60 month horizon differs in a statistically significant way from the percentage of males. To explore this more deeply and more precisely, Table 8 reports the results of a multinomial logit regression across the three most populated horizons in our sample: the twenty-four month, thirty-six month, and the sixty month horizons.

Columns (1) and (2) report the impact of personal characteristics such as analyst gender, race, education and region, as well as the industry of the firm being covered. The coefficients are relative to the base horizon (twenty-four month horizon). Once again, for single horizons, there are a few coefficients that are significantly different than zero (e.g., gender and race for the thirty-six month horizon beta relative to the twenty-four month horizon). However, Column (3) reports the tests of equality across *all* horizons that an analyst could choose. In each case, the coefficients are not significantly different than zero. Column (3) provides the strongest evidence that the distributions of chosen CAPM beta horizons do not significantly depend on the personal characteristics of the analyst.

4.4.3 Does Estimation Ambiguity Matter for Real Outcomes?

In the final section, we study the relationship between estimation ambiguity and stock market outcomes. Establishing this link at the micro-level is challenging because it requires observing an agent's model, its input parameters, and estimation outcomes, and such a combination is rarely feasible outside of a controlled experiment (Asparouhova et al., 2015). Our empirical setting allows us to meet these conditions for an important set of financial experts at top financial institutions who are tasked with information discovery for other market participants.

Our analysis is rooted in a large theoretical literature that predicts a positive link between the heterogeneity in agents' private valuations and their trading volume in secondary markets. Theory postulates that investors trade securities primarily because they have different private valuations ((Milgrom and Stokey, 1982; Karpoff, 1986; Banerjee and Kremer, 2010). Consistent with these predictions, empirical work finds a surprisingly high amount of trading for U.S. stocks, given the relative information transparency in the U.S. market and a modest amount of portfolio rebalancing. In a survey of this literature, Hong and Stein (2007) conclude that the bulk of trading must come from differences in investors' valuation models "that lead traders to disagree about the value of a stock even when they have access to the same information sets" (p. 112). Since nearly all analysts use the same estimation model for the cost of equity, and since its inputs are based on market data, we have a convenient setting to test these predictions within the same modeling framework and a common information set. To execute this analysis, we introduce a measure of a stock's scaled trading volume, $FVOL$, defined as the total number of shares traded in a month scaled by the total number of shares outstanding, following prior work (Ajinkya et al. (1991). The independent variable of interest in this analysis is a measure of discount rate heterogeneity. Because our analysis is conducted at the firm-year level, this measure is defined as the spread between the maximum and minimum estimates of WACC by all analysts covering a firm in a given year with available WACC data.

The discount rate heterogeneity corresponds to $\max_{a \in A}(WACC_{i,t_1,T}) - \min_{a \in A}(WACC_{i,t_1,T})$, where a represents an analyst in the entire set of analysts A for a given firm-year observation, i indexes firm, and times t is the month when the second forecast is published (e.g., the time when the disagreement was created). Our results are robust to other measures of the discount rate dispersion, such as the standard deviation.

Table 9 Panel A shows the results using total trading volume scaled by shares ouststanding. Column 1 reports the estimates from a regression that includes no control variables. The evidence indicates a positive and significant association between the WACC dispersion and trading volume, and this result is significant at 1%. To account for temporal persistence in a stock's trading volume, Column 2 adds controls for the lagged trading volume, following (Cookson and Niessner, 2020). The positive coefficient on the WACC dispersion shrinks in magnitude but remains significant at 1%. The R^2 jumps from 1% to 80%, suggesting that last month's trading volume explains the vast majority of the variation in this month's volume.

Columns 3 and 4 add year-month and firm fixed effects, respectively. The R^2 in column 4 with firm fixed effects is 88%, and the coefficient on the variable of interest is 0.154, still significant at the 1% level. Column 5 adds additional control variables to capture the common drivers of trading volume identified in prior work (see the discussion in Cookson and Niessner, 2020). These controls include the natural logarithm of the firm's market capitalization, the cumulative return and return volatility over the three trailing months, the cumulative return from 4 to 12 months prior, and finally, the number of months between the first and last analyst forecasts. After including these controls, the coefficient on WACC heterogeneity is 0.120, and it remains significant at 1%.

Column 6 shows that the relationship between the discount rate dispersion and trading volume is robust to controlling for disagreement in growth expectations. As a proxy for disagreement in long-run growth rates, we use an analogously-constructed spread between the terminal growth rates across analysts covering the same stock.

Panel B in Table 9 shows the results using an alternative measure of trading volume akin to Cookson and Niessner (2020). In particular, *Abnormal FVOL* is equal to the monthly trading volume minus the mean monthly trading volume over the previous twelve months, or one year of trading volume, scaled by the the total common shares outstanding in month t. The results in Panel B are qualitatively and quantitatively similar to those in Panel A. The economic magnitudes in both Panels A and B are both significant. In particular, a one-standard deviation increase in WACC disagreement is associated with a 1-2% increase in total trading volume, while the same increase in WACC disagreement is associated with a 3-4% increase in abnormal trading volume. While smaller in absolute terms than the results in Cookson and Niessner (2020), their results are at the daily level. Thus, our results at the monthly level are the same order of magnitude.

Finally, Table IA1 tests the external validity of the discount rate estimates derived in analyst reports. This table evaluates the assumption that the dispersion in the analysts' discount rates captures the inherent ambiguity in measuring a firm's cost of capital outside of the analyst sample—that is, the dispersion that arises in an econometrician's estimates of the cost of capital based on market data. In this column, a firm's dispersion in WACC is measured as the spread between the largest and smallest econometrician betas estimated in the month the second analyst forecast is published. The econometrician betas are based on commonly-prescribed methodological choices in model estimation: trailing estimation windows of 24, 36, 48, 60, and 72 months, the 10-year Treasury rate for the risk-free rate, and the S&P 500 as the market proxy. Using this combination of parameters, the independent variable is defined asMax($CAPM\ Beta_{E,i,t_1}$)− Min($CAPM\ Beta_{E,i,t_1}$), where the set of econometrician betas (E) for which we estimate the maximum and minimum values cover different trailing estimation windows, such as 24, 36, and 48 months.

Table IA1 reports the results for our sample firms in 2000-2023. The dependent variable is a firm's total trading volume scaled by shares outstanding, measured at monthly frequency, and the independent variable is a measure of dispersion in an econometrician's betas. The evidence shows that the spread between the maximum and minimum econometrician betas is positively related to a firm's trading volume, and this relationship is statistically significant at 1%. The point estimate of 0.004 in Column 5 suggests an economically meaningful effect. According to this point estimate, a one standard deviation increase in the spread between the largest and smallest econometrician betas is associated with an increase in a stock's trading volume of 2.8%.

In summary, cross-analyst dispersion in a firm's cost of capital is positively associated with trading volume, consistent with models of investor disagreement. This relationship holds with alternative measures of parameter dispersion estimated independently of analyst forecasts.

5 Conclusion

This paper has studied how finance professionals deal with estimation uncertainty when calculating a firm's required rate of return. When confronted with an array of feasible estimation methods, analysts appear to adopt one empirical model and adhere to a consistent set of estimation parameters throughout their careers. Such persistence in methodological choices generates large cross-analyst dispersion in the estimated discount rate for the same stock and correlates with market-based measures of investor disagreement.

While we use securities valuation as a convenient laboratory to study the inner workings of agents' modeling choices, the concept of estimation ambiguity extends beyond financial economics. Since many economic decisions require model estimation, they routinely confront agents with similar methodological challenges, such as selecting the appropriate empirical model, choosing parameter values, and adapting their choices in response to the arrival of new information or new estimation tasks.

While our paper makes a step towards understanding the micro-foundations of decision-making under estimation uncertainty, it leaves many open questions. One of the lingering questions deals with the factors that lead agents with similar backgrounds to adopt different estimation methodologies, ranging from private preferences to the role of formative experiences, such as mentorship, on-the-job training, or academic coursework. We hope that the growing interest in agents' decision-making under estimation uncertainty will continue to yield novel insights on this topic.

References

Ajinkya, B. B., R. K. Atiase, and M. J. Gift. 1991. Volume of Trading and the Dispersion in Financial Analysts' Earnings Forecasts. *The Accounting Review* 66:389–401.

Andrei, D., B. Carlin, and M. Hasler. 2019. Asset Pricing with Disagreement and Uncertainty about the Length of Business Cycles. *Management Science* 65:2900–2923.

Asparouhova, E., P. Bossaerts, J. Eguia, and W. Zame. 2015. Asset Pricing and Asymmetric Reasoning. *Journal of Political Economy* 123:66–122.

Asquith, P., M. B. Mikhail, and A. S. Au. 2005. Information Content of Equity Analyst Reports. *Journal of Financial Economics* 75:245–282.

Banerjee, S., and I. Kremer. 2010. Disagreement and Learning: Dynamic Patterns of Trade. *Journal of Finance* 65:1269–1302.

Bertrand, M., and A. Schoar. 2003. Managing with Style: The Effect of Managers on Firm Policies. *Quarterly Journal of Economics* 118:1169–1208.

Bewley, T. F. 2002. Knightian decision theory. Part I. *Decisions in Economics and Finance* 25:79–110.

Cookson, J. A., and M. Niessner. 2020. Why Don't We Agree? Evidence from a Social Network of Investors. *Journal of Finance* 75:173–228.

Décaire, P. H. 2023. Capital budgeting and Idiosyncratic Risk. Working paper, Arizona State University.

Décaire, P. H., and H. Bessembinder. 2021. Discount Rate Uncertainty and Capital Investment. Working paper, Arizona State University.

Décaire, P. H., and J. R. Graham. 2023. Valuation Fundamentals. Working paper, Arizona State University.

Dessaint, O., C. Otto, and D. Thesmar. 2021. CAPM-Based Company (Mis)valuations. *Review of Financial Studies* .

Fox, C. R., and A. Tversky. 1995. Ambiguity Aversion and Comparative Ignorance. *The Quarterly Journal of Economics* 110:585–603.

Giacomini, R., T. Kitagawa, and H. Uhlig. 2019. Estimation Under Ambiguity. *Working paper* .

Giacomini, R., T. Kitagawa, and A. Volpicella. 2022. Uncertain identification. *Quantitative Economics* 13:95–123.

Giglio, S., M. Maggiori, J. Stroebel, and S. Utkus. 2021. Five facts about beliefs and portfolios. *American Economic Review* 111:1481–1522.

Gilardi, F., M. Alizadeh, and M. Kubli. 2023. ChatGPT Outperforms Crowd-Workers for Text-Annotation Tasks. *Proceedings of the National Academy of Sciences* 120:1–3.

Gilboa, I., and D. Schmeidler. 1989. Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics* 18:141–153.

Gormsen, N. J., and K. Huber. 2023. Corporate Discount Rates. Working Paper 31329, National Bureau of Economic Research.

Graham, J. R., and C. Harvey. 2001. The Theory and Practice of Corporate Finance: Evidence from the Field. *Journal of Financial Economics* 60:187–243.

Hansen, L. P., and T. J. Sargen. 2001. Robust control and model uncertainty. *American Economic Review* 91:60–66.

Hashim, N. A., and N. C. Strong. 2018. Do Analysts' Cash Flow Forecasts Improve Their Target Price Accuracy? *Contemporary Accounting Research* 35:1816–1842.

Heath, C., and A. Tversky. 1991. Preference and Belief: Ambiguity and Competence in Choice under Uncertainty. *Journal of Risk and Uncertainty* 4:5–28.

Hong, H., and J. C. Stein. 2007. Disagreement and the Stock Market. *Journal of Economic Perspectives* 21:109–128.

Huang, S., H. Tan, X. Wang, and C. Yu. 2023. Valuation Uncertainty and Analysts' Use of DCF Models. *Review of Accounting Studies* 28:827–861.

Huntington-Klein, N., A. Arenas, E. Beam, M. Bertoni, J. R. Bloem, P. Burli, N. Chen, P. Grieco, G. Ekpe, T. Pugatch, M. Saavedra, , and Y. Stopnitzky. 2021. The influence of hidden researcher decisions in applied microeconomics. *Economic Inquiry* 59:944–960.

Kahneman, D., O. Sibony, and C. R. Sunstein. 2021. *Noise: A Flaw in Human Judgment*. William Collins, 1 London Bridge Street, London.

Karpoff, J. M. 1986. A Theory of Trading Volume. *Journal of Finance* 41:1069–1087.

Keynes, J. M. 1921. *A Treatise on Probability*. Macmillan.

Kruger, P., A. Landier, and D. Thesmar. 2015. The WACC fallacy: The real effects of using a unique discount rate. *Journal of Finance* 70:1253–1285.

Lintner, J. 1965. The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics* 47:13–37.

Meeuwis, M., J. A. Parker, A. Schoar, and D. I. Simester. 2022. Belief Disagreement and Portfolio Choice. *Journal of Finance* 77:3191–3247.

Menkveld, A. J., A. Dreber, F. Holzmeister, J. Huber, M. Johannesson, M. Kirchler, S. Neusüss, M. Razen, and U. Weitzel. 2023. Non-Standard Errors. *Journal of Finance*, Forthcoming.

Milgrom, P., and N. Stokey. 1982. Information, Trade and Common Knowledge. *Journal of Economic Theory* 26:17–27.

Mitton, T. 2022. Methodological Variation in Empirical Corporate Finance. *Review of Financial Studies* 35:527–575.

Raiffa, H. 1961. Risk Ambiguity and the Savage Axioms: Comment. *Quarterly Journal of Economics* 69:690–94.

Sadka, R., and A. Scherbina. 2007. Analyst Disagreement, Mispricing, and Liquidity. *The Journal of Finance* 62:2367–2403.

Sharpe, W. F. 1964. Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. *Journal of Finance* 19:425–442.

Shiller, R. J. 2014. Speculative Asset Prices. *American Economic Review* 104:1486–1517.

Smith, C. E., and R. Cribbie. 2014. Factorial ANOVA with Unbalanced Data: A Fresh Look at the Types of Sums of Squares. *Journal of Data Science* 12:385–403.

Tsoy, A. M. A. 2023. Asymmetric Information and Security Design under Knightian Uncertainty. *Journal of Finance (forthcoming)* .

Tversky, A., and D. Kahneman. 1974. Judgment under Uncertainty: Heuristics and Biases. *Science* 185:1124–1131.

Figure 1: Spread in Econometrician Estimated CAPM Betas Across Horizons. This figure displays the average spread between the maximum and minimum econometrician estimated CAPM betas across different horizon estimation windows (e.g., 24 month, 36 month,..., 72 month) in universe of CRSP firms from 1932 through 2022. The betas are estimated using the CAPM with monthly returns where the market factor is from Ken French's website. The gray bars depict NBER recessions.

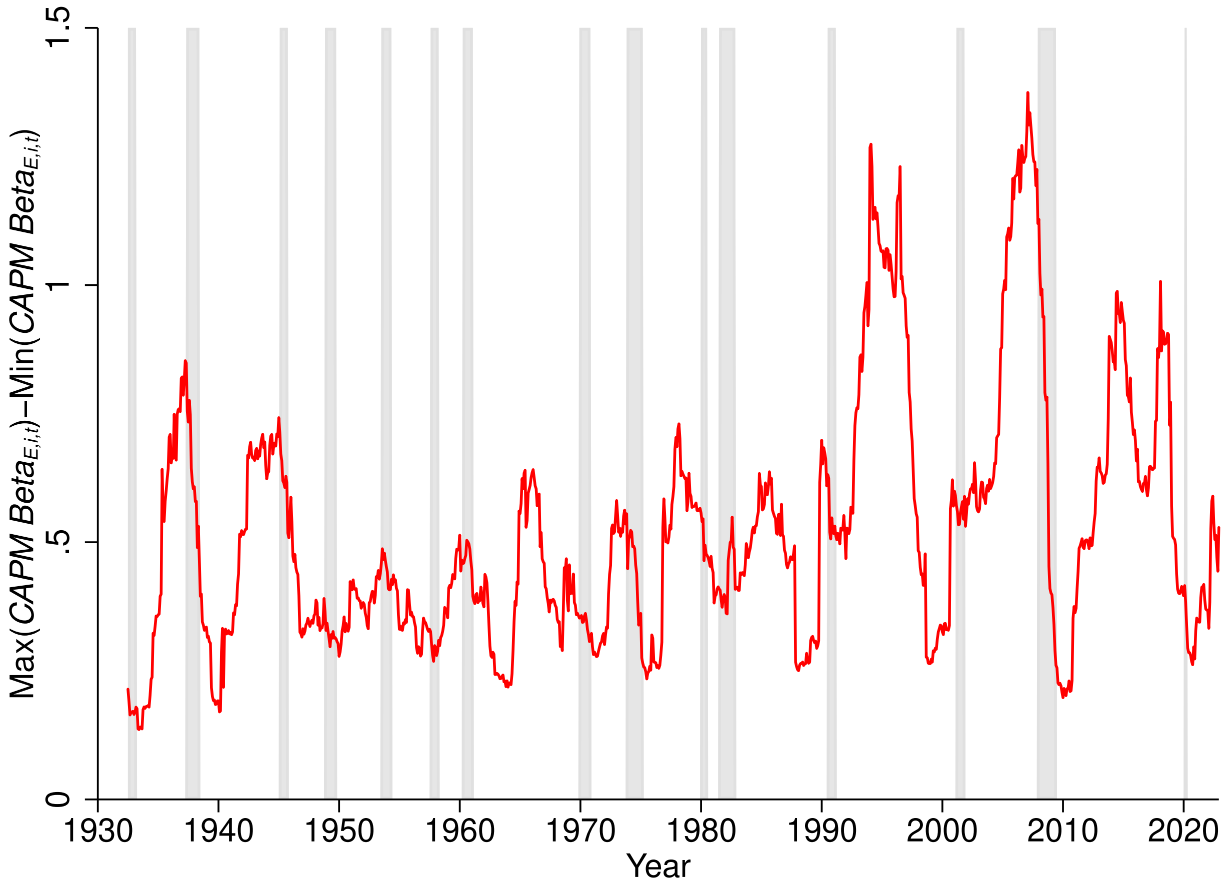


Figure 2: CAPM Beta Horizon Selection This figure highlights the ambiguity that analysts face when selecting the horizon of the estimation window in the CAPM formula. Analysts know that the true beta of the firm can be estimated with one, or a combination of several of the horizons, however, they do not know the weights to apply (e.g., P(h=1), P(h=2), and so on).

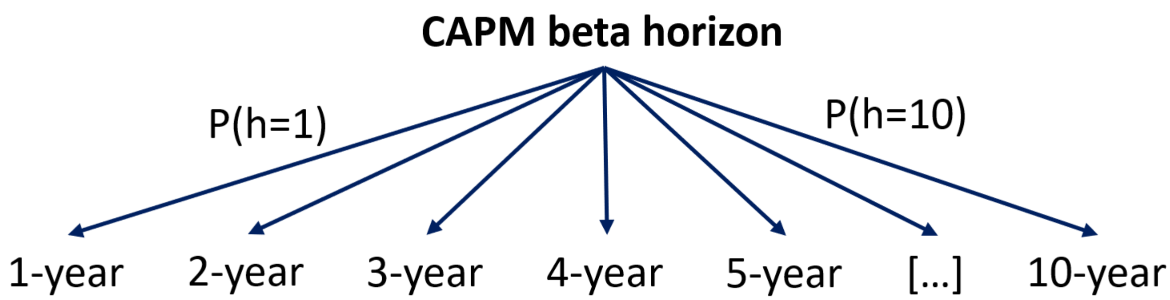


Figure 3: Heterogeneity in Analysts' Weighted Average Cost of Capital Estimates. This figure displays the cross-sectional distribution of the absolute difference between two analysts' weighted average cost of capital (WACC) estimates for the same firm at the same point in time ($WACC_{|A-B|,i,t}$). The sample period is 2000 through 2023. Data on individual analysts' estimates of discount rate are hand-collected from sell-side analyst equity research reports.

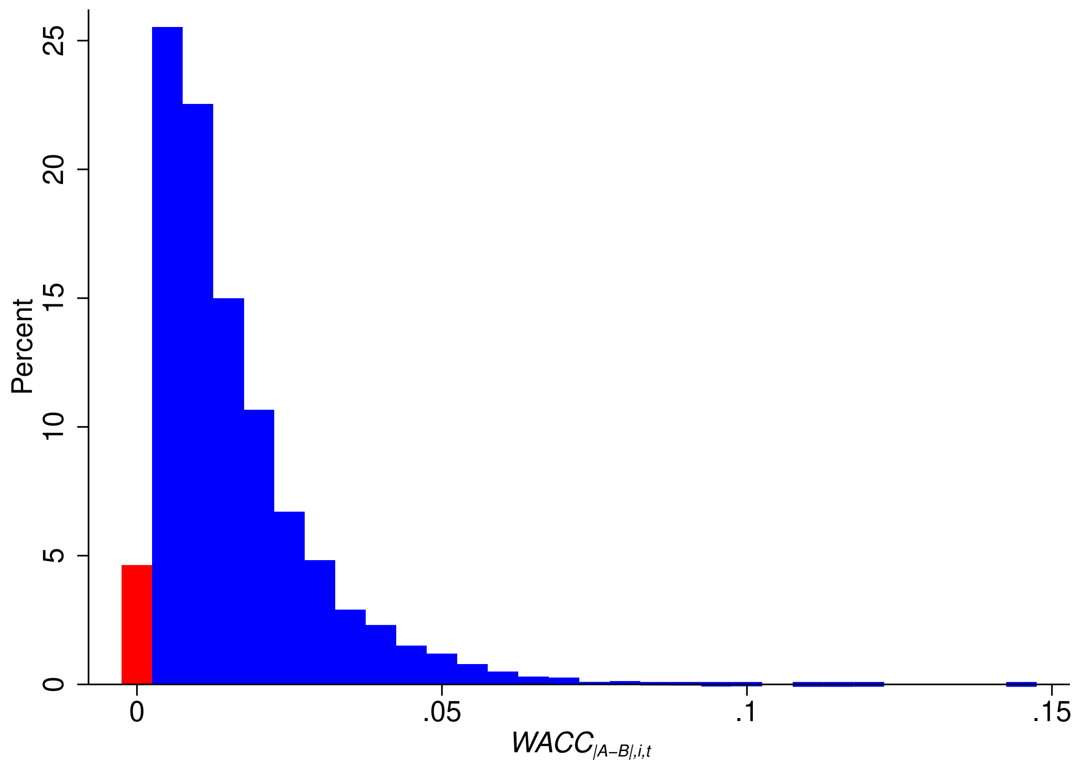
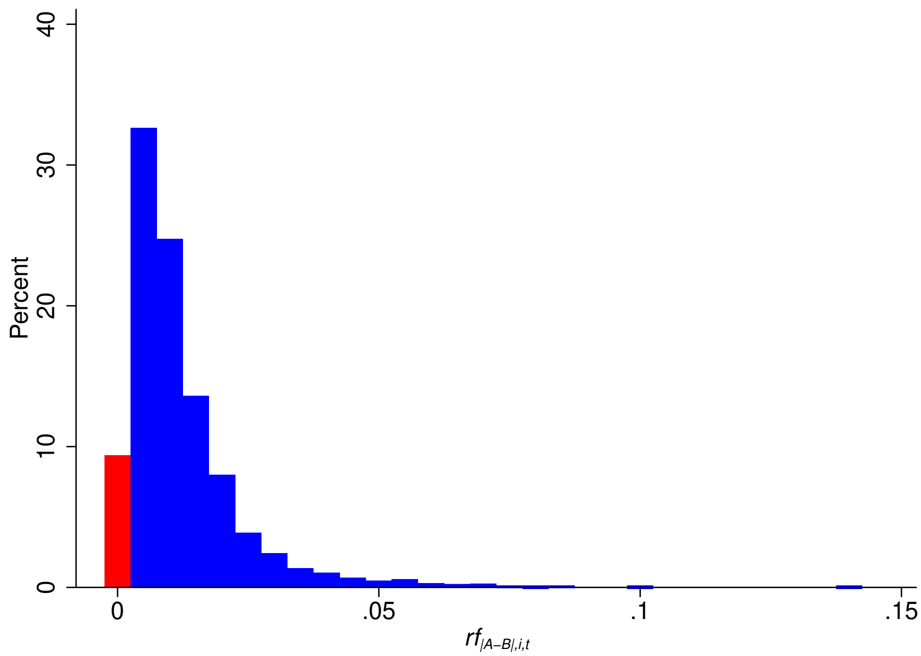


Figure 4: Heterogeneity in Analysts' CAPM Parameter Estimates. These figures display the cross-sectional distributions of the absolute differences between two analysts' CAPM input estimates for the same firm at the same point in time. In particular, Panel A focuses on the absolute difference in analysts' risk-free rate estimates ($rf_{|A-B|,i,t}$), and Panel B focuses on the absolute difference in analysts' CAPM beta multiplied by the equity risk premium ($(CAPM\ Beta \times ERP)_{|A-B|,i,t}$). The sample period is 2000 through 2023. Data on individual analysts' estimates of the CAPM inputs are hand-collected from sell-side analyst equity research reports.

(A) Risk-Free Rate



(B) CAPM Beta × Equity Risk Premium

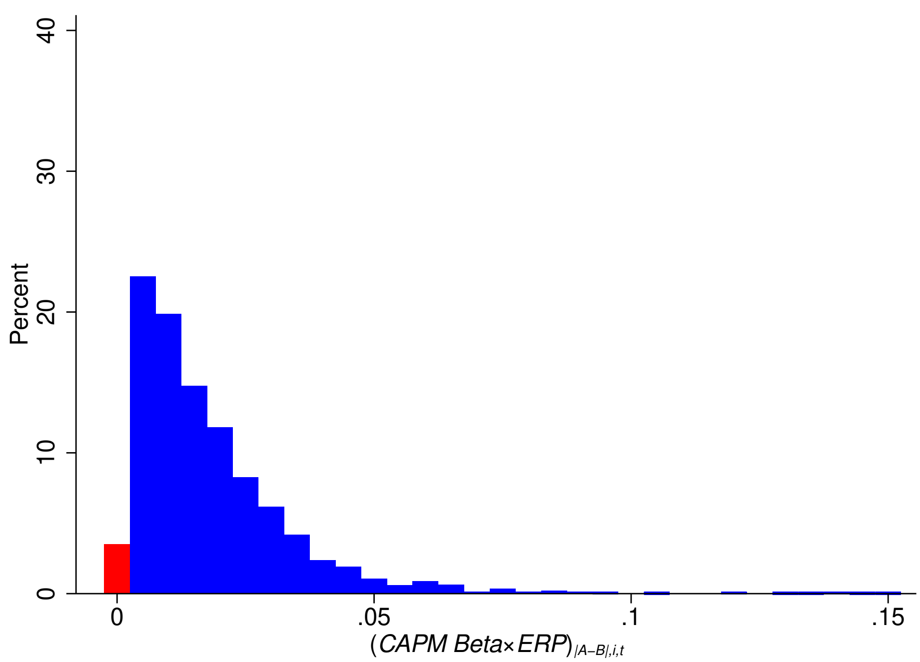


Figure 5: Heterogeneity in Analysts' Benchmark CAPM Betas. These figures display cross-sectional distributions of the details of analysts' chosen benchmarks CAPM beta. In particular, Panel (A) shows the distribution of the chosen return frequency for the beta estimation. Panel (B) shows the distribution of the horizons for the beta estimations. In both panels, red bars represent the most common approach in our sample. The sample period is 2000 through 2023. Data on individual analysts' choice of risk-free rate securities and horizons are hand-collected from sell-side analyst equity research reports.

(A) CAPM Beta Estimation Return Frequency

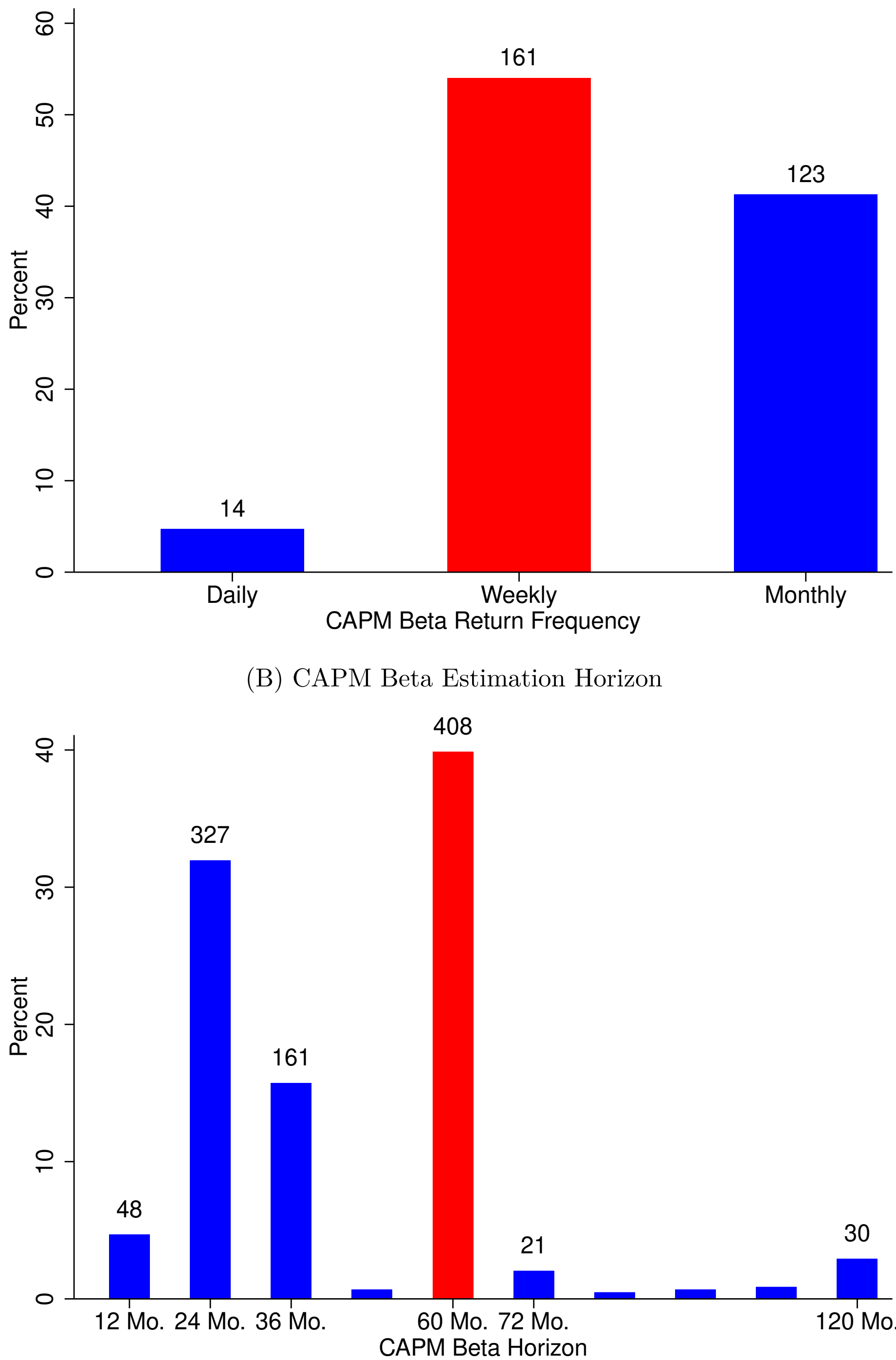
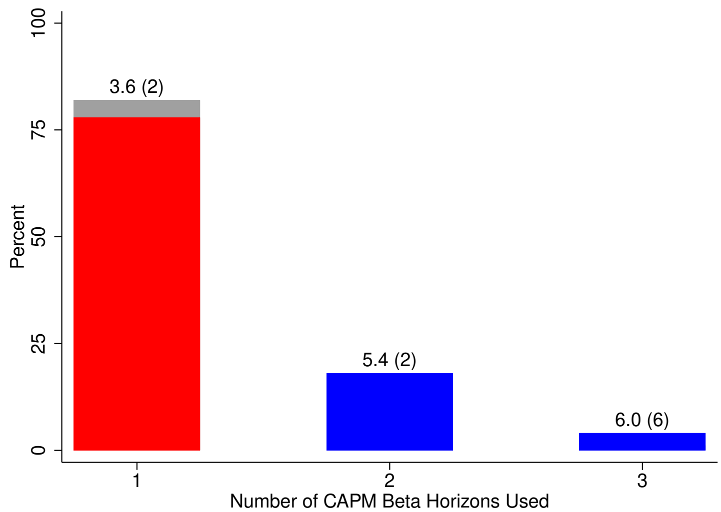
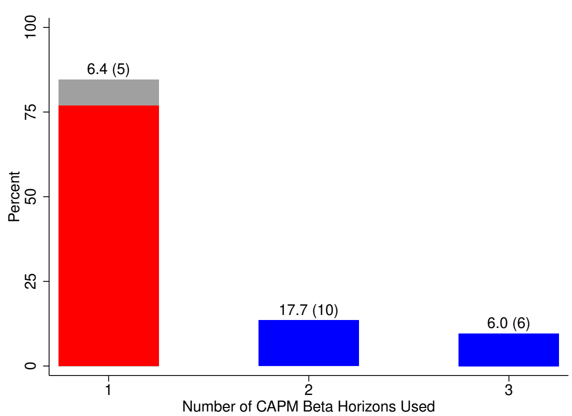


Figure 6: CAPM Beta Horizons Strategies. These figures display cross-sectional distributions of the strategies analysts' employ when choosing benchmarks CAPM beta. In particular, each shows the percentage of individual analysts that use only 1 benchmark beta horizon, the percentage that use 2, and the percentage that use 3 (the maximum in our sample). Panel (A) shows the distributions with all the analysts in our sample in which we observe at least two separate beta horizon discussions, Panel (B) shows the distributions with only the analysts in which we observe at least 4 separate beta horizon discussions, and Panel (C) shows the distribution with only the analysts in which we observe beta horizon discussions for multiple firms across multiple years. In all three panels, red bars represent the most common approach in our sample (only 1 chosen horizon) and the gray bars represent that percentage of analysts that use only 1 strategy, but average benchmark betas using multiple estimation horizons. The mean number of observations per analyst in each strategy bucket is displayed above each bar, and the median number of observations per analyst in each strategy bucket is displayed above each bar in parentheses. The sample period is 2000 through 2023. Data on individual analysts' choice of beta horizon length are hand-collected from sell-side analyst equity research reports.

(A) All Analysts Identifying Beta Horizon.



(B) 4+ Forecasts.



(C) Forecasts Across Multiple Firms and Years

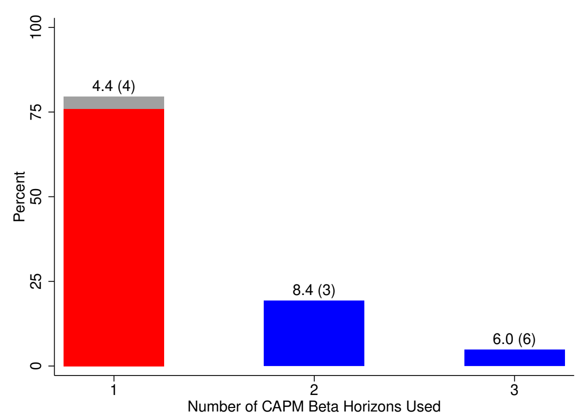
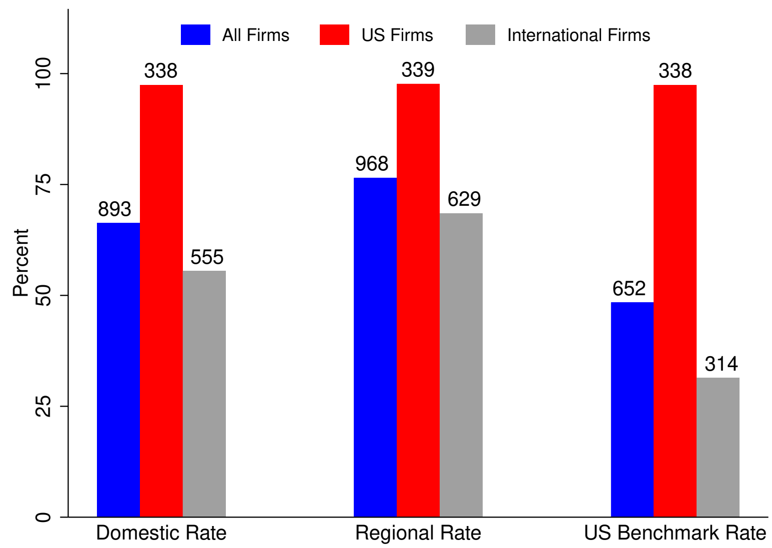


Figure 7: Heterogeneity in Analysts' Benchmark Risk-Free Rates. These figures display cross-sectional distributions of the details of analysts' chosen benchmarks for the risk-free rate. In particular, Panel (A) shows the distirbution of the chosen region for the benchmark security used by analysts in setting their risk-free rate proxy. Domestic rates represent the treasury rate for the country where the firm is headquartered (e.g., the U.K. treasury security for a firm headquartered in England, regional rates represent the treasury rate from one of the countries in the same continent where the firm is headquartered (e.g., the U.K. treasury security for a firm headquartered in France), and the U.S. benchmark rate, which is a U.S. treasury security. Panel (B) shows the distribution of the horizons of the risk-free securities chosen by analysts. In both panels, blue bars represent all the firms in our sample, red bars represent U.S. firms and gray bars represent international firms. The number of observations in each category appears above each bar. In Panel (B), the number of observations is displayed only for all firms using each horizon. The sample period is 2000 through 2023. Data on individual analysts' choice of risk-free rate securities and horizons are hand-collected from sell-side analyst equity research reports.

(A) Benchmark Security



(B) Benchmark Horizon

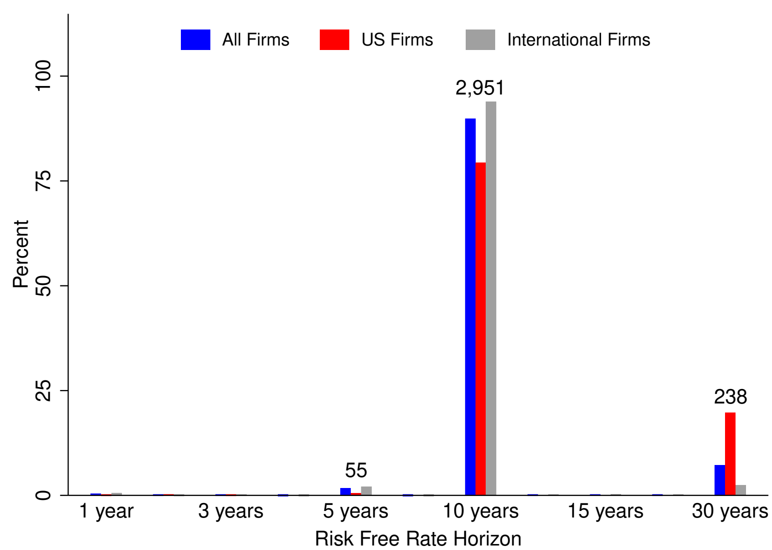
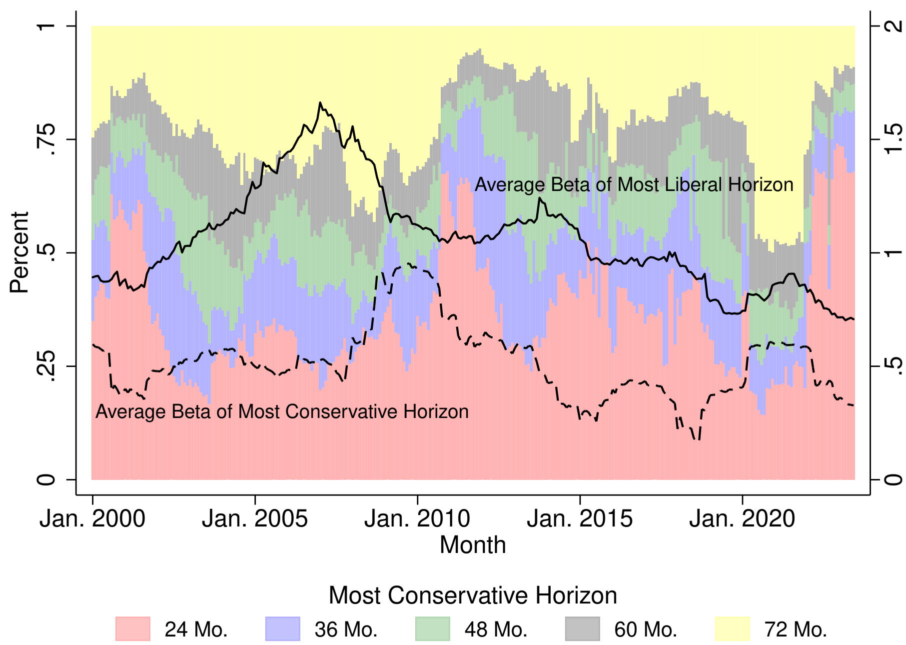


Figure 8: Heterogeneity in the Most Conservative and Liberal Beta Strategies. These figures display cross-sectional variation in the most conservative and most liberal beta strategies across each month of our sample. In particular, if an analyst used a strategy to give the most conservative cost of capital (e.g., lowest) or most liberal (e.g., highest), he or she would use the CAPM beta horizon that returned the lowest or highest beta, respectively. The black lines depict the patterns of the average of the conservative and liberal strategies across firms each month, while the different colored shaded regions portray the percentage of firms in which the most conservative (Panel A) and the most liberal (Panel B) strategy was the one corresponding to each particular horizon (e.g., 24 months, 36 months, etc.). The sample period is 2000 through 2023. Data on firms' stock returns is from Datastream.

(A) Most Conservative Strategy



(B) Most Liberal Strategy

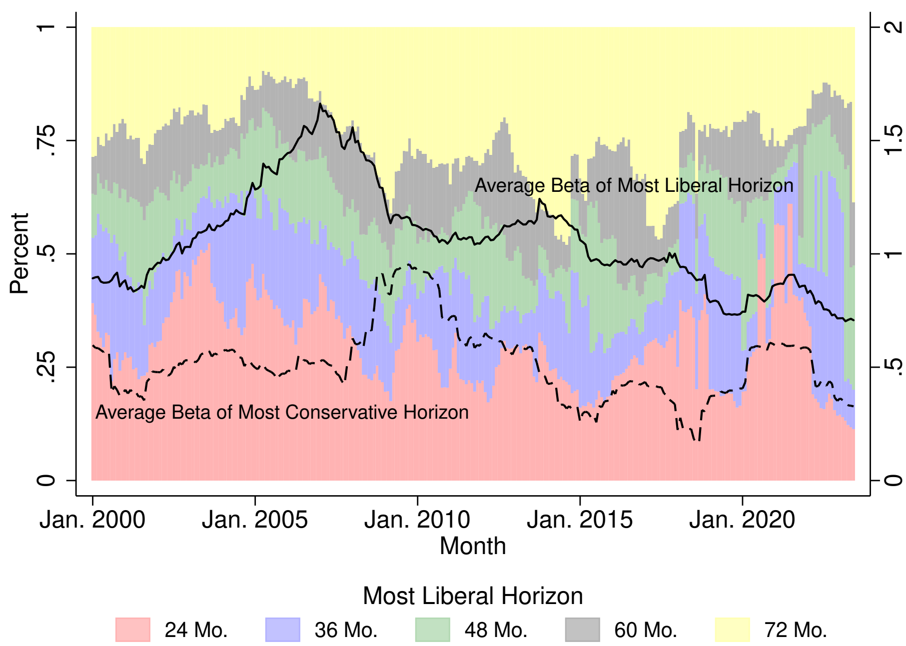
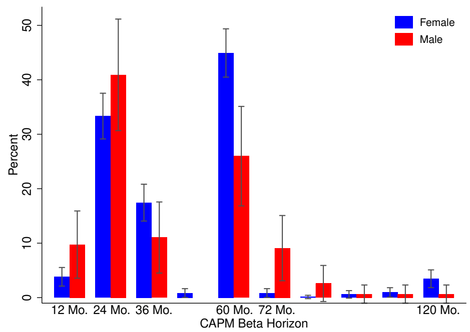
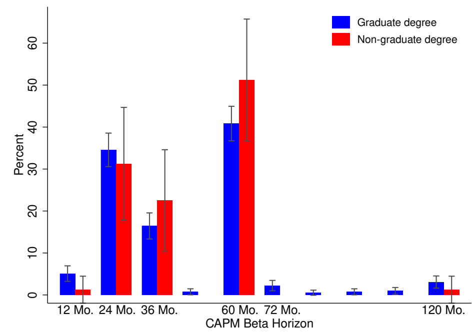


Figure 9: Disagreement in Analyst Equity Beta and Equity Risk Premium in the Time-Series. These figures display cross-sectional distributions of analysts chosen CAPM beta horizons across personal and firm characteristics. In particular, Panel (A) shows the horizon distributions across analyst gender, Panel (B) across analyst education, Panel (C) across analyst race, Panel (D) across analyst region, and Panel (D) across key firm industries. In all panels, 90% standard errors are displayed. The sample period is 2000 through 2023. Data on personal characteristics were graciously shared by Marius Guenzel. Data on individual analysts' choice of CAPM beta horizons are hand-collected from sell-side analyst equity research reports.

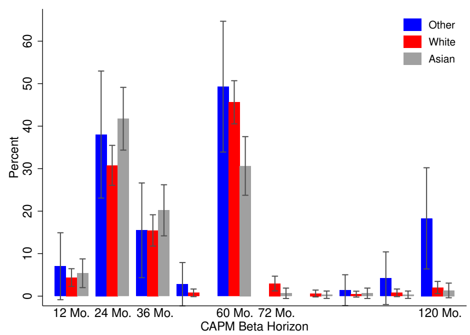
(A) Analyst Gender



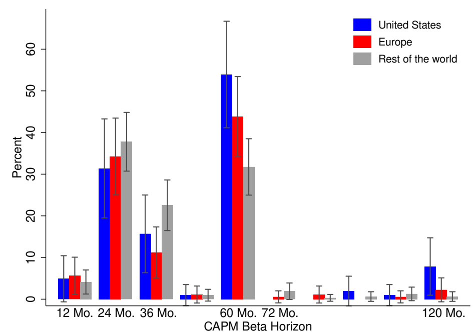
(B) Analyst Education



(C) Analyst Race



(D) Analyst Region



(E) Firm Industry

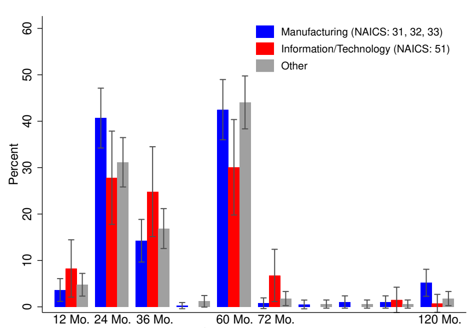


Table 1: Sample Details and Summary Statistics. This table reports the sample details and summary statistics for our three separate but complimentary samples. Panel A reports the sample details and summary statistics for the overlapping WACC sample. To be included in this sample, a firm must be covered by at least 2 analysts in the same year that report an estimate for the firm's weight average cost of capital (WACC). Not every analyst reports every detail of how the WACC is calculated, so the individual components will have less observations than the full sample (e.g., there are only 29,244 reports in the overlapping WACC sample that have estimates for the firm's terminal growth rate (TGR). Panels B and C report the sample details for the textual analysis samples: beta (Panel B) and the risk-free rate (Panel C), respectively. The full sample period is 2000-2023. Data on individual analysts' estimates of the WACC and its components are hand-collected from sell-side analyst equity research reports. Data on other firm characteristics is from Refinitiv/Datastream.

Panel A: Overlapping WACC Sample						
Number of Total Observations			45,992			
Number of Total Firms			4,261			
Number of Total Firm HQ Countries			63			
Number of Total Brokerage Houses			42			
Number of Total Identified Analysts			4,566			
Number of Total Analyst Countries			45			
	Mean	Std. Dev.	25^{th} Pct.	Median	$75^{th} Pct.$	Obs.
Firm Details						
Analyst Coverage (#)	10.0	7.4	4.0	8.0	15.0	12,060
Sample Coverage (years)	4.0	4.2	1.0	2.0	5.0	4,261
Equity Report Details						
$WACC_{a,i,t}$	0.089	0.019	0.076	0.087	0.100	45,992
$rf_{a,i,t}$	0.040	0.017	0.030	0.040	0.050	10,921
$CAPM\ Beta_{a,i,t}$	1.089	0.280	0.900	1.050	1.200	12,409
$ERP_{a,i,t}$	0.057	0.014	0.050	0.055	0.064	11,052
$r^E_{a,i,t}$	0.101	0.024	0.085	0.099	0.114	7,833
$TGR_{a,i,t}$	0.022	0.020	0.015	0.020	0.030	29,244
Pairwise Differences						
$WACC_{\|A-B\|,i,t}$	0.014	0.013	0.005	0.010	0.019	48,019
$rf_{\|A-B\|,i,t}$	0.010	0.011	0.003	0.008	0.013	3,247
$CAPM\ Beta_{\|A-B\|,i,t}$	0.219	0.211	0.080	0.170	0.300	4,170
$ERP_{\|A-B\|,i,t}$	0.013	0.013	0.005	0.010	0.016	2,921
$(CAPM\ Beta \times ERP)_{\|A-B\|,i,t}$	0.016	0.016	0.005	0.011	0.022	2,059
$r^E_{\|A-B\|,i,t}$	0.018	0.017	0.006	0.013	0.024	1,498
Panel B: CAPM Beta Textual Analysis Sample						
Number of Total Observations			1,023			
Number of Total Firms			794			
Number of Total Brokerage Houses			36			
Number of Total Analysts			508			
	Mean	Std. Dev.	25^{th} Pct.	Median	75^{th} Pct.	Obs.
$CAPM\ Beta_{a,i,t}$	1.124	0.395	0.870	1.050	1.300	828
Panel C: Risk-Free Rate Textual Analysis Sample						
Number of Total Observations			3,284			
Number of Total Firms			1,679			
Number of Total Brokerage Houses			48			

Table 2: Frequency of Above and Below Consensus Estimates for WACC and TGR. This table reports the frequency of analysts estimates for analyst estimates of weighted average costs of capital and terminal growth rates (TGR) that are above and below the consensus (mean) estimate for a given firm-year pair. Data on individual analysts' estimates of discount rates and TGRs are hand-collected from sell-side analyst equity research reports.

		Discount Rate$_{a,i,t}$		
		Above Consensus	Below Consensus	Total
$TGR_{a,i,t}$	Above Consensus	29.5% 7,037	25.1% 6,004	54.6% 13,041
	Below Consensus	19.7% 4,706	25.7% 6,136	45.4% 10,842
	Total	49.2% 11,743	50.8% 12,140	100.0% 23,883

Table 3: Cost of Equity Campbell-Shiller Decomposition. This table reports the results of a Cambell-Shillder Decomposition of the pairwise differences between to analysts estimates firm's equity cost of capital. In other words, this table reports the results of linear regression models in which the dependent variables are either the absolute difference in risk-free rates or CAPM betas× the equity risk premium (ERP) estimated by two analysts covering the same firm at the same time. The dependent variable of interest is the absolute difference in equity cost of capital estimated by two analysts covering the same firm at the same time. The sample period is 2000-2023. Panel A displays the results for the entire sample, while Panels B and C split between firms that are headquartered in the United States (Panel A) and the rest of the world (Panel B). Data on individual analysts' estimates of the WACC and its components are hand-collected from sell-side analyst equity research reports. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Dep. variable =	$rf_{A-B,i,t}$				$(\beta \times ERP)_{A-B,i,t}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Full Sample								
$r^E_{A-B,i,t}$	0.227***	0.227***	0.215***	0.214***	0.773***	0.773***	0.785***	0.786***
	(0.033)	(0.032)	(0.040)	(0.040)	(0.033)	(0.032)	(0.040)	(0.040)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	1,498	1,497	1,119	1,117	1,498	1,497	1,119	1,117
F Statistic	46.57	49.93	29.01	29.15	542.02	576.98	386.54	393.63
Panel B: United States Sample								
$r^E_{A-B,i,t}$	0.075	0.083	0.113	0.113	0.925***	0.917***	0.887***	0.887***
	(0.053)	(0.051)	(0.078)	(0.071)	(0.053)	(0.051)	(0.078)	(0.071)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	229	228	160	154	229	228	160	154
F Statistic	2.01	2.70	2.13	2.53	309.10	329.37	130.73	156.88
Panel C: International Sample								
$r^E_{A-B,i,t}$	0.257***	0.256***	0.233***	0.225***	0.743***	0.744***	0.767***	0.775***
	(0.039)	(0.037)	(0.045)	(0.045)	(0.039)	(0.037)	(0.045)	(0.045)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	1,269	1,268	959	957	1,269	1,268	959	957
F Statistic	43.61	47.18	26.46	25.53	364.05	399.78	287.42	301.57

Table 4: Frequency of Above and Below Consensus Estimates for CAPM Beta and ERP. This table reports in Panel A, the frequency of analysts estimates for the CAPM beta and equity risk premium that are above and below the consensus (mean) estimates for a given firm year-pair. Data on individual analysts' estimates of CAPM betas and equity risk premiums are hand-collected from sell-side analyst equity research reports.

		$CAPM\ Beta_{a,i,t}$		
		Above Consensus	Below Consensus	Total
$Equity\ Risk\ Premium_{a,i,t}$	Above Consensus	24.2% 815	31.2% 1,049	55.4% 1,864
	Below Consensus	26.6% 895	18.0% 604	44.6% 1,499
	Total	50.8% 1,710	49.2% 1,653	100.0% 3,363

Table 5: Auto-correlation in the Most Conservative and Most Liberal Beta Strategies. This table the results of linear regression models in which the dependent variable is an indicator variable equal to one if the respective beta horizon (e.g., 24 months, 36 months, etc.) corresponds to the most conservative (Panel A) or the most liberal (Panel B) beta strategy. In particular, if an analyst used a strategy to give the most conservative cost of capital (e.g., lowest) or most liberal (e.g., highest), he or she would use the CAPM beta horizon that returned the lowest or highest beta, respectively. The main independent variables of interest are the lagged versions of the dependent variables. The models include both firm and year-month fixed effects. The sample period is 2000 through 2003. Data on firms' stock returns is from Datastream. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: Most Conservative Beta Horizon					
Dependent variable =	I(24 Mo.)$_{i,t}$	I(36 Mo.)$_{i,t}$	I(48 Mo.)$_{i,t}$	I(60 Mo.)$_{i,t}$	I(72 Mo.)$_{i,t}$
	(1)	(2)	(3)	(4)	(5)
I(24 Mo.)$_{i,t-1}$	0.036***				
	(0.002)				
I(36 Mo.)$_{i,t-1}$		-0.209***			
		(0.001)			
I(48 Mo.)$_{i,t-1}$			-0.166***		
			(0.001)		
I(60 Mo.)$_{i,t-1}$				-0.153***	
				(0.001)	
I(72 Mo.)$_{i,t-1}$					0.205***
					(0.003)
Year-Month FE	✓	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓	✓
Observations	150,071	150,071	150,071	150,071	150,071
F Statistic	247.80	21893.15	17395.51	16872.76	5788.80
R^2	0.12	0.10	0.09	0.08	0.16
Panel B: Most Liberal Beta Horizon					
I(24 Mo.)$_{i,t-1}$	0.090***				
	(0.002)				
I(36 Mo.)$_{i,t-1}$		-0.209***			
		(0.002)			
I(48 Mo.)$_{i,t-1}$			-0.191***		
			(0.001)		
I(60 Mo.)$_{i,t-1}$				-0.173***	
				(0.001)	
I(72 Mo.)$_{i,t-1}$					0.249***
					(0.003)
Year-Month FE	✓	✓	✓	✓	✓
Firm FE	✓	✓	✓	✓	✓
Observations	150,071	150,071	150,071	150,071	150,071
F Statistic	1555.88	18927.52	20992.95	23037.35	8522.89
R^2	0.13	0.14	0.12	0.09	0.16

Table 6: ANOVA Variance Decomposition of the Level of Analysts' CAPM Equity Betas. This table reports the results of an analysis of variance (ANOVA) for the level analysts' estimates of equity beta. The independent variables of interest are indicator variables for firm, the brokerage house covering the firm, and for the analyst completing the equity report. The full sample includes only observations in in which there are at least 2 estimates of an equity beta at the firm, brokerage house and analyst level. Data on individual analysts' estimates of equity betas are hand-collected from sell-side analyst equity research reports.

	Sum of Squares (% of Model)	Degrees of Freedom	Adjusted Partial R^2
	(1)	(2)	(3)
Panel A: Full Sample			
Firm Indicators	78%	1,947	0.38
Brokerage Indicators	2%	36	0.01
Analyst Indicators	19%	1,120	0.28
Observations		6,411	
R^2		0.78	
Adjusted R^2		0.57	
Panel B: ≥ 5 Observations per Analyst			
Firm Indicators	86%	1,686	0.38
Brokerage Indicators	2%	28	0.00
Analyst Indicators	13%	395	0.24
Observations		4,475	
R^2		0.78	
Adjusted R^2		0.58	
Panel C: ≥ 5 Observations per Analyst, ≥ 5 Observations per Firm			
Firm Indicators	74%	464	0.34
Brokerage Indicators	4%	28	0.00
Analyst Indicators	22%	353	0.28
Observations		2,180	
R^2		0.75	
Adjusted R^2		0.59	

Table 7: ANOVA Variance Decomposition of Individual Fixed Effects from the Beta Regress. This table reports the results of an analysis of variance (ANOVA) for the individual fixed effects we extract from a linear regression on the level of analysts' CAPM equity betas. The independent variables of interest are indicator variables for the analysts gender, race, whether they have a master's degree, and country of analysts' location. Panel B adds an indicator for young. Young takes a value of 1 if the analyst is under the sample median for age, and 0 otherwise. The full sample includes only observations in in which there are at least 2 estimates of an equity beta at the firm, brokerage house and analyst level. Data on individual analysts' estimates of equity betas are hand-collected from sell-side analyst equity research reports. Personal characteristics of the analysts in our sample is collected from a social networking site and was graciously shared by Marius Guenzel.

	Sum of Squares (% of Model)	Degrees of Freedom	Adjusted Partial R^2
	(1)	(2)	(3)
Panel A			
Gender Indicator	6%	1	0.01
Race Indicators	18%	4	-0.02
Master's Degree Indicator	0%	1	-0.01
Analyst Country Indicators	76%	8	0.02
Observations		154	
R^2		0.09	
Adjusted R^2		-0.00	
Panel B			
Gender Indicator	15%	1	0.00
Race Indicators	16%	3	-0.03
Master's Degree Indicator	0%	1	-0.01
Analyst Country Indicators	69%	8	-0.02
Young Indicator	1%	1	-0.01
Observations		96	
R^2		0.09	
Adjusted R^2		-0.07	

Table 8: Multinomial Probit of Analysts' Choice of CAPM Beta Horizon. This table reports the results of a multinomial logit model in which the categories for the left hand size are indicators that take the value of 1 if the chosen beta horizon is 24 months (base case), 36 months, or 60 months, which represent the three most commonly chosen horizons in our sample . The independent variables of interest are indicator variables for the analysts gender, race, whether they have a master's degree, and country of analysts' location. Moreover, we include indicator variables for the firm's industry. Data on individual analysts' chosen horizon for the CAPM equity betas are collected from sell-side analyst equity research reports using textual analysis.

CAPM Beta Horizon (Base = 24 Months)	36 Months	60 Months	Equality of Distributions Across Horizons
	(1)	(2)	(3)
Binary Characteristics			
I(Gender = Male)	-0.379*	-0.146	1.06
	(0.368)	(0.436)	(0.589)
I(Education = Graduate degree)	-0.078	0.218	0.49
	(0.479)	(0.471)	(0.782)
I(Race = Non-white)	0.012**	-0.055	0.05
	(0.313)	(0.277)	(0.974)
Categorical Characteristics			
I(Region = United States)	Base Category		
I(Region = Europe)	-0.481	-0.276	
	(0.438)	(0.420)	5.25
I(Region = Other)	-0.093	-0.556	(0.263)
	(0.372)	(0.418)	
I(Industry = Manufacturing)	Base Category		
I(Industry = Info/Tech)	0.356	-0.234	
	(0.429)	(0.475)	3.95
I(Industry = Other)	0.425	0.262	(0.413)
	(0.288)	(0.262)	
Constant	-0.566	0.449	
	(0.513)	(0.518)	
Observations	517		
Log pseudolikelihood	-523.17		
χ^2	12.07		
p-Value (χ^2))	0.601		

Table 9: WACC Disagreement and Trading Volume. This table reports the results of linear regression models in which the dependent variables are the monthly trading volume scaled by total common shares outstanding (Panel A) and the monthly trading volume minus the mean of the previous 12 months of trading volume, scaled by total common shares outstanding (Panel B) . The independent variable of interest is the difference between the maximum WACC estimate and the minimum WACC estimate by different analysts covering the same firm in the first quarter of the same year. The scaled trading volume and WACC disagreement variables are measured in the month in which the second analyst forecast is released (e.g., the month in which the disagreement is created). The sample period is 2000-2023. Data on analyst estimates of WACC and TGR is hand-collected from sell-side analyst equity reports. Data on trading volume, market capitalization, and stock returns is from Datastream. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A	$\text{FVOL}_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\text{Max}_{a\in A}(WACC_{i,t})-\text{ Min}_{a\in A}(WACC_{i,t})$	1.057***	0.229***	0.236***	0.152***	0.121***	0.133**
	(0.128)	(0.047)	(0.048)	(0.045)	(0.042)	(0.057)
$\text{Max}_{a\in A}(TGR_{i,t})-\text{ Min}_{a\in A}(TGR_{i,t})$						0.046
						(0.067)
$\text{FVOL}_{i,t-1}$		1.018***	1.019***	0.785***	0.761***	0.742***
		(0.010)	(0.010)	(0.019)	(0.021)	(0.030)
log(Market $\text{Capitalization}_{i,t}$)					-0.009***	-0.012***
					(0.002)	(0.003)
Cumulative $\text{Return}_{i,t-3,t-1}$					-0.010*	-0.006
					(0.006)	(0.008)
Cumulative $\text{Return}_{i,t-12,t-4}$					0.001	-0.000
					(0.003)	(0.004)
Return $\text{Volatility}_{i,t-3,t-1}$					0.000	0.000
					(0.000)	(0.000)
Months Between $\text{Forecasts}_{i,t0,T}$					-0.002***	-0.002**
					(0.001)	(0.001)
Year-Month FE			✓	✓	✓	✓
Firm FE				✓	✓	✓
Observations	16,193	15,840	15,840	14,435	12,354	6,979
F Statistic	68.56	5545.55	5512.10	834.84	211.13	104.29
R^2	0.01	0.80	0.80	0.88	0.88	0.89

Panel B	Abnormal $\text{FVOL}_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\text{Max}_{a\in A}(WACC_{i,t})-\text{ Min}_{a\in A}(WACC_{i,t})$	0.090**	0.162***	0.170***	0.089**	0.083**	0.113**
	(0.040)	(0.039)	(0.039)	(0.038)	(0.037)	(0.049)
$\text{Max}_{a\in A}(TGR_{i,t})-\text{ Min}_{a\in A}(TGR_{i,t})$						0.061
						(0.063)
Abnormal $\text{FVOL}_{i,t-1}$		0.425***	0.412***	0.482***	0.508***	0.516***
		(0.021)	(0.021)	(0.023)	(0.024)	(0.033)
log(Market $\text{Capitalization}_{i,t}$)					-0.004***	-0.006**
					(0.002)	(0.002)
Cumulative $\text{Return}_{i,t-3,t-1}$					-0.021***	-0.015*
					(0.006)	(0.009)
Cumulative $\text{Return}_{i,t-12,t-4}$					-0.000	0.000
					(0.002)	(0.003)
Return $\text{Volatility}_{i,t-3,t-1}$					-0.000	0.000
					(0.000)	(0.000)
Months Between $\text{Forecasts}_{i,t0,T}$					-0.002**	-0.002**
					(0.001)	(0.001)
Year-Month FE			✓	✓	✓	✓
Firm FE				✓	✓	✓
Observations	15,854	14,797	14,797	13,501	12,353	6,978
F Statistic	5.20	226.84	210.45	219.15	83.48	45.59
R^2	0.00	0.14	0.16	0.41	0.42	0.44

Appendix A: Variable Definitions

Table A1: Variable Definitions

Subscript a indicates a specific analyst, i indicates a specific firm, and t indicates a year.

Variable	**Definition**
Analysts' $\text{WACC}_{a,i,t}$	The weighted average cost of capital (WACC) used by analysts to evaluate firm cash flows in equity reports.
Analysts' terminal growth $\text{rate}_{a,i,t}$ (TGR)	The terminal growth rate used by equity analysts in their DCF models, measured from the equity reports.
Analysts' CAPM equity $\text{beta}_{a,i,t}$	The equity beta used by analysts when computing their discount rate in equity reports.
Analysts' equity risk $\text{premium}_{a,i,t}$ (ERP)	The equity risk premium used by analysts when computing their discount rate in equity reports.
Analysts' risk-free $\text{rate}_{a,i,t}$ (Rf)	The risk-free rate used by analysts when computing their discount rate in equity reports.

Internet Appendix
for
"Resolving Estimation Ambiguity"

Paul H. Décaire[1] , Denis Sosyura[2], and Michael D. Wittry[3]

This Internet Appendix reports results that are mentioned but not tabulated in the main paper. We report 1 table, as outlined below:

1. Table IA1: Beta Horizon Disagreement and Trading Volume

   Reference in the main paper: "" (Section)

[1]W.P. Carey School of Business, Arizona State University, email: paul.decaire@asu.edu.
[2]W.P. Carey School of Business, Arizona State University, email: dsosyura@asu.edu.
[3]Fisher College of Business, Ohio State University, email: wittry.2@osu.edu.

Table IA1: Beta Horizon Disagreement and Trading Volume. This table reports the results of linear regression models in which the dependent variable is the monthly trading volume scaled by total common shares outstanding. The independent variable of interest is disagreement over a set of econometrican estimate CAPM betas. This CAPM Beta disagreement variable is calculated as the difference between the largest and smallest CAPM Beta in month t when using different forecast horizons (e.g., 24-month, 36-month, 60-month, etc) to estimate each beta. The scaled trading volume and CAPM beta disagreement variables are measured in the month in which the second analyst forecast is released (e.g., the month in which the disagreement is created). The sample period is 2000-2023. Data on trading volume, market capitalization, and stock returns is from Datastream. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Dependent variable =	$\text{FVOL}_{i,t}$				
	(1)	(2)	(3)	(4)	(5)
Max($CAPM\ Beta_{E,i,t}$)− Min($CAPM\ Beta_{E,i,t}$)	0.041***	0.005***	0.006***	0.004***	0.004***
	(0.002)	(0.000)	(0.000)	(0.001)	(0.001)
$\text{FVOL}_{i,t-1}$		0.891***	0.889***	0.640***	0.622***
		(0.003)	(0.003)	(0.008)	(0.010)
log(Market Capitalization$_{i,t}$)					-0.005***
					(0.001)
Cumulative Return$_{i,t-3,t-1}$					-0.007***
					(0.001)
Cumulative Return$_{i,t-12,t-4}$					-0.001
					(0.001)
Return Volatility$_{i,t-3,t-1}$					-0.000***
					(0.000)
Year-Month FE			✓	✓	✓
Firm FE				✓	✓
Observations	468,352	454,475	454,475	454,429	304,137
F Statistic	293.65	52939.04	53340.06	3300.80	834.77
R^2	0.01	0.80	0.80	0.83	0.83