Resolving Estimation Ambiguity*

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Abstract

Economic models develop conceptual frameworks for fundamental decisions but rarely prescribe a specific estimation approach. Using novel data on the inputs and assumptions in professional stock valuations, we study how financial analysts address estimation uncertainty when calculating a firm's cost of capital. Analysts use the same return-generating model (CAPM) but diverge in their estimation choices for key inputs, such as equity betas. Such estimation choices are driven by idiosyncratic analyst-specific criteria, persist throughout their career and across brokerages, and generate large cross-analyst variation in discount rates for the same stock. The dispersion in discount rates is associated with higher market measures of investor disagreement, such as trading volume. Overall, we provide micro evidence on how financial experts resolve estimation uncertainty.

JEL classification: G30, G31, G41, D25, D82, D83, O13, Q15, R14

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1 Introduction

Economic theory provides models that offer insights for fundamental decisions. However, it rarely prescribes a specific approach to estimate them. This methodological leeway gives rise to estimation ambiguity in the sense that different parameterizations produce distinct versions of the same conceptual model (Giacomini et al., 2019). Agents know that there exists a combination of methods that might best approximate a parameter's true value, but they rarely know the correct weights of each approach. Thus, even if all agents agree on the underlying conceptual model and have access to the same data, variation in their methodological choices due to estimation ambiguity can produce different modeling outcomes.

When agents estimate the same model but face a variety of feasible methods and plausible assumptions, how do they arrive at their decisions and how do their estimation choices affect real aggregate outcomes? Multiple theories generate insights about how agents should address estimation ambiguity. However, obtaining direct empirical evidence to pin down the most likely theories has remained elusive because agents' inputs, assumptions, and methodological selection processes are difficult to discern.

We examine these questions by studying the inner workings of estimating a firm's required rate of return—a key valuation driver and a foundation for investment decisions. We collect over 40,000 stock valuation models of financial analysts from top brokerage firms worldwide. We observe the discount rate model selected, its inputs and values, as well as a discussion of the information sources and methodological choices. Our panel data structure allows us to track each analyst over time and across the different settings they analyze, enabling us to investigate whether and why an agent's methodological choices might vary or exhibit persistence. This setting also generates useful variation in the degree of uncertainty across model inputs. In particular, the cost of equity estimation combines inputs that are observable in financial markets, such as the risk-free rate, and those that require estimation, such as equity betas. As a result, we can exploit the variation in estimation uncertainty across different inputs for the same firm in the same year, while holding constant the selected estimation model.

Securities valuation offers a convenient laboratory to study estimation ambiguity. Most finance professionals rely on the same model of the return-generating process—namely, the

CAPM, to estimate a firm's cost of capital, as shown in surveys (Graham and Harvey, 2001) and field data (Kruger et al., 2015; Dessaint et al., 2021), but theory provides no guidance on estimating the model's inputs. Since the CAPM inputs are based on public data, a natural question is whether finance professionals obtain comparable results while estimating the same model.

We find that financial analysts arrive at significantly different discount rates for the same firm at the same time. For a given firm, the average difference in the estimated cost of equity for two different analysts publishing equity reports at the same time is 180 basis points or 18 percent of the mean cost of equity in our sample. Similarly, the average difference in the weighted average cost of capital (WACC) for two analysts covering the same firm at the same time is 140 basis points or 16 percent of the mean WACC. Given the outsized quantitative importance of the discount rate in financial models, these magnitudes are somewhat unexpected because most prior work rarely considers analyst dispersion in the context of discount rates.

The discount rate dispersion is associated with a large variation in a firm's private valuations. For the median firm, a one standard deviation increase in the estimated cost of equity is associated with a 22% drop in intrinsic value, comparable to the difference between a buy and hold recommendation, when using a simple dividend discount model. This effect is quantitatively important because higher discount rates are not offset by other modeling choices, such as higher explicit or terminal growth rates. For example, in 45% of the valuation models, an above-consensus estimate of the discount rate is associated with a below-consensus estimate of the terminal growth rate, suggesting that they diverge in the opposite direction. As a result, the dispersion in the estimated discount rate appears to arise independently from cash flow disagreement.

Next, we study why professional agents with comparable tools arrive at different results for the same estimation task to understand the underlying factors of this particular channel of disagreement. We find that the discount rate variation is driven not by the return generating model, but by the methodological choices in model estimation. Décaire and Graham (2023) show that when estimating discount rates nearly all analysts (97%) use the CAPM, but our results suggest they diverge sharply on its inputs.

We investigate the sources of variation in model inputs and find that they are driven by variables that require estimation and have more estimation uncertainty. The largest sources of variation come from the differences in the estimation of betas and the equity risk premium (ERP). Using a Campbell-Shiller decomposition, we show that estimation ambiguity is a key driver of heterogeneity in the discount rate by decomposing it into the estimates of the risk-free rate and beta×ERP. The evidence shows that nearly 80% of the variation in the cost of equity is driven by the estimates of beta×ERP, and only a small minority of the variation (21%) is attributable to the choice of the risk-free rate.

The variation in the estimated betas and ERP is surprisingly large. For example, the average difference in the estimated betas for two different analysts publishing equity reports at the same time is 0.219 or 20% of the unconditional sample mean for CAPM beta. The variation in the estimated ERP is similarly large. During the median year in our sample (2000-2023), the 25th percentile of the ERP estimate is 5 percent per year, and the 75th percentile is 6.4 percent, indicating an interquartile range of 1.4 percentage points or 25% of the mean. Similarly, during the median sample year, the average difference in the ERP for two analysts covering the same firm at the same time is 130 basis points or 23% of the mean (5.7 percent per year).

To understand the drivers of divergence in model inputs, we conduct a contextual analysis of valuation reports and extract the description of estimation procedures and data sources for the main inputs. We manually read a sample of reports to construct a training algorithm and develop a machine-learning procedure to obtain key insights from analysts' qualitative discussions. We augment this approach with an advanced analysis that incorporates artificial intelligence and big-data contextual structures.

We find that a key driver of the differences in beta estimates for the same stock is the trailing estimation horizon, the number of years of monthly data used to estimate the beta. The estimation horizon varies from one to ten years, and the most common horizons use monthly stock returns over the trailing five years (40%), two years (32%), or three years (16%). The choice of the estimation horizon is particularly important for valuation outcomes during periods of strong market returns and economic growth, as the spread between the maximum and minimum beta estimated with different rolling windows is pro-cyclical (e.g., see Figure 1).

In contrast to the large dispersion in beta and ERP estimates, analysts tend to converge on the inputs linked to observable values, such as the risk-free rate. In the U.S., the majority (98%) of valuations rely on the 10-year Treasury as a proxy for the risk-free rate, while a minority of reports use the 30-year Treasury (1%). Overseas, valuation reports show more variation in the choice of the risk-free asset, ranging from government bonds issued by the foreign firm's country, another country in the firm's region (e.g., Germany in the Eurozone), or the United States. These choices generate modest cross-sectional variation in the risk-free rate and rationalize the evidence from the Campbell-Shiller decomposition.

Our findings suggest that analysts' estimation outcomes reflect persistent methodological choices rather than ad-hoc tweaking of financial models to yield a particular result. For example, valuation models with higher betas do not systematically use smaller ERP, as would be expected if analysts attempted to reverse engineer estimation outcomes by adjusting estimation procedures.

A natural question is why critical valuation inputs vary widely across financial experts who work for similar financial firms and have comparable professional backgrounds. Does the variation in the modeling choices reflect the rules of the brokerage house, the institutional features of the firm, the common practice of its industry, or the modeling preferences of an analyst?

To address this question, we estimate a variance decomposition (ANOVA) for the dispersion in the CAPM betas observed for a given firm-year into several components attributable to institutional norms, personal preferences, or the focal firm. By including firm fixed effects, we absorb time-invariant differences in betas across firms driven by their different exposure to systematic factors, and exploit within-firm variation across personal preferences and institutional norms respectively captured with analysts and brokerage houses fixed effects. We find that personal preferences explain 28% of the within-firm variation in beta estimates, several times more than those related to institutional norms (1%).

Consistent with a key role of an analyst in selecting estimation parameters, we find that over 80% of the analysts use the same estimation horizon across all reports in their sample. These methodological choices persist when the same analyst covers different stocks or moves across brokerage houses. In contrast, we do not find similar consistencies in estimation choices for analysts within the same brokerage house or for stocks within the same industry. This evidence suggests that the choice of model parameters rests with the individual analyst, and this choice is persistent throughout an analyst's career. This persistency squares with the analysis in Bewley (2002) applied to the role of ambiguity in parameter selection. In the absence of a clearly dominant alternative, individuals exposed to ambiguity will maintain the status quo.

Next, we study how analyst characteristics are related to their choice of model estimation parameters. Using a wide array of analyst characteristics, such as education, seniority, location, and demographics, we find that they have modest effects in explaining modeling choices. Overall, our evidence suggests that estimation choices are persistent and driven by idiosyncratic individual-specific criteria, such that the same analyst applies a consistent set of estimation parameters over time and across firms, but the choice of model is unrelated to a set of common observables.

Our findings help distinguish among the most common theories on resolving estimation ambiguity proposed in the literature. First, robust methodological choices, such as the max-min criteria discussed in Gilboa and Schmeidler (1989) and Hansen and Sargen (2001), suggest selecting a method that would be optimal under a justifiable worst-case scenario.[1] For example, an analyst guided by this approach could select a trailing horizon for beta estimation that would yield the most liberal (conservative) beta such that a firm's cash flows are never overestimated (underestimated). A second group of theories suggests agents address estimation ambiguity by aggregating a model's outcomes over its likely specifications following a Bayesian selection criterion (Giacomini et al., 2019, 2022). Lastly, behavioral theories offer a promising avenue to interpret our results. Ambiguity-averse agents might favor familiar strategies (Heath and Tversky, 1991; Fox and Tversky, 1995), such as those covered during their academic training or the ones they were exposed to during their formative professional years, instead of considering all possible methods. A more extreme version of this criterion relates to Raiffa (1961)'s critique, which suggests that one can resolve estimation ambiguity using a random draw over the possible (justifiable) methods under consideration.

[1] Existing empirical evidence must discipline the set of justifiable scenarios that can be reasonably considered.

Our results are broadly consistent with a behavioral explanation. First, the data suggests that the adoption of a max-min criterion is unlikely in our setting as such a strategy would result in using different beta estimation horizons across firms and, over time, when selecting the most conservative or liberal estimates for each valuation exercise (e.g., see Figure 8). In fact, in only 21% of cases would an analyst adopting such decision criteria end up using the same beta horizon for two consecutive years for the average firm. Rather, we observe that analysts' methodological preferences are extremely persistent and seem to be driven by idiosyncratic individual-specific criteria. Simple heuristics and anchoring (Tversky and Kahneman, 1974) driven by early career influences or randomly selecting a method and sticking with it is more consistent with the patterns we document in the data than the sophisticated approaches discussed in the literature.[2] Indeed, that 80% of analysts consistently employ the same methodology, coupled with the fact that only 3-4% engage in any form of aggregation, indicates that the alternative criteria explored in the literature are, at best, adopted by a minority of professionals.

In our final analyses, we study how the disagreement in the estimated discount rates is associated with market outcomes. In the cross-section, while the average disagreement in estimated cash flows declines with a greater number of valuations that diversify away extreme estimates, the differences in the discount rate need not cancel out and may be augmented with a greater number of valuations due to the convexity in the discount rate's effect on the share price—Jensen's inequality. Consistent with this mechanism, we find that stocks with a greater disagreement in the discount rate have significantly higher trading volume. A one standard deviation in the discount rate disagreement (153 bps) is associated with a 4 percent increase in the abnormal trading volume.

Overall, our findings suggest that a large share of the dispersion in stock valuation is attributable to modeling choices in estimating a firm's discount rate. Most of the variation results from estimating the same return-generating model but with different trailing horizons (beta estimates) and under different assumptions about the equity risk premium. These methodological choices reside with the financial analyst and provide micro foundation for prior evidence on the importance of individuals' factors in asset valuation.

[2] At the moment, our evidence cannot distinguish between a role for familiarity or a simple random draw. We are currently collecting data to address this.

2 Literature Review

The central contribution of this paper is to provide granular evidence on the inner workings of estimating financial models by sophisticated intermediaries. Our paper contributes to prior research on (1) ambiguity in professional decisions, (2) estimating the cost of capital, and (3) disagreement in stock valuations.

First, our findings relate to the literature investigating the effect of ambiguity on the estimation outcomes of practitioners. Kahneman et al. (2021) discusses how judges can give different judgments on similar cases due to different mental models. Multiple recent papers investigate whether estimation ambiguity affects estimate dispersion in science, such as Huntington-Klein et al. (2021) and Menkveld et al. (2023), showing that even well-intentioned researchers can arrive at different outcomes in the absence of clear guidance. In parallel work, Mitton (2022) posits that academic researchers might exploit estimation ambiguity to obtain significant results in their work. Using "what-if" scenarios and simulations, he shows one can obtain drastically different outcomes by selecting different ways to process the data. Our work extends these papers' insights in three ways. First, we shed light on the likely mechanism used by practitioners to resolve estimation ambiguity, offering avenues to refine theories studying how agents address ambiguity in the real world. Second, we investigate the factors that can explain why individuals select a particular approach when resolving ambiguity. Lastly, we present evidence about how estimation ambiguity likely impacts aggregate outcomes, by studying its relation to the financial market.

Second, we expand our understanding of how key market agents determine and apply the cost of capital in their professional decisions. Since the discount rate is rarely observable to the econometrician, most research has inferred the discount rate from surveys (Graham and Harvey, 2001), observed investment actions (Kruger et al., 2015; Dessaint et al., 2021; Décaire, 2023), or select disclosures (Gormsen and Huber, 2023). While these approaches yield useful WACC measures, they remain silent about how agents arrive at their estimates and update them in response to market conditions. Our paper complements this work by providing evidence on how equity analysts estimate each component of the discount rate.

The discount rates in analyst reports are important for both corporate managers and investors. Décaire and Graham (2023) find that analyst discount rates are unbiased predictors of future stock returns, indicating that they provide useful forward-looking information. Décaire and Bessembinder (2021) show that the discount rate uncertainty has significant real effects on project valuation and corporate investment. We supplement these findings by identifying the drivers of variation in analyst discount rates.

Third, our paper is also a part of the broader literature that studies the origins of disagreement in private valuations. Theory makes an important distinction between market participants' disagreement in information inputs and valuation methods (see Hong and Stein, 2007 for a review), but this distinction has been difficult to test. Andrei et al. (2019) and Meeuwis et al. (2022) show that investors react differently to common information shocks and suggest that they rely on different mental models. Using investor activity on a social media platform, Cookson and Niessner (2020) find that investor disagreement is evenly split between a self-declared investment approach and different information sets. Using a survey, Giglio et al. (2021) find that investors disagree about both cash flows and expected returns and conclude that it is crucial to jointly model their disagreement about both parameter groups and their comovement. Rather than relying on indirect measures of disagreement in valuation, such as surveys, social media, or trades, our paper provides direct evidence on the inner workings of agents' financial models. We show why financial experts arrive at different outcomes even when estimating the same model with public data. While most prior literature on analyst disagreement (see Sadka and Scherbina, 2007 for a review) has focused on the dispersion in cash flow forecasts, we show that the dispersion in the estimated discount rates is also an important determinant of private valuations disagreement.

Our work responds to a call in the empirical literature for opening up the black box of stock valuations by financial intermediaries. In a survey of this work, Bradshaw (2011) concludes that "For this literature to progress, research that provides any kind of penetration of the 'black box' of how analysts actually process information should be encouraged, even if methods or approaches are imperfect" (p. 43). Several papers make progress on this question by revealing analysts' information sources, such as management interactions (Green et al., 2014), company visits (Cheng et al., 2016), and non-deal roadshows (Bradley et al., 2022). Using data on the online status of Bloomberg terminals, Ben-Rephael et al. (2023) study analysts' decision to collect hard and soft information and its effect on forecast precision.

We complement prior work that studies analysts' information acquisition by dissecting the next step in their valuations—the incorporation of data into financial models. By holding the model fixed across analysts, we provide evidence about the drivers of their different methodological choices as well as the real effects of those differences on financial markets.

Finally, results also contribute to the discussion linking ambiguity in valuation with the role of subjectivity in finance (Keynes, 1921; Shiller, 2014), by showing how individuals' idiosyncratic preferences about the methods used to estimate financial models can be associated with market-wide disagreement.

3 Conceptual Framework

We start this section by introducing the conceptual model used by equity analysts to estimate firms' discount rate and cost of equity. We then use those models to precisely illustrate the sources of estimation ambiguity that we investigate in the paper.

We can express discount rates, the WACC, as:

$$WACC_{i,t} = W^E_{i,t} * (rf_t + \beta_{i,t} * ERP_t) + W^D_{i,t} * (1 - \tau_{i,t}) * r^D_{i,t} \tag{1}$$

Where W^E and W^D are the weights of equity and debt in the firm's capital structure, respectively. rf is the risk-free rate, ERP denotes the equity risk premium, β corresponds to the CAPM beta, τ is the firm's marginal tax rate, and r^D is the cost of debt. While this model is conceptually clear, and it provides guidelines about how these variables interact together in the overall computation of a firm's cost of capital, substantial ambiguity remains in the measurement and estimation of some of its parameters, making room for individuals' subjective interpretation. In that sense, studying how financial professionals determine their CAPM beta is well suited for our analysis: there is a clear theoretical framework underpinning its estimation, but limited guidance about its exact estimation.

Estimation ambiguity materializes in many aspects of the CAPM beta measurement. Focusing on the basic formula:

$$r_{i,t} - rf_t = \alpha + \beta * (r^M_t - rf_t) \tag{2}$$

Where $r_{i,t}$ is the firm's stock market return at a predetermined frequency, and r_t^M is the market return. While the underlying theory is simple, analysts have limited guidance in setting up the regression design when it comes to choosing (i) the number of days, months, or years over which the analysis should be performed (i.e., beta's horizon), (ii) the market benchmark to be used to proxy for the market returns r_t^M, and (iii) the frequency of the returns (i.e., daily, weekly, or monthly). Different assumptions about these elements of the CAPM regression can yield material differences in the estimate, and thus, generate different costs of capital for the firm.

We focus our analysis on the effect of beta's horizon, as this dimension of estimation ambiguity can be both directly investigated in the numerical data and equity analysts generally discuss this modeling choice in equity reports. This added level of visibility allows us to benchmark our regression results with a textual analysis from discussions in equity reports to verify our conclusions. This permits us to confirm the conclusions of our empirical analysis with precise discussions about how professionals determine their cost of capital. In the context of our analysis, estimation ambiguity arises because analysts know that distinct horizons, or combination of horizons, can best represent the firm's true beta, but they do not know the weight to put on each of those cases (Figure 2).

Lastly, we note that not all financial professionals personally estimate their CAPM betas. Mainstream data providers such as Bloomberg, Refinitiv, and Yahoo also provide their own estimates. While not explicitly acknowledged by some of the analysts, taking the CAPM beta from those databases implies that professionals implicitly adopt the provider's distinct methodology. For example, Bloomberg by default uses a 2-year horizon at the weekly frequency, while Refinitiv favors using a 5-year horizon at the monthly frequency.

In contrast, determining other components of the WACC, such as the risk-free rate, is a more direct process, which offers a natural benchmark to evaluate the nature of estimation ambiguity. For example, Figure 7 shows that analysts nearly unanimously (91%) use the 10-year domestic treasury yield to estimate the risk-free rate in their models. Ultimately, our setting allows us to hold the model used by our financial agents fixed and determine if inputs suffering from estimation ambiguity exhibit greater heterogeneity.

4 Methodology and Data

4.1 Data Source

The bulk of our data is sourced from equity research reports (i.e., the original documents) published by sell-side analysts. We start with an initial batch of 157,549 equity reports with mentions of the keywords “DCF”, “discounted cash flow”, and “weighted average cost of capital”, or “WACC” from 42 major equity research firms. We restrict the time window to reports published in the first quarter of the calendar year (January 1^{st} to April 1^{st}) from 2000 to 2023. This ensures that our data are systematically measured at a similar time point in the year. In cases where analysts publish more than one report on the same firm during the first quarter, we systematically keep the earliest publication in that calendar year to avoid duplicates for a given analyst-firm-year pair. This procedure results in 45,992 equity reports that include at least one of our variables of interest, and for which each firm-year pair is covered by at least 2 analysts.

We collect numerical values for each of the inputs in four steps. First, documents are pre-processed using a Python program to identify sections of text, tables, and figures containing relevant information for the study (e.g., discount rates, risk-free rate, or equity betas). Second, we convert each of these forms of publication into text snippets. Third, for each variable, we use artificial intelligence to extract the numerical value from the snippets. Fourth, we export the text snippets and the numerical values extracted by artificial intelligence to Excel – and our research team manually verifies every single number. This last step of the collection effort (manual verification) is crucial to ensure the integrity of the data used in the analysis. While artificial intelligence is an efficient tool for text extraction, error rates in the processing of complex sentences can be well above acceptable levels when AI is left unsupervised (Gilardi et al., 2023).

The disclosure of this information is done on a purely voluntary basis. However, prior literature has found that the intensity of information disclosure of DCF modeling assumptions is positively associated with report accuracy (Asquith et al., 2005; Hashim and Strong, 2018), and more detailed information disclosure leads to larger market reactions following changes in recommendations (Huang et al., 2023). Moreover, by detailing their valuation theses, informed analysts have the opportunity to differentiate their work from their uninformed rivals, gaining credibility in the process. More closely related to the study of analysts' discount rates and their inputs, Décaire and Graham (2023) shows that the average discount rate reported by analysts is an unbiased predictor of firms' one-year realized returns. They interpret this result as suggestive evidence that data is informative and captures inherent features associated with stocks' returns.

4.2 Firms and Coverage

The equity reports are produced by 42 of the largest equity research departments operating throughout the world. Because we aim to understand disagreement in the valuations of the same security during the same time period, we restrict our sample to firm-year observations with equity reports produced by at least two analysts. Panel A of Table 1 highlights the specific coverage for each of the DCF inputs in terms of number of firms and firm-year observations that meet this requirement. For example, in the discount rate sample, the 45,992 reports cover 4,261 firms located in 63 countries during the 2000-2023 period. Panel A also shows that as the DCF inputs increase in specificity, e.g., moving from discount rates to the components of a standard weighted average cost of capital (WACC) calculation, our sample sizes reduce as fewer equity reports include every such input.

Table 1 Panel A also reports on a limited set of firm characteristics for our discount rate sample. For example, the average firm is covered by 10 analysts that we can identify over its life and is included in the sample for 4.0 years. As with standard commercial databases that report on analyst expectations (e.g., EPS forecasts via I/B/E/S), our sample skews older and larger than the full universe of publicly traded firms that could be downloaded from Compustat or CRSP. However, Décaire and Graham (2023) show that the characteristics of the firms in their sample (which closely matches out sample) are similar to the firms from I/B/E/S data in terms of size and investment intensity.

In terms of geographic coverage, 34% in North America, 33% of firms have their headquarters located in Europe,18% in Asia, 11% in Oceania, 3% in South America, and 1% in Africa. The reports are produced by equity analysts located in Europe in 45% of the cases, 22% Asia, 18% in North America, 11% in Oceania, 2% in South America, and 2% in Africa.

Two dozen NAICS industry sectors (2-digit) are represented in our sample, with the eight largest broad sectors accounting for 74% of the total coverage: 34% for manufacturing (NAICS 31-32-33), 16% for information (NAICS 51), 9% for professional services (NAICS 54), 6% for retail trades (NAICS 44-45), 6% for mining and oil & gas (NAICS 12), 4% for transportation (NAICS 48-49), 5% for utilities (NAICS 22), and 4% for finance and insurance (NAICS 52). Overall, these statistics suggest that our sample is comprehensive, representative, and comparable to its commercial counterparts.

5 Results

5.1 Required Rate of Return Heterogeneity and its Sources

This subsection examines the dispersion in the analysts' estimates of corporate discount rates. First, we document the magnitude of within-firm variation in contemporaneous WACC estimates across analysts. Second, we decompose the sources of this variation and evaluate their relative importance.

Table 1 provides descriptive evidence on the inputs and outcomes in WACC estimation from analyst reports. The bottom pane of Panel A tabulates the absolute values of average pairwise differences between analysts evaluating the same firm in the same calendar quarter. The table reveals significant cross-sectional dispersion. For example, the mean pairwise difference in WACC estimates ($WACC_{|A-B|,i,t}$) for a given firm is 1.4%. This differential is statistically significant and economically important. For the median sample firm, a 1.4% increase in WACC corresponds to a 17.3% drop in the present value of expected cash flows and price per share.

Figure 3 displays the distribution of the absolute values of pairwise differences in estimated WACCs for the same firm in the same quarter. The black vertical bar (centered on the absolute difference of zero) shows that fewer than 5% of analyst pairs arrive at the same WACC estimates, after correcting for the rounding error. In contrast, nearly 50% of the pairwise differences in the estimated WACC exceed 1 percentage point, and over 10% of the differences exceed 3 percentage points.

Table 2 underscores the quantitative importance of WACC differentials by showing that higher discount rates are not systematically offset by higher cash flow growth rates in the same valuation reports. This table reports the frequency of WACC and terminal growth rate (TGR) estimates from the same valuation models that fall above or below their respective consensus (mean) values for the same firm in the same year. The right-hand side column shows that in nearly *half* (45.4%) of the valuation models, an above-consensus estimate of the discount rate is associated with a below-consensus estimate of TGR (or vice versa), suggesting that they diverge in the opposite directions. This diverging pattern speaks against the joint manipulation of these valuation drivers to achieve desired outcomes. Overall, the dispersion in WACC appears to be distinct from the dispersion in cash flows.

The stark variation in within-firm WACC estimation raises the question of why financial analysts with comparable analytical tools arrive at different results when performing a routine task that relies largely on public information. We conjecture that the variation in WACC outcomes is related to estimation uncertainty. Since theory does not prescribe the preferred estimation method, analysts face a multitude of plausible methodological choices, and these choices have profound effects on estimation outcomes. To study the drivers of dispersion in WACC estimates, we decompose the estimated discount rates into two components: (1) the risk-free rate and (2) the compensation for exposure to market risk ($\beta \times ERP$). As before, we compute the absolute pairwise differences between analyst reports for the same firm in the same calendar quarter. Table 1 reports that the mean (median) values for $rf_{|A-B|,i,t}$ and $(CAPM\ Beta \times ERP)_{|A-B|,i,t}$ are 1.0% (0.8%) and 1.6% (1.1%), respectively. That is, the absolute magnitude of the mean spread in CAPM Beta multiplied by the equity risk premium is 60% larger than that of the risk-free rate, a difference significant at the 1% level.

Figure 4 depicts the distribution of the absolute values of pairwise differences in the estimated risk-free rate and estimated ($CAPM\ Beta \times ERP$), displayed in Panels A and B, respectively. Panel A in Figure 4 shows that the distribution of the risk-free rate differentials is significantly tighter, as expected. For example, nearly 40% of the pairwise differences in the risk-free rate estimates do not exceed 50 basis points.

Panel B suggests that the distribution of heterogeneity in CAPM betas and ERP differs sharply from that of the risk-free rate. The black vertical bar centered on zero shows that only 2% of the analysts arrive at the same contemporaneous estimates of beta and ERP for the same firm. The overall distribution $(CAPM\ Beta \times ERP)_{|A-B|,i,t}$ is significantly wider than that of the risk-free rate. Only 20% of the differentials do not exceed 50 basis points (vs. nearly 35% of such cases for the risk-free rate), and the flatter shape of the distribution indicates greater dispersion.

In summary, analysts produce significantly different WACC estimates for the same firm. This variation is economically important and distinct from other modeling inputs. Analysts converge on the estimates of the risk-free rate, but diverge on the compensation for a stock's exposure to market risk.

5.2 Estimated Cost of Equity Capital

The conceptual framework in Section 3 suggests that estimation ambiguity introduces more uncertainty for model inputs that require estimation (such as betas) than for the inputs that are observable in public data (such as the risk-free rate). The distributions of the pairwise differences for $(CAPM\ Beta \times ERP)$ and rf presented in the previous section appear to align with the predicted empirical pattern. This subsection provides additional evidence on this prediction by studying which model parameters explain the dispersion in estimation outcomes.

To isolate the effects of model estimation from firm characteristics and temporal factors, we exploit variation in estimation uncertainty across different model parameters for the same focal firm, while holding constant the estimated model and the time of its estimation. Our first test decomposes the discount rate heterogeneity between two analysts evaluating the same firm i at the same point in time t.

Using the definition of WACC, we can express this decomposition as follows:

$$\underbrace{r^E_{A,i,t} - r^E_{B,i,t}}_{r^E_{A-B,i,t}} = \underbrace{rf_{A,i,t} - rf_{B,i,t}}_{rf_{A-B,i,t}} + \underbrace{\beta_{A,i,t} * ERP_{A,i,t} - \beta_{B,i,t} * ERP_{B,i,t}}_{[\beta * ERP]_{A-B,i,t}} \tag{3}$$

where $r^E_{\cdot,i,t}$ denotes the analyst's estimate of the cost of equity, and the terms below the underbraces correspond to the notation shorthands to facilitate exposition. This decomposition allows us to determine whether changes in the cost of equity are driven by the differences in the estimated risk-free rate or the compensation for market risk. The order of subtraction,

$r^E_{A,i,t} - r^E_{B,i,t}$ or $r^E_{B,i,t} - r^E_{A,i,t}$, is inconsequential for the decomposition results.

To capture the cost of equity differential attributable to each model input, we estimate the following equation:

$$var(r^E_{A-B,i,t}) = cov(rf_{A-B,i,t}, r^E_{A-B,i,t}) + cov([\beta * ERP]_{A-B,i,t}, r^E_{A-B,i,t}) \tag{4}$$

$$1 \approx \frac{cov(rf_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})} + \frac{cov([\beta * ERP]_{A-B,i,t}, r^E_{A-B,i,t})}{var(r^E_{A-B,i,t})} \tag{5}$$

By construction, the combined variation attributable to the risk-free rate and the compensation for market risk ($\beta \times ERP$) explains 100% of the heterogeneity in the estimated cost of equity. The two right-hand side terms in Equation (5) are estimated using a univariate linear regression of their respective right-hand side terms in Equation (3).

Table 3 shows the estimation results. Panel A corresponds to the full sample, while Panels B and C refer to the subsamples of domestic and international firms, respectively. Across all panels, columns 1-3 focus on the share of the heterogeneity in the cost of equity attributable to the choice of the risk-free rate. Columns 4-6 perform an equivalent decomposition for the compensation for market risk ($\beta \times ERP$).

The results in Panel A of Table 3 show that the dominant majority (79%) of the within-firm variation in the cost of equity is attributable to analyst dispersion in the compensation for risk ($\beta \times ERP$). The remaining one-fifth (21%) of the variation is attributable to the differences in the estimated risk-free rate. This relationship remains robust as we gradually saturate specifications with fixed effects that absorb temporal variation in the cost of equity (year fixed effects) and the cross-firm heterogeneity (firm fixed effects). With the inclusion of both groups of fixed effects, the results are robust to exploiting only within-firm variation in analyst estimates derived in the same calendar year.

Panels B and C split the sample between U.S. and foreign firms. These splits are motivated by the variation in the choice set for the risk-free proxy between domestic and international settings. While the selection of the risk-free rate in the U.S. is usually confined to U.S. Treasuries of different maturities, the risk-free asset for foreign firms is far less obvious. Common risk-free proxies in analyst reports for foreign firms include sovereign bonds issued by the government of the U.S, the firm's own country, or another country in the firm's region (e.g., Germany in the Eurozone). These choices generate additional cross-sectional variation in the risk-free rate.

Panels B and C show that the choice of the risk-free proxy explains twice as much variation in the cost of equity for international firms (22.5%) than it does for U.S. firms (11.3%). Moreover, for U.S. firms, the risk-free rate dispersion is a statistically insignificant factor in explaining the discrepancies in analysts' estimated cost of equity capital.

In summary, most of the cross-analyst variation in a firm's cost of capital is attributable to parameters that require estimation—namely, beta and the equity risk premium. The choice of the risk-free rate explains a modest share of the variation. The quantitative importance of the risk-free parameter increases in an international setting, consistent with an expanded choice set of risk-free proxies.

5.3 Textual Analysis of Analyst Methodologies

This section uses textual analysis to gain insights into analysts' methodological choices and provide evidence on their data sources, estimation horizons, and parameter values. We evaluate how these methodological choices vary across analysts and across different reports of the same analyst.

5.3.1 CAPM Beta

Our first analysis studies analysts' methodological choices in the estimation of CAPM betas. In these tests, we restrict our sample to the subset of equity reports that provide sufficient detail on beta estimation. This sample includes 1,023 beta estimates for 794 firms derived by 508 analysts across 36 brokerage houses. Panel B in Table 1 provides descriptive statistics for this sample and shows the distribution of beta estimates.

Figure 5 describes analysts' decisions on key estimation parameters in deriving CAPM betas: return frequency (Panel A) and trailing estimation horizon (Panel B). The evidence indicates substantial variation in analysts' methodological choices. Panel A shows an almost even split between a preference for weekly vs. monthly return frequency, which appear in 54% and 42% of beta estimations, respectively. A small minority of reports use daily frequency, but this approach is far less common (4.7%).

Panel B focuses on the trailing estimation horizon—that is, the historical sample period for beta estimation. The distribution of the trailing horizon is bimodal, and the majority of analysts elect to use either the trailing 24 months (32%) or the trailing 60 months (40%). Aside from these common approaches, a significant minority of reports (16%) select the estimation period of the trailing 36 months, and the remaining 12% are scattered across a wide range of estimation windows—from the trailing 12 to 120 months.

The trailing estimation horizon reveals a strong analyst preference for multiples of 12 months. For example, none of the analyst reports uses a trailing estimation horizon corresponding to an uneven number of years, such as 18 months or 50 months, although theory does not prescribe an annual reference point. Such a unanimous preference for an annual count in the trailing horizon could reflect a simplifying heuristic to reduce the dimensionality of the estimation task, consistent with anchoring (Tversky and Kahneman, 1974). Another possible explanation is an attempt to account for intra-year seasonality in market returns and product sales.

After identifying significant variation in estimation methods across analysts, we turn to the within-analyst analyses and study how estimation methods vary for the same analyst over time and across stocks. Figure 6 shows the number of different trailing estimation horizons used by the same analyst across different reports.

Figure 6 reveals remarkable persistence in an analyst's estimation choices over time and across firms. Over three quarters of analysts use the same trailing horizon in beta estimation across all of their valuation reports in our sample. Panels A restricts the sample to analysts with multiple reports. The results in Panel A show that 78% of the analysts use only one, invariant estimation horizon (e.g., trailing 60 months) across all of their reports in our sample. The individual-level persistence in estimation horizon is even greater, since an additional 4% of the analysts apply the same weighted average method in evaluating the trailing horizon, for example, by equally weighting their estimates from the trailing 24 months and the trailing 60 months.

Panel B shows that the fraction of analysts using the same estimation horizon across all of their reports remains similar if we focus only on analysts with at least four valuation reports. For the average analyst in this subsample, we observe 567 reports with complete estimation data. The results show that 84% of the analysts use only one estimation horizon across all reports. A minority of analysts switch between two horizons (12%), and relatively few use three (4%).

Panel C restricts the sample to analysts with complete valuation reports for multiple stocks and over multiple years. The results reveal a similar pattern. Over three-quarters of analysts apply the same estimation horizon across all of their covered stocks and over the entire sample period.

In summary, we find large cross-analyst variation in beta estimation methods. Analysts use a variety of trailing estimation horizons and return frequencies, and no approach accounts for the dominant majority of observations, consistent with estimation ambiguity. In contrast, estimation choices of the same analysts are remarkably persistent over time and across firms. The majority of analysts appear to resolve estimation ambiguity by selecting one set of estimation parameters and consistently applying it across all settings.

5.3.2 Risk-Free Rate

This subsection studies analysts' parameter choices for the risk-free rate. Theory postulates that the risk-free parameter captures an annualized rate of return that can be earned without taking any risk (Sharpe, 1964; Lintner, 1965). Yet, while the CAPM is a one-period model, it does not specify the length of the one period or the appropriate asset type, leaving leeway for subjective interpretation. This section offers evidence on the analysts' choice of two key parameters in selecting the risk-free rate: (1) the risk-free security and (2) the maturity of the risk-free asset.

Empirical work often questions the mere existence of a riskless asset and diverges on the recommended proxies (e.g., see Blitz, 2020). Early work on CAPM recommends short-maturity T-bills, emphasizing their liquidity and low duration. Later studies suggest medium-term Treasuries as a better match for an investor's horizon. Yet another strand of the literature proposes Treasury Inflation Protected Securities (TIPS), stressing the importance of inflation risk. The debate on the risk-free asset is even more contentious in an international setting, where agents can choose from domestic, regional, or global proxies. As a result, although the risk-free rate does not require estimation, this input is subject to meaningful uncertainty.

To study analysts' inputs for the risk-free rate, we focus on a subsample of equity reports that provide sufficient detail on this parameter. This sample consists of valuation reports for 1,679 firms across 3,284 firm-years and 48 brokerage houses. Panel C in Table 1 shows summary statistics for this subsample.

Panel A in Figure 7 shows the distribution of the risk-free benchmarks and compares analysts' methodological choices between U.S. firms and foreign firms. Blue bars describe the entire sample, while grey and red bars correspond to firms headquartered in the U.S. and abroad, respectively. The panel depicts the frequency of three common groups of benchmarks: (1) domestic, (2) regional, and (3) American, which correspond to government securities issues by the focal country, another large economy in the region, or the United States, respectively. For example, in the context of a hypothetical firm in the Netherlands, these three categories correspond to the government bonds of the Netherlands (domestic), Germany (regional), and the United States (American).

The results reveal a stark contrast in the risk-free benchmark between domestic and foreign firms. As expected, nearly all reports for American firms select U.S. Treasuries as the risk-free asset. In contrast, there is much less consensus across foreign firms. Only a slight majority of the valuation models for foreign firms select the government bonds of the firm's own country (55%). This choice is more common for large and well-developed economies, such as Germany, France, and Japan. Another 31% of valuation reports for foreign firms rely on the U.S. Treasuries as the risk-free benchmark. Yet another 8% use the government bonds of the leading economy in their region outside the firm's own country. These choices are more frequent for firms in smaller countries with less liquid government bond markets, such as Austria and Belgium, as well as for firms in large economies that have non-negligible default risk, such as Brazil and India.

Panel B in Figure 7 focuses the distribution of maturities of the risk-free assets. The results show that 90% of the reports select the 10-year maturity of the risk-free asset. This preference holds robustly for firms headquartered in the United States (80%) and abroad (94%). The second most popular maturity choice is 30 years (7% of all firms), and it is significantly more common for firms in the U.S. (20%) than those overseas (2%). Finally, a small minority of the valuation models select other maturities for the risk-free asset, mostly around the five-year horizon.

In summary, analysts' inputs for the risk-free rate show less variation than their beta estimates. In the U.S., most analysts select the 10-year U.S. Treasury as their riskless proxy. Abroad, analysts converge to a similar maturity preference but diverge on their choices of the riskless asset.

5.3.3 Implications for Estimation Ambiguity

This subsection studies the implications of our evidence for professional decision making under ambiguity. We first discuss three common strategies proposed in the theoretical literature as agents' responses to estimation ambiguity: (1) justifiable worst-case scenario, (2) outcome aggregation, and (3) reliance on heuristics. Next, we juxtapose our findings with these predictions to identify the framework that appears most consistent with the observed analyst behaviors.

The results in Section 5.3 yield three insights about analyst responses to ambiguity in estimating a firm's discount rate. First, analysts estimate the same empirical model, but diverge on their methodological choices and parameter inputs. Second, the divergence in analysts' inputs is greater for parameters that require estimation rather than those that are observable in the data. Third, despite the large variation in methodological choices across analysts, the estimation choices of each agent are remarkably persistent. That is, analysts respond to estimation ambiguity by adopting one set of estimation parameters and applying this approach consistently across their estimation tasks. Overall, the persistence in analysts' methodological choices underscores the importance of understanding which theoretical model best describes their revealed preferences.

The first group of theories posits that agents respond to ambiguity by adopting robust strategies (Tsoy, 2023). Under this approach, ambiguity-averse agents evaluate estimation tasks by selecting a method that would be optimal under a justifiable worst-case scenario (Gilboa and Schmeidler, 1989). For example, an analyst following a robust strategy would select a beta estimation period that generates the most liberal (conservative) beta estimate to prevent an overestimation (underestimation) of a firm's cash flows.

Figure 8 provides evidence suggesting that a justifiable worst-case strategy is unlikely to be a common approach in beta estimation. Panels A and B show the cross-sectional distribution of firms for which a given trailing horizon in beta estimation would result in the most conservative (i.e., lowest) beta estimates or the most liberal (i.e., highest) beta estimates, respectively. These distributions are estimated for each month between 2000 and 2023, and the x-axis corresponds to the timing of estimation. The five most common estimation horizons (from 24 to 72 months) are marked by bars of different colors.

The results in Figure 8 indicate that analysts are unlikely to select a trailing horizon that would be optimal under a justifiable worst-case scenario. First, none of the common trailing horizons in beta estimation consistently produces the most conservative or the most liberal beta estimates. For example, the 24-month and 72-month horizons make up the highest percentage of both the most conservative estimates (Panel A) and the most liberal estimates (Panel B). Second, the distribution shows substantial time-series variation, suggesting that the choice of the most conservative or liberal estimation horizon varies significantly over time. Thus, an analyst following an optimal strategy under a justifiable worst-case scenario would frequently switch their estimation horizons. In contrast, over three quarters of the analysts select only one, time-invariant estimation horizon for all of their reports.

Table 5 corroborates the conclusion that the choice of the most conservative or liberal estimation horizon varies significantly over time. This table shows the annual autocorrelations for the most conservative estimation horizon (Panel A) and the most liberal horizon (Panel B), which correspond to the lowest and highest beta estimates, respectively. Across columns 1-5, the dependent variable is a binary indicator equal to one of the five common estimation horizons (24, 36, 48, 60, and 72 months). The main independent variable is the one-year lagged value of the dependent variable. The results in Table 5 show that no single estimation horizon would correspond to a worst-case strategy consistently, year-over-year. For example, the highest autocorrelation across all of the estimation horizons is 0.299, an estimate that corresponds to the 72-month horizon as the most liberal strategy (column 5 of Panel B). Moreover, the majority of autocorrelation coefficients across both panels are negative, and these estimates are significant at 1%. Thus, a beta estimation horizon that captures the most liberal or conservative strategy during a given year is less likely to capture this strategy the following year. These results confirm that agents who respond to estimation ambiguity by adopting an optimal worst-case strategy should frequently alter their estimation horizon, and this pattern is inconsistent with our evidence.

A second group of theories argues that agents respond to estimation ambiguity by aggregating a model's outcomes across its likely specifications, following a Bayesian selection criterion Giacomini et al. (2019, 2022). We find only sporadic evidence of outcome aggregation in analyst reports and conclude that this approach is relatively uncommon. In particular, only a small minority of analysts (3-4%) adopt any form of outcome aggregation, such as computing a weighted average of beta estimates obtained from different trailing horizons. In most such cases, the weighting schemes are simplistic and anchored on round numbers, such as an equal weighting of beta estimates derived from short and long horizons.

A third group of theories suggests that agents respond to ambiguity by restricting their consideration set to a subset of familiar scenarios and relying on simplifying heuristics (Heath and Tversky, 1991; Fox and Tversky, 1995). Our evidence is broadly consistent with these predictions. First, despite a multitude of possible estimation periods, the dominant majority of analysts (85%) restrict their choice set to a small subset of trailing horizons—namely, two, three, or five years. Second, while theory does not prescribe annual reference points in model estimation, all analysts unanimously select estimation horizons in multiples of 12 months, consistent with a simplifying heuristic. Third, analysts' methodological preferences show strong intertemporal anchoring across multiple settings. These decisions appear to be driven by idiosyncratic analyst-specific criteria and remain persistent over time. These empirical patterns are consistent with behavioral models of resolving ambiguity with a random draw.[3]

In summary, our evidence is most consistent with behavioral models of resolving ambiguity. When faced with ambiguity, analysts appear to use heuristics, rely on intertemporal anchoring, and make idiosyncratic agent-specific choices. In contrast, we find only modest support for theories predicting aggregation of estimation outcomes, and only a small fraction of analysts matches these predictions. Finally, we find no evidence of a widespread use of

[3]For example, Raiffa (1961) argues that when agents face ambiguity and existing strategies are hardly applicable (e.g., Gilboa and Schmeidler (1989) or Giacomini et al. (2019)), they can arrive at a decision by using a random draw over the methods under consideration

max-min criteria.

5.4 What Explains Analysts' Choices of CAPM Beta?

The wide cross-sectional variation in analysts' methodological choices raises the question about the drivers of their decisions. This section seeks to understand whether the analysts' estimation choices reflect the rules of the brokerage house, the attributes of the focal firm, the characteristics of the analyst, or the idiosyncratic individual responses to estimation uncertainty.

5.4.1 Analyst and Brokerage Effects

As a first step, we estimate a variance decomposition (ANOVA) for the dispersion in the estimated CAPM betas across three cross-sectional drivers: (1) focal firm, (2) brokerage house, and (3) analyst. Specifically, the dependent variables are the indicators that identify the firm evaluated in the report, the brokerage house issuing the report, and the lead analyst performing the valuation.

Table 6 reports the results of the variance decomposition, and Panel A shows the estimates for the full sample. The evidence in Panel A indicates a strong model fit, with an R^2 of 78% and an adjusted R^2 of 57%. As expected, firm-level indicators account for the majority (78%) of the variation because they capture the differences in firms' exposure to systematic risk.

The evidence on the role of analysts and brokerage houses is perhaps less expected. We find that the effect of individual analysts is significantly more important than that of their brokerage houses. For example, analyst indicators account for 19% of the variation explained by the model and produce an adjusted partial R^2 of 28%. In contrast, brokerage house indicators explain a miniscule share of the variation (2%) and generate an adjusted partial R^2 of just 0.01

We alert the reader to two methodological caveats in interpreting the results in Panel A of Table 6. First, the order in which cross-sectional indicators are included in the empirical model can affect the results. To mitigate this effect, we estimate the sum of squares sequentially, following the approach in (Smith and Cribbie, 2014). As another precaution, we also include the brokerage house indicators before the analyst indicators. If the order of variable addition induces a bias, such a bias would result in *overestimating* the explanatory power of the brokerage house relative to the analyst.

A second caveat is that the number of firms and analysts in the model is far greater than that of brokerages. This imbalance leaves more degrees of freedom for firms and analysts and, as a result, mechanically increases their portion of model-explained variation relative to brokerages. While the partial R^2s in column 3 mitigate this concern, we perform two additional ANOVA decompositions in restricted samples to validate the estimation results. Panel B repeats the ANOVA decomposition for a subset of analysts with at least five valuation reports. Panel C performs the same decomposition after restricting the sample to analysts and firms with at least five valuation reports.

The results in Panels B and C in Table 6 are concistent with the evidence in the full sample. The adjusted partial R^2 for the analyst indicators remains between 24 and 28% across each specification, consistent with the estimate of 28% in the full sample. Similarly, the adjusted partial R^2 for brokerage houses never exceeds 1%, consistent with prior evidence. The contrast in the decomposition results for analysts and brokerages suggests that beta estimation methods reside with the analysts rather than being dictated by their brokerages.

In summary, analyst fixed effects explain significantly more within-firm variation in beta estimates than brokerage fixed effects. This evidence suggests that model estimation choices reflect the decisions of analysts rather than merely follow the templates of their brokerages.

5.4.2 Analyst Characteristics

Given the importance of analyst fixed effects for beta estimates, this subsection offers a closer look at analyst characteristics. We examine whether analysts' estimation choices vary with their personal attributes, such as education, location, and demographics.

To collect information on analyst characteristics, we match the lead analysts in our sample with a major professional networking database. To validate the matches, we compare the analyst's employer listed in the valuation report against his or her work history in the professional networking database. After confirming the matches, we obtain information on analysts' demographics (such as age, gender, and race), education level, and office location (inferred from the office phone number in the valuation report).

To extract the variation in beta estimates attributable to analysts, we estimate the following fixed effect regression:

$$CAPM\ Beta_{a,i,t} = \alpha + \beta_a + \gamma_i + \delta_b \tag{6}$$

where a denotes an individual analyst, i denotes a firm, and b denotes a brokerage.

After estimating this regression, we save the estimated coefficients $\hat{\beta}_a$, following the approach in Bertrand and Schoar (2003). These coefficients capture an analyst's unobserved impact on beta estimation, conditional on firm and brokerage fixed effects.

We use the estimated analyst coefficients as the dependent variable in our next level of variance decomposition. In this ANOVA decomposition, we estimate a model with a cross-section of the analysts' personal characteristics: gender, race, graduate degree indicator, and office location (country indicator). Since the coefficients on analyst fixed effects are estimated across firms and over time, we collapse our data to the analyst level and arrive at a sample of 154 observations. In this subsample, the estimated discount rates are statistically indistinguishable from those in the full sample.

Panel A in Table 7 shows the results of the ANOVA decomposition. The evidence indicates that the observed analyst characteristics have little explanatory power, whether considered jointly or separately. For example, the model generates an R^2 of 9% and an adjusted R^2 of close to 0%. The near-zero R^2 suggests that beyond the mechanical effect of the categorical variables, the model with analyst characteristics explains no meaningful variation in the analyst fixed effects. A similar conclusion emerges from the analysis of specific analyst characteristics. In particular, none of the analyst attributes contributes a meaningful explanatory power, as shown by the range of partial R^2 from -2% to 2%.

Panel B in Table 7 adds an additional indicator variable that identifies young analysts. This binary indicator is equal to one for analysts whose age is below the sample median (48 years) at the time of the valuation report. The results in Panel B are very similar to those in Panel A and suggest that observable analyst characteristics do not explain the variation in the estimated betas.

Figure 9 provides additional evidence on each of the examined analyst characteristics.

Panels A through E show the distributions of beta estimation horizon for analysts sorted by gender, graduate education, race, office location, and the focal firm's industry, respectively. Thin vertical lines correspond to the 90% confidence intervals.

Panel A compares beta estimation horizons between male and female analysts. While there are some differences across the frequency of several estimation horizons, most of them fall short of being statistically significant. The overall distribution for *all* beta horizons for males and females look strikingly similar.

Other panels yield similar conclusions. For example, Panel B finds no significant differences in the estimation horizon between analysts with graduate education and their peers without graduate degrees. This conclusion holds separately for all beta estimation horizons. Overall, the evidence in Figure 9 does not reveal a reliable pattern between analyst estimation choices and any of the examined personal characteristics.

Table 8 provides regression evidence on the relationship between analyst characteristics and their methodological choices in beta estimation. Using a multinomial logit specification, this table studies whether analyst characteristics predict the likelihood of selecting one of the three most common beta estimation horizons, which correspond to the trailing 24 (base case), 36, or 60 months.

The results in Table 8 suggest that most analyst characteristics have little predictive power for the choice of the estimation horizon. For example, most of the coefficients are statistically insignificant, have small point estimates, and flip signs between columns 1 and 2. Column 3 provides a test of the equality of distributions across *all* horizons. The results in column 3 show that the distributions of beta horizons do not significantly depend on analyst attributes, a conclusion that holds for all examined analyst characteristics.

In summary, common professional attributes of equity analysts, such as their education, location, and demographics, have little explanatory power for explaining their estimation choices. It is possible that analysts' modeling choices reflect idiosyncratic agent-specific factors unrelated to common observables, such as private preferences, formative experiences, or on-the-job mentorship.

5.4.3 Does Estimation Ambiguity Matter for Real Outcomes?

In the final section, we study the relationship between estimation ambiguity and stock market outcomes. Establishing this link at the micro-level is challenging because it requires observing an agent's model, its input parameters, and estimation outcomes, and such a combination is rarely feasible outside of a controlled experiment (Asparouhova et al., 2015). Our empirical setting allows us to meet these conditions for an important set of financial experts tasked with information production for other market agents.

Our analysis is rooted in a large theoretical literature that predicts a positive link between the heterogeneity in agents' private valuations and their trading volume in secondary markets. Theory postulates that investors trade securities primarily because they have different private valuations ((Milgrom and Stokey, 1982; Karpoff, 1986; Banerjee and Kremer, 2010). Consistent with these predictions, empirical work finds a surprisingly high amount of trading for U.S. stocks, given the relative information transparency in the U.S. market and a modest amount of portfolio rebalancing. In a survey of this literature, Hong and Stein (2007) conclude that the bulk of trading must come from differences in investors' valuation models "that lead traders to disagree about the value of a stock even when they have access to the same information sets" (p. 112).

Since nearly all analysts use the same estimation model for the cost of equity, and since its inputs are based on market data, we have a convenient setting to test these predictions within the same modeling framework and a common information set. To execute this analysis, we introduce a measure of a stock's scaled trading volume, $FVOL$, defined as the total number of shares traded in a month scaled by the total number of shares outstanding, following prior work (Ajinkya et al., 1991). The independent variable of interest in this analysis is a measure of discount rate heterogeneity. Because our analysis is conducted at the firm-year level, this measure is defined as the spread between the maximum and minimum estimates of WACC by all analysts covering a firm in a given year with available WACC data.

The discount rate heterogeneity corresponds to $\max_{a \in A}(WACC_{i,t_1,T}) - \min_{a \in A}(WACC_{i,t_1,T})$, where a represents an analyst in the entire set of analysts A for a given firm-year observation, i indexes firm, and times t is the month when the second forecast is published (e.g., the time when the disagreement was created). Our results are robust to other measures of the discount rate dispersion, such as the standard deviation.

Panel A in Table 9 shows the results using total trading volume scaled by shares outstanding. Column 1 reports the estimates from a regression that includes no control variables. The evidence indicates a positive and significant association between the WACC dispersion and trading volume, and this result is significant at 1%. To account for temporal persistence in a stock's trading volume, column 2 adds controls for the lagged trading volume, following (Cookson and Niessner, 2020). The positive coefficient on the WACC dispersion shrinks in magnitude but remains significant at 1%. The R^2 jumps from 1% to 80%, suggesting that last month's trading volume explains the vast majority of the variation in this month's volume.

Columns 3 and 4 add year-month and firm fixed effects, respectively. The R^2 in column 4 with firm fixed effects is 88%, and the coefficient on the variable of interest is 0.154, still significant at the 1% level. Column 5 adds additional control variables to capture the common drivers of trading volume identified in prior work (see the discussion in Cookson and Niessner, 2020). These controls include the natural logarithm of the firm's market capitalization, the cumulative return and return volatility over the three trailing months, the cumulative return from 4 to 12 months prior, and finally, the number of months between the first and last analyst forecasts. After including these controls, the coefficient on WACC heterogeneity is 0.120, and it remains significant at 1%.

Column 6 shows that the relationship between the discount rate dispersion and trading volume is robust to controlling for disagreement in growth expectations. As a proxy for disagreement in long-run growth rates, we use an analogously-constructed spread between the terminal growth rates across analysts covering the same stock.

Panel B in Table 9 shows the results using an alternative measure of trading volume akin to Cookson and Niessner (2020). In particular, *Abnormal FVOL* is equal to the monthly trading volume minus the mean monthly trading volume over the previous twelve months, or one year of trading volume, scaled by the the total common shares outstanding in month t. The results in Panel B are qualitatively and quantitatively similar to those in Panel A. The economic magnitudes in both Panels A and B are both significant. In particular, a one-standard deviation increase in WACC disagreement is associated with a 1-2% increase in total trading volume, while the same increase in WACC disagreement is associated with a 3-4% increase in abnormal trading volume. While smaller in absolute terms than the results in Cookson and Niessner (2020), their results are at the daily level. Thus, our results at the monthly level are the same order of magnitude.

Finally, Table IA1 tests the external validity of the discount rate estimates derived in analyst reports. This table evaluates the assumption that the dispersion in the analysts' discount rates captures the inherent ambiguity in measuring a firm's cost of capital outside of the analyst sample—that is, the dispersion that arises in an econometrician's estimates of the cost of capital based on market data. In particular, this dispersion is measured as the spread between the largest and smallest econometrician betas estimated in the month the second analyst forecast is published. The econometrician betas are based on commonly-prescribed methodological choices in model estimation: trailing estimation windows of 24, 36, 48, 60, and 72 months, the 10-year Treasury rate for the risk-free rate, and the S&P 500 as the market proxy. Using this combination of parameters, the independent variable is defined as Max($CAPM\ Beta_{E,i,t_1}$)$-$ Min($CAPM\ Beta_{E,i,t_1}$), where the set of econometrician betas (E) for which we estimate the maximum and minimum values cover different trailing estimation windows, such as 24, 36, and 48 months.

Table IA1 reports the results for our sample firms in 2000-2023. The dependent variable is a firm's total trading volume scaled by shares outstanding, measured at monthly frequency, and the independent variable is a measure of dispersion in an econometrician's betas. The evidence shows that the spread between the maximum and minimum econometrician betas is positively related to a firm's trading volume, and this relationship is statistically significant at 1%. The point estimate of 0.004 in Column 5 suggests an economically meaningful effect. According to this point estimate, a one standard deviation increase in the spread between the largest and smallest econometrician betas is associated with an increase in a stock's trading volume of 2.8%.

In summary, cross-analyst dispersion in a firm's cost of capital is positively associated with trading volume, consistent with models of investor disagreement. This relationship holds with alternative measures of parameter dispersion estimated independently of analyst forecasts.

6 Conclusion

This paper has studied how finance professionals deal with estimation uncertainty when calculating a firm's required rate of return. When confronted with an array of feasible estimation methods, analysts appear to adopt one empirical model and adhere to a consistent set of estimation parameters throughout their careers. Such persistence in methodological choices generates large cross-analyst dispersion in the estimated discount rate for the same stock and correlates with market-based measures of investor disagreement.

While we use securities valuation as a convenient laboratory to study the inner workings of agents' modeling choices, the concept of estimation ambiguity extends beyond financial economics. Since many economic decisions require model estimation, they routinely confront agents with similar methodological challenges, such as selecting the appropriate empirical model, choosing parameter values, and adapting their choices in response to the arrival of new information or new estimation tasks.

While our paper makes a step towards understanding the micro-foundations of decision-making under estimation uncertainty, it leaves many open questions. One of the lingering questions deals with the factors that lead agents with similar backgrounds to adopt different estimation methodologies, ranging from private preferences to the role of formative experiences, such as mentorship, on-the-job training, or academic coursework. We hope that the growing interest in agents' decision-making under estimation uncertainty will continue to yield novel insights on this topic.

References

Ajinkya, B. B., R. K. Atiase, and M. J. Gift. 1991. Volume of Trading and the Dispersion in Financial Analysts' Earnings Forecasts. *The Accounting Review* 66:389–401.

Andrei, D., B. Carlin, and M. Hasler. 2019. Asset Pricing with Disagreement and Uncertainty about the Length of Business Cycles. *Management Science* 65:2900–2923.

Asparouhova, E., P. Bossaerts, J. Eguia, and W. Zame. 2015. Asset Pricing and Asymmetric Reasoning. *Journal of Political Economy* 123:66–122.

Asquith, P., M. B. Mikhail, and A. S. Au. 2005. Information Content of Equity Analyst Reports. *Journal of Financial Economics* 75:245–282.

Banerjee, S., and I. Kremer. 2010. Disagreement and Learning: Dynamic Patterns of Trade. *Journal of Finance* 65:1269–1302.

Ben-Rephael, A., B. I. Carlin, Z. Da, and R. D. Israelsen. 2023. Security Analysts and the Collection of Hard and Soft Information. NBER working paper 31936.

Bertrand, M., and A. Schoar. 2003. Managing with Style: The Effect of Managers on Firm Policies. *Quarterly Journal of Economics* 118:1169–1208.

Bewley, T. F. 2002. Knightian decision theory. Part I. *Decisions in Economics and Finance* 25:79–110.

Blitz, D. 2020. The Risk-Free Asset Implied by the Market: Medium-Term Bonds instead of Short-Term Bills. *Journal of Portfolio Management* 46:120–132.

Bradley, D., R. Jame, and J. Williams. 2022. Non-Deal Roadshows, Informed Trading, and Analyst Conflicts of Interest. *The Journal of Finance* 77:265–315.

Bradshaw, M. T. 2011. Analysts' Forecasts: What Do We Know after Decades of Work? Working paper, Boston College.

Cheng, Q., F. Du, X. Wang, and Y. Wang. 2016. Seeing is Believing: Analysts' Corporate Site Visits. Working paper, Singapore Management Univeristy.

Cookson, J. A., and M. Niessner. 2020. Why Don't We Agree? Evidence from a Social Network of Investors. *Journal of Finance* 75:173–228.

Décaire, P. H. 2023. Capital budgeting and Idiosyncratic Risk. Working paper, Arizona State University.

Décaire, P. H., and H. Bessembinder. 2021. Discount Rate Uncertainty and Capital Investment. Working paper, Arizona State University.

Décaire, P. H., and J. R. Graham. 2023. Valuation Fundamentals. Working paper, Arizona State University.

Dessaint, O., C. Otto, and D. Thesmar. 2021. CAPM-Based Company (Mis)valuations. *Review of Financial Studies* .

Fox, C. R., and A. Tversky. 1995. Ambiguity Aversion and Comparative Ignorance. *The Quarterly Journal of Economics* 110:585–603.

Giacomini, R., T. Kitagawa, and H. Uhlig. 2019. Estimation Under Ambiguity. Working paper, Unversity College London.

Giacomini, R., T. Kitagawa, and A. Volpicella. 2022. Uncertain identification. *Quantitative Economics* 13:95–123.

Giglio, S., M. Maggiori, J. Stroebel, and S. Utkus. 2021. Five facts about beliefs and portfolios. *American Economic Review* 111:1481–1522.

Gilardi, F., M. Alizadeh, and M. Kubli. 2023. ChatGPT Outperforms Crowd-Workers for Text-Annotation Tasks. *Proceedings of the National Academy of Sciences* 120:1–3.

Gilboa, I., and D. Schmeidler. 1989. Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics* 18:141–153.

Gormsen, N. J., and K. Huber. 2023. Corporate Discount Rates. Working Paper 31329, National Bureau of Economic Research.

Graham, J. R., and C. Harvey. 2001. The Theory and Practice of Corporate Finance: Evidence from the Field. *Journal of Financial Economics* 60:187–243.

Green, T. C., R. Jame, S. Markov, and M. Subasi. 2014. Access to management and the informativeness of analyst research. *Journal of Financial Economics* 114:239–255.

Hansen, L. P., and T. J. Sargen. 2001. Robust control and model uncertainty. *American Economic Review* 91:60–66.

Hashim, N. A., and N. C. Strong. 2018. Do Analysts' Cash Flow Forecasts Improve Their Target Price Accuracy? *Contemporary Accounting Research* 35:1816–1842.

Heath, C., and A. Tversky. 1991. Preference and Belief: Ambiguity and Competence in Choice under Uncertainty. *Journal of Risk and Uncertainty* 4:5–28.

Hong, H., and J. C. Stein. 2007. Disagreement and the Stock Market. *Journal of Economic Perspectives* 21:109–128.

Huang, S., H. Tan, X. Wang, and C. Yu. 2023. Valuation Uncertainty and Analysts' Use of DCF Models. *Review of Accounting Studies* 28:827–861.

Huntington-Klein, N., A. Arenas, E. Beam, M. Bertoni, J. R. Bloem, P. Burli, N. Chen, P. Grieco, G. Ekpe, T. Pugatch, M. Saavedra, , and Y. Stopnitzky. 2021. The influence of hidden researcher decisions in applied microeconomics. *Economic Inquiry* 59:944–960.

Kahneman, D., O. Sibony, and C. R. Sunstein. 2021. *Noise: A Flaw in Human Judgment.* William Collins, 1 London Bridge Street, London.

Karpoff, J. M. 1986. A Theory of Trading Volume. *Journal of Finance* 41:1069–1087.

Keynes, J. M. 1921. *A Treatise on Probability.* Macmillan.

Kruger, P., A. Landier, and D. Thesmar. 2015. The WACC fallacy: The real effects of using a unique discount rate. *Journal of Finance* 70:1253–1285.

Lintner, J. 1965. The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *Review of Economics and Statistics* 47:13–37.

Meeuwis, M., J. A. Parker, A. Schoar, and D. I. Simester. 2022. Belief Disagreement and Portfolio Choice. *Journal of Finance* 77:3191–3247.

Menkveld, A. J., A. Dreber, F. Holzmeister, J. Huber, M. Johannesson, M. Kirchler, S. Neusüss, M. Razen, and U. Weitzel. 2023. Non-Standard Errors. *Journal of Finance*, Forthcoming.

Milgrom, P., and N. Stokey. 1982. Information, Trade and Common Knowledge. *Journal of Economic Theory* 26:17–27.

Mitton, T. 2022. Methodological Variation in Empirical Corporate Finance. *Review of Financial Studies* 35:527–575.

Raiffa, H. 1961. Risk Ambiguity and the Savage Axioms: Comment. *Quarterly Journal of Economics* 69:690–94.

Sadka, R., and A. Scherbina. 2007. Analyst Disagreement, Mispricing, and Liquidity. *The Journal of Finance* 62:2367–2403.

Sharpe, W. F. 1964. Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk. *Journal of Finance* 19:425–442.

Shiller, R. J. 2014. Speculative Asset Prices. *American Economic Review* 104:1486–1517.

Smith, C. E., and R. Cribbie. 2014. Factorial ANOVA with Unbalanced Data: A Fresh Look at the Types of Sums of Squares. *Journal of Data Science* 12:385–403.

Tsoy, A. M. A. 2023. Asymmetric Information and Security Design under Knightian Uncertainty. *Journal of Finance (forthcoming)* .

Tversky, A., and D. Kahneman. 1974. Judgment under Uncertainty: Heuristics and Biases. *Science* 185:1124–1131.

Figure 1: Spread in Econometrician Estimated CAPM Betas Across Horizons. This figure displays the average spread between the maximum and minimum econometrician estimated CAPM betas across different horizon estimation windows (e.g., 24 month, 36 month,..., 72 month) in universe of CRSP firms from 1932 through 2022. The betas are estimated using the CAPM with monthly returns where the market factor is from Ken French's website. The gray bars depict NBER recessions.

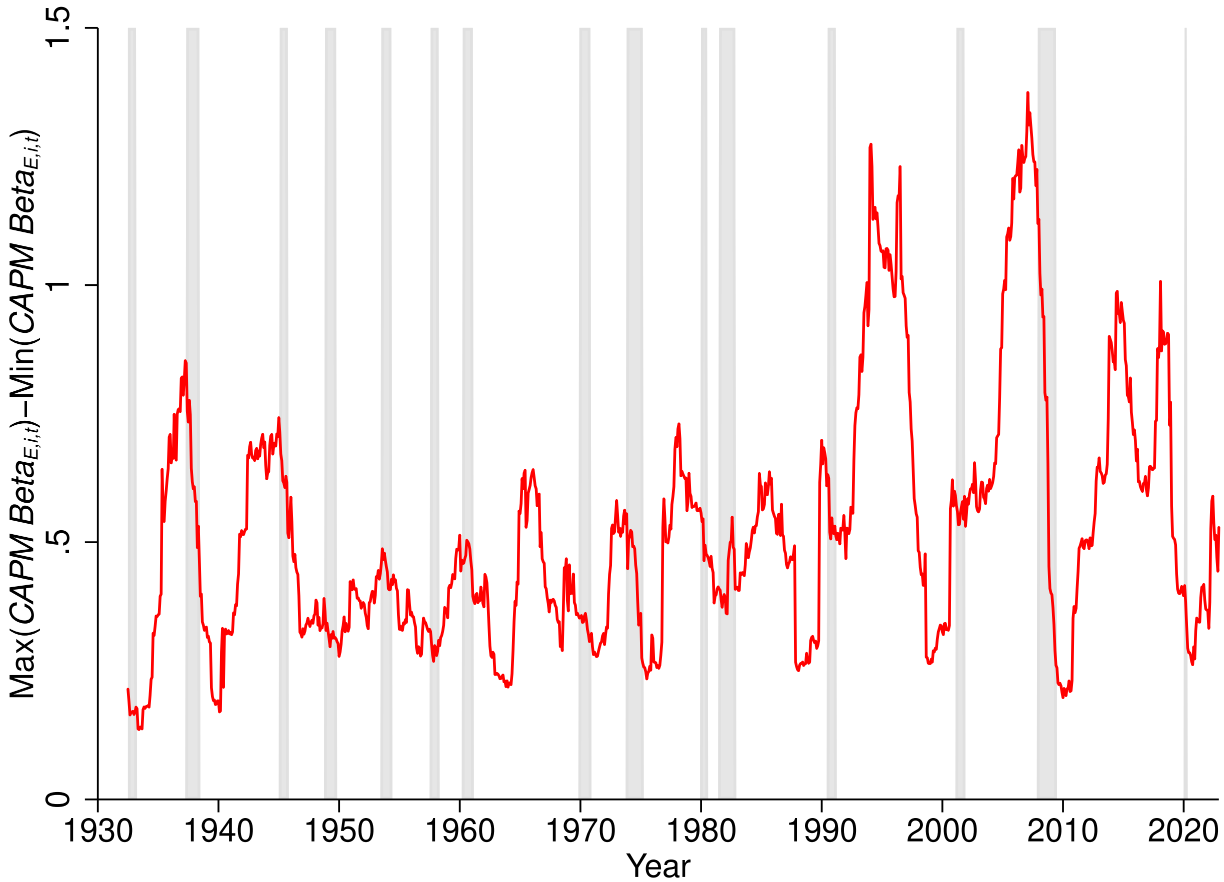


Figure 2: CAPM Beta Horizon Selection This figure highlights the ambiguity that analysts face when selecting the horizon of the estimation window in the CAPM formula. Analysts know that the true beta of the firm can be estimated with one, or a combination of several of the horizons, however, they do not know the weights to apply (e.g., P(h=1), P(h=2), and so on).

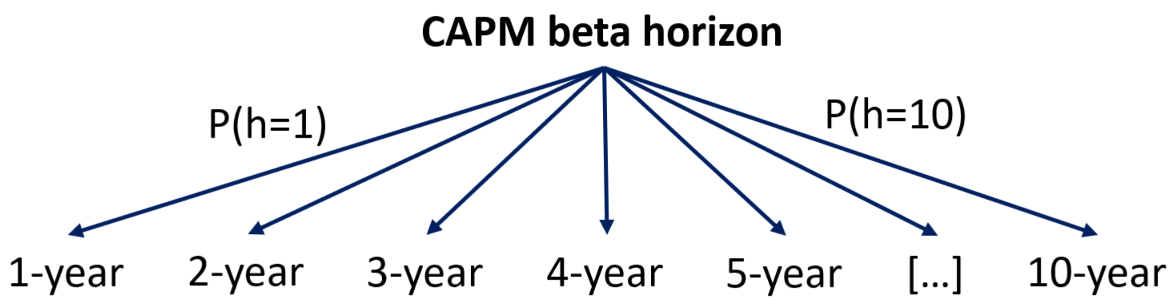


Figure 3: Heterogeneity in Analysts' Weighted Average Cost of Capital Estimates. This figure displays the cross-sectional distribution of the absolute difference between two analysts' weighted average cost of capital (WACC) estimates for the same firm at the same point in time ($WACC_{|A-B|,i,t}$). The sample period is 2000 through 2023. Data on individual analysts' estimates of discount rate are hand-collected from sell-side analyst equity research reports.

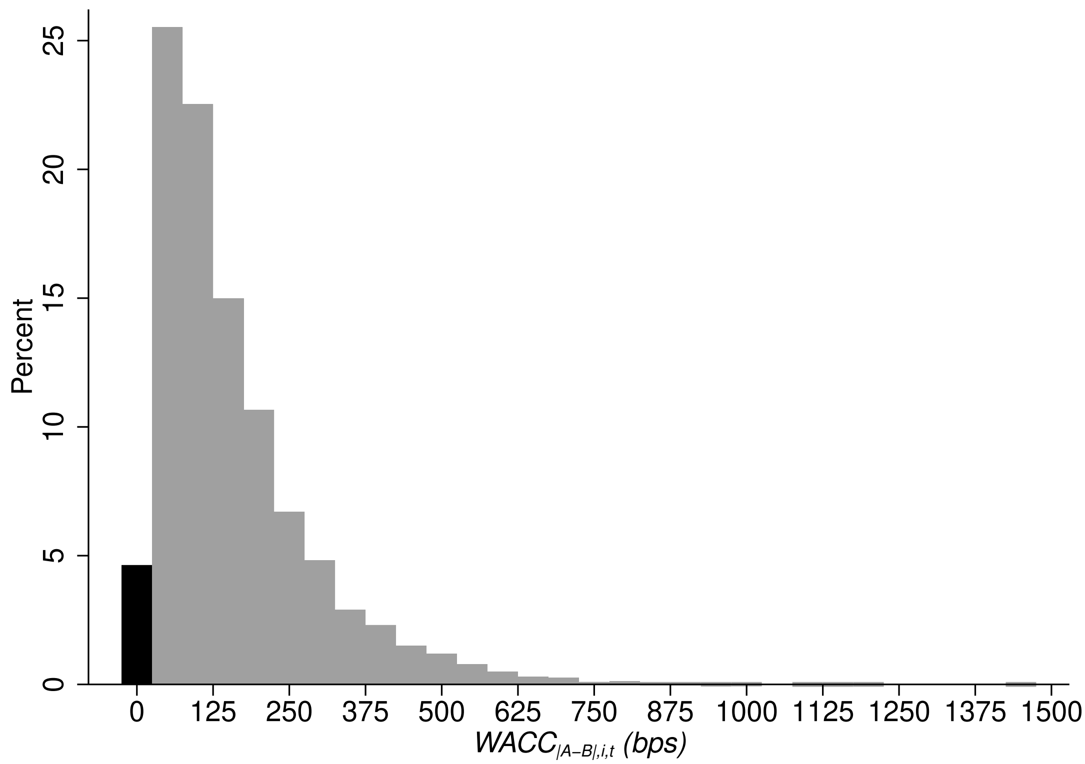
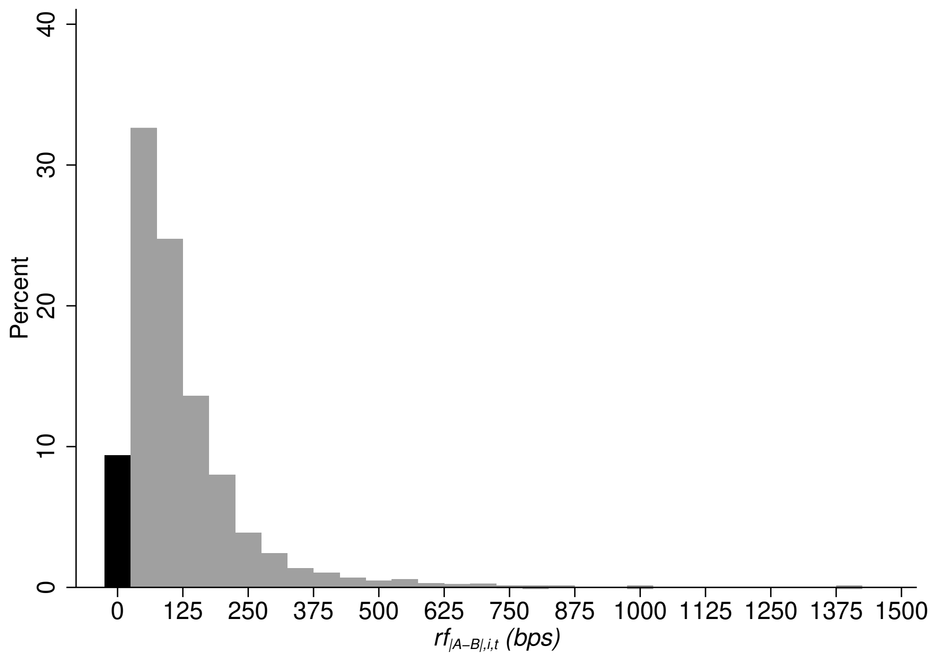


Figure 4: Heterogeneity in Analysts' CAPM Parameter Estimates. These figures display the cross-sectional distributions of the absolute differences between two analysts' CAPM input estimates for the same firm at the same point in time. In particular, Panel A focuses on the absolute difference in analysts' risk-free rate estimates ($rf_{|A-B|,i,t}$), and Panel B focuses on the absolute difference in analysts' CAPM beta multiplied by the equity risk premium ($(CAPM\ Beta \times ERP)_{|A-B|,i,t}$). The sample period is 2000 through 2023. Data on individual analysts' estimates of the CAPM inputs are hand-collected from sell-side analyst equity research reports.

(A) Risk-Free Rate



(B) CAPM Beta × Equity Risk Premium

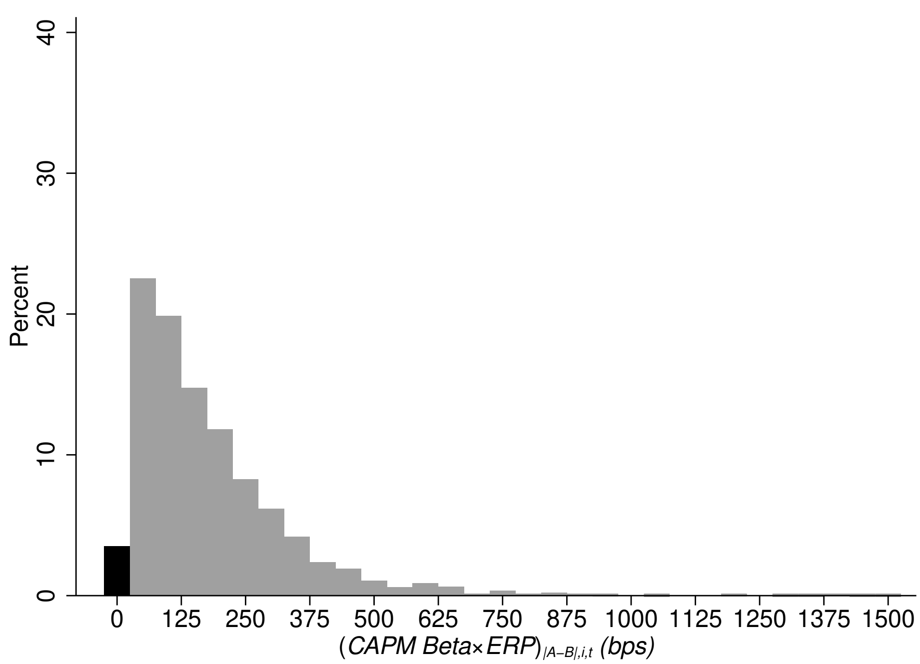
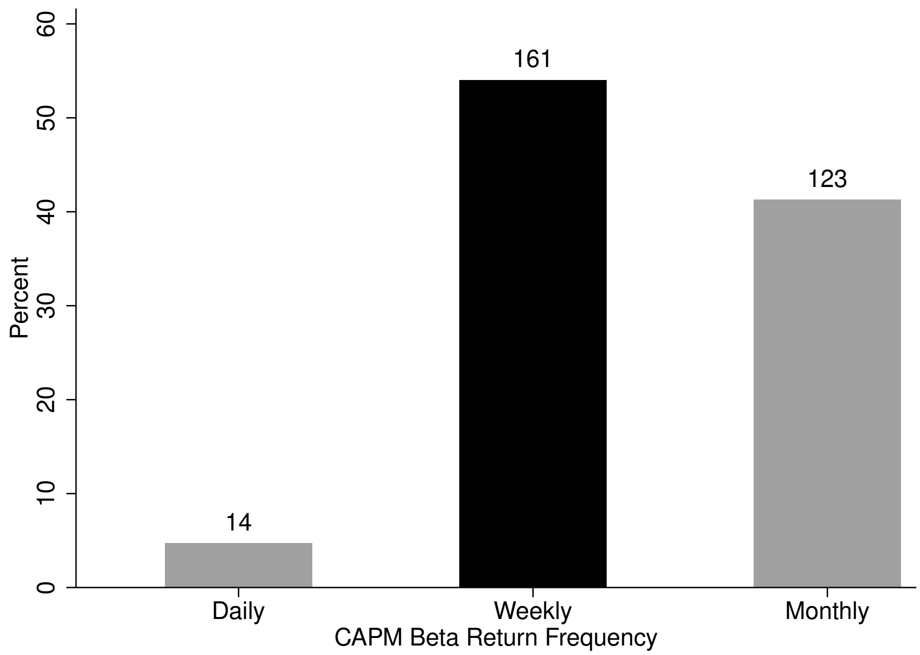


Figure 5: Heterogeneity in Analysts' Benchmark CAPM Betas. These figures display cross-sectional distributions of the details of analysts' chosen benchmarks CAPM beta. In particular, Panel (A) shows the distribution of the chosen return frequency for the beta estimation. Panel (B) shows the distribution of the horizons for the beta estimations. In both panels, black bars represent the most common approach in our sample. The sample period is 2000 through 2023. Data on individual analysts' choice of risk-free rate securities and horizons are hand-collected from sell-side analyst equity research reports.

(A) CAPM Beta Estimation Return Frequency



(B) CAPM Beta Estimation Horizon

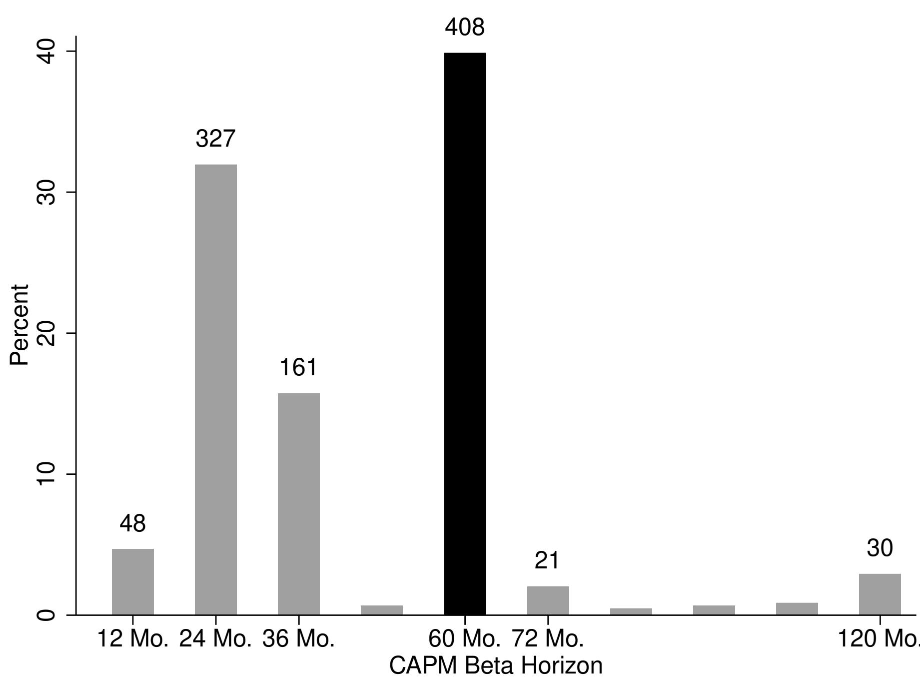
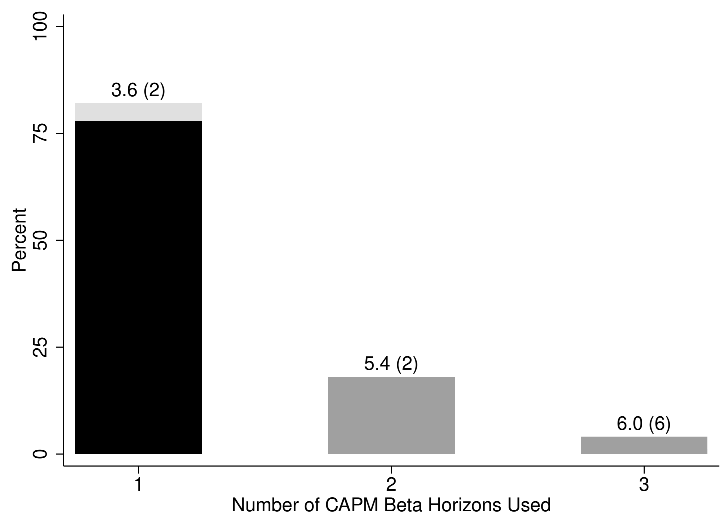
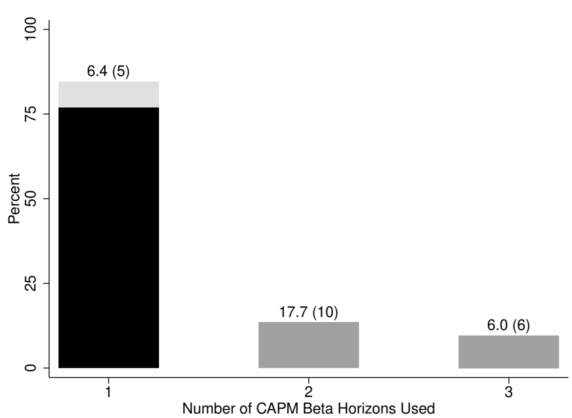


Figure 6: CAPM Beta Horizons Strategies. These figures display cross-sectional distributions of the strategies analysts' employ when choosing benchmarks CAPM beta. In particular, each shows the percentage of individual analysts that use only 1 benchmark beta horizon, the percentage that use 2, and the percentage that use 3 (the maximum in our sample). Panel (A) shows the distributions with all the analysts in our sample in which we observe at least two separate beta horizon discussions, Panel (B) shows the distributions with only the analysts in which we observe at least 4 separate beta horizon discussions, and Panel (C) shows the distribution with only the analysts in which we observe beta horizon discussions for multiple firms across multiple years. In all three panels, red bars represent the most common approach in our sample (only 1 chosen horizon) and the gray bars represent that percentage of analysts that use only 1 strategy, but average benchmark betas using multiple estimation horizons. The mean number of observations per analyst in each strategy bucket is displayed above each bar, and the median number of observations per analyst in each strategy bucket is displayed above each bar in parentheses. The sample period is 2000 through 2023. Data on individual analysts' choice of beta horizon length are hand-collected from sell-side analyst equity research reports.

(A) All Analysts Identifying Beta Horizon.



(B) 4+ Forecasts.



(C) Forecasts Across Multiple Firms and Years

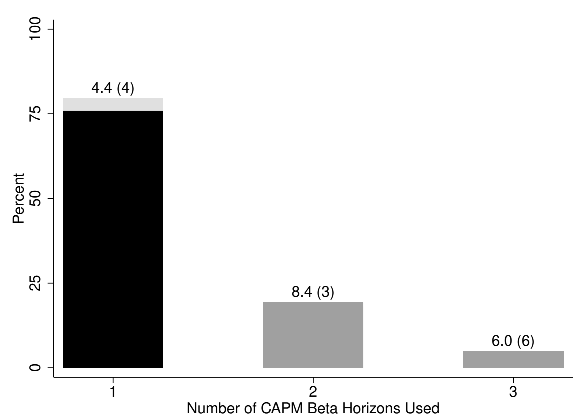
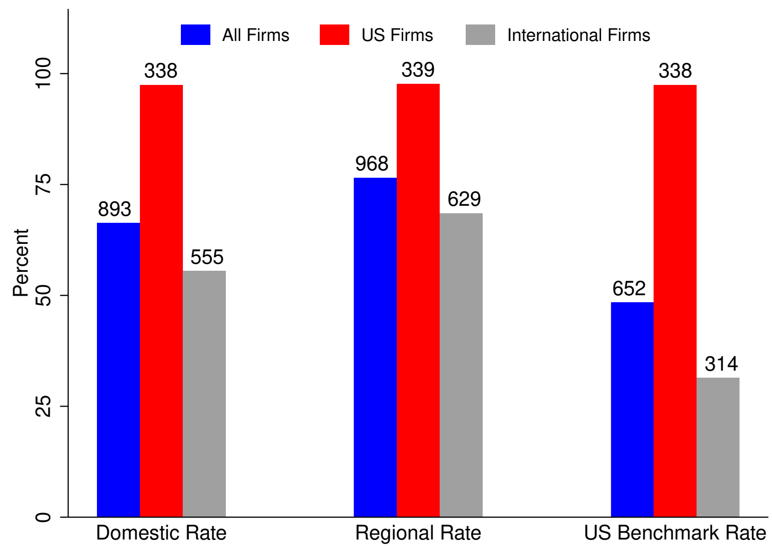


Figure 7: Heterogeneity in Analysts' Benchmark Risk-Free Rates. These figures display cross-sectional distributions of the details of analysts' chosen benchmarks for the risk-free rate. In particular, Panel (A) shows the distirbution of the chosen region for the benchmark security used by analysts in setting their risk-free rate proxy. Domestic rates represent the treasury rate for the country where the firm is headquartered (e.g., the U.K. treasury security for a firm headquartered in England, regional rates represent the treasury rate from one of the countries in the same continent where the firm is headquartered (e.g., the U.K. treasury security for a firm headquartered in France), and the U.S. benchmark rate, which is a U.S. treasury security. Panel (B) shows the distribution of the horizons of the risk-free securities chosen by analysts. In both panels, blue bars represent all the firms in our sample, red bars represent U.S. firms and gray bars represent international firms. The number of observations in each category appears above each bar. In Panel (B), the number of observations is displayed only for all firms using each horizon. The sample period is 2000 through 2023. Data on individual analysts' choice of risk-free rate securities and horizons are hand-collected from sell-side analyst equity research reports.

(A) Benchmark Security



(B) Benchmark Horizon

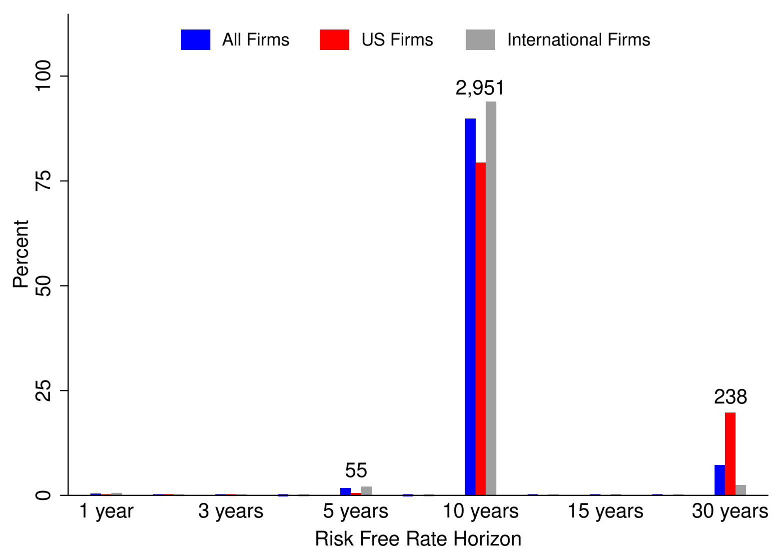
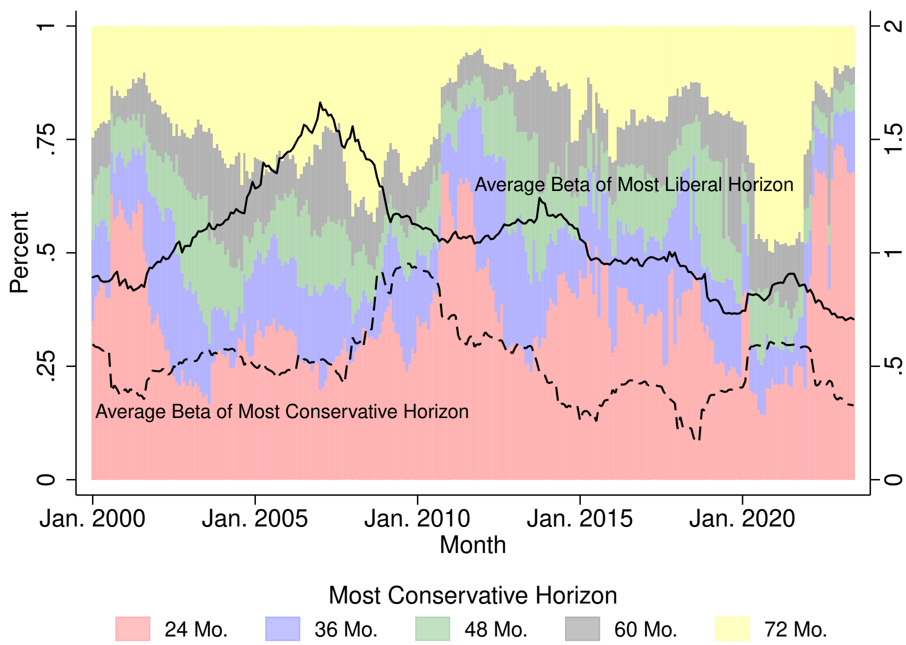


Figure 8: Heterogeneity in the Most Conservative and Liberal Beta Strategies. These figures display cross-sectional variation in the most conservative and most liberal beta strategies across each month of our sample. In particular, if an analyst used a strategy to give the most conservative cost of capital (e.g., lowest) or most liberal (e.g., highest), he or she would use the CAPM beta horizon that returned the lowest or highest beta, respectively. The black lines depict the patterns of the average of the conservative and liberal strategies across firms each month, while the different colored shaded regions portray the percentage of firms in which the most conservative (Panel A) and the most liberal (Panel B) strategy was the one corresponding to each particular horizon (e.g., 24 months, 36 months, etc.). The sample period is 2000 through 2023. Data on firms' stock returns is from Datastream.

(A) Most Conservative Strategy



(B) Most Liberal Strategy

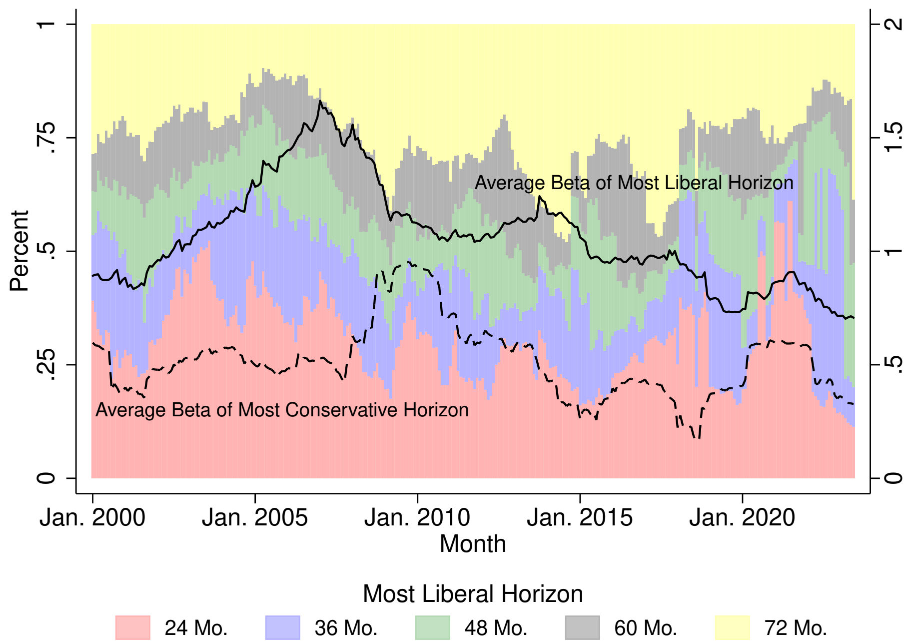
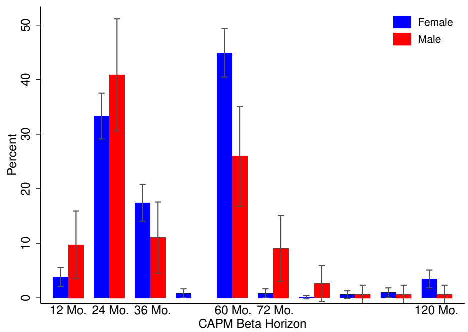
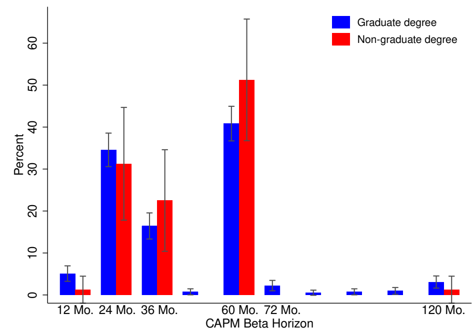


Figure 9: Disagreement in Analyst Equity Beta and Equity Risk Premium in the Time-Series. These figures display cross-sectional distributions of analysts chosen CAPM beta horizons across personal and firm characteristics. In particular, Panel (A) shows the horizon distributions across analyst gender, Panel (B) across analyst education, Panel (C) across analyst race, Panel (D) across analyst region, and Panel (D) across key firm industries. In all panels, 90% standard errors are displayed. The sample period is 2000 through 2023. Data on personal characteristics were graciously shared by Marius Guenzel. Data on individual analysts' choice of CAPM beta horizons are hand-collected from sell-side analyst equity research reports.

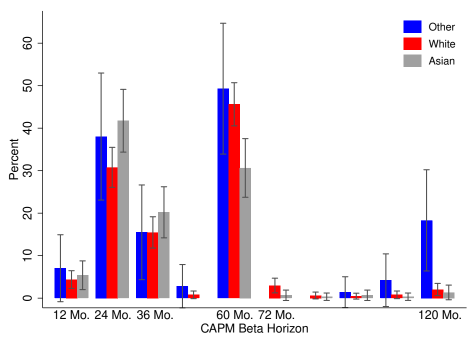
(A) Analyst Gender



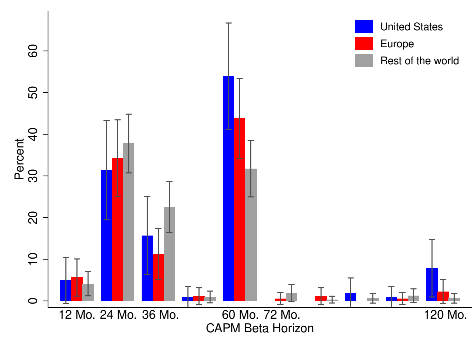
(B) Analyst Education



(C) Analyst Race



(D) Analyst Region



(E) Firm Industry

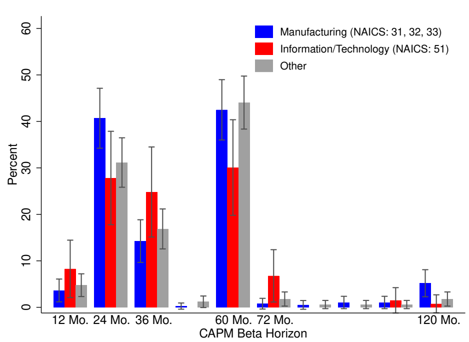


Table 1: Sample Details and Summary Statistics. This table reports the sample details and summary statistics for our three separate but complimentary samples. Panel A reports the sample details and summary statistics for the overlapping WACC sample. To be included in this sample, a firm must be covered by at least 2 analysts in the same year that report an estimate for the firm's weight average cost of capital (WACC). Not every analyst reports every detail of how the WACC is calculated, so the individual components will have less observations than the full sample (e.g., there are only 29,244 reports in the overlapping WACC sample that have estimates for the firm's terminal growth rate (TGR). Panels B and C report the sample details for the textual analysis samples: beta (Panel B) and the risk-free rate (Panel C), respectively. The full sample period is 2000-2023. Data on individual analysts' estimates of the WACC and its components are hand-collected from sell-side analyst equity research reports. Data on other firm characteristics is from Refinitiv/Datastream.

Panel A: Overlapping WACC Sample						
Number of Total Observations			45,992			
Number of Total Firms			4,261			
Number of Total Firm HQ Countries			63			
Number of Total Brokerage Houses			42			
Number of Total Identified Analysts			4,566			
Number of Total Analyst Countries			45			
	Mean	Std. Dev.	25^{th} Pct.	Median	$75^{th} Pct.$	Obs.
Firm Details						
Analyst Coverage (#)	10.0	7.4	4.0	8.0	15.0	12,060
Sample Coverage (years)	4.0	4.2	1.0	2.0	5.0	4,261
Equity Report Details						
$WACC_{a,i,t}$	0.089	0.019	0.076	0.087	0.100	45,992
$rf_{a,i,t}$	0.040	0.017	0.030	0.040	0.050	10,921
$CAPM\ Beta_{a,i,t}$	1.089	0.280	0.900	1.050	1.200	12,409
$ERP_{a,i,t}$	0.057	0.014	0.050	0.055	0.064	11,052
$r^E_{a,i,t}$	0.101	0.024	0.085	0.099	0.114	7,833
$TGR_{a,i,t}$	0.022	0.020	0.015	0.020	0.030	29,244
Pairwise Differences						
$WACC_{\|A-B\|,i,t}$	0.014	0.013	0.005	0.010	0.019	48,019
$rf_{\|A-B\|,i,t}$	0.010	0.011	0.003	0.008	0.013	3,247
$CAPM\ Beta_{\|A-B\|,i,t}$	0.219	0.211	0.080	0.170	0.300	4,170
$ERP_{\|A-B\|,i,t}$	0.013	0.013	0.005	0.010	0.016	2,921
$(CAPM\ Beta \times ERP)_{\|A-B\|,i,t}$	0.016	0.016	0.005	0.011	0.022	2,059
$r^E_{\|A-B\|,i,t}$	0.018	0.017	0.006	0.013	0.024	1,498
Panel B: CAPM Beta Textual Analysis Sample						
Number of Total Observations			1,023			
Number of Total Firms			794			
Number of Total Brokerage Houses			36			
Number of Total Analysts			508			
	Mean	Std. Dev.	25^{th} Pct.	Median	75^{th} Pct.	Obs.
$CAPM\ Beta_{a,i,t}$	1.124	0.395	0.870	1.050	1.300	828
Panel C: Risk-Free Rate Textual Analysis Sample						
Number of Total Observations			3,284			
Number of Total Firms			1,679			
Number of Total Brokerage Houses			48			

Table 2: Frequency of Above and Below Consensus Estimates for WACC and TGR. This table reports the frequency of analysts estimates for analyst estimates of weighted average costs of capital and terminal growth rates (TGR) that are above and below the consensus (mean) estimate for a given firm-year pair. Data on individual analysts' estimates of discount rates and TGRs are hand-collected from sell-side analyst equity research reports.

		Discount Rate$_{a,i,t}$		
		Above Consensus	Below Consensus	Total
$TGR_{a,i,t}$	Above Consensus	29.5% 7,037	25.1% 6,004	54.6% 13,041
	Below Consensus	19.7% 4,706	25.7% 6,136	45.4% 10,842
	Total	49.2% 11,743	50.8% 12,140	100.0% 23,883

Table 3: Cost of Equity Campbell-Shiller Decomposition. This table reports the results of a Cambell-Shillder Decomposition of the pairwise differences between to analysts estimates firm's equity cost of capital. In other words, this table reports the results of linear regression models in which the dependent variables are either the absolute difference in risk-free rates or CAPM betas× the equity risk premium (ERP) estimated by two analysts covering the same firm at the same time. The dependent variable of interest is the absolute difference in equity cost of capital estimated by two analysts covering the same firm at the same time. The sample period is 2000-2023. Panel A displays the results for the entire sample, while Panels B and C split between firms that are headquartered in the United States (Panel A) and the rest of the world (Panel B). Data on individual analysts' estimates of the WACC and its components are hand-collected from sell-side analyst equity research reports. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Dep. variable =	$rf_{A-B,i,t}$				$(\beta \times ERP)_{A-B,i,t}$			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Panel A: Full Sample								
$r^E_{A-B,i,t}$	0.227***	0.227***	0.215***	0.214***	0.773***	0.773***	0.785***	0.786***
	(0.033)	(0.032)	(0.040)	(0.040)	(0.033)	(0.032)	(0.040)	(0.040)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	1,498	1,497	1,119	1,117	1,498	1,497	1,119	1,117
F Statistic	46.57	49.93	29.01	29.15	542.02	576.98	386.54	393.63
Panel B: United States Sample								
$r^E_{A-B,i,t}$	0.075	0.083	0.113	0.113	0.925***	0.917***	0.887***	0.887***
	(0.053)	(0.051)	(0.078)	(0.071)	(0.053)	(0.051)	(0.078)	(0.071)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	229	228	160	154	229	228	160	154
F Statistic	2.01	2.70	2.13	2.53	309.10	329.37	130.73	156.88
Panel C: International Sample								
$r^E_{A-B,i,t}$	0.257***	0.256***	0.233***	0.225***	0.743***	0.744***	0.767***	0.775***
	(0.039)	(0.037)	(0.045)	(0.045)	(0.039)	(0.037)	(0.045)	(0.045)
Year FE		✓		✓		✓		✓
Firm FE			✓	✓			✓	✓
Observations	1,269	1,268	959	957	1,269	1,268	959	957
F Statistic	43.61	47.18	26.46	25.53	364.05	399.78	287.42	301.57

Table 4: Frequency of Above and Below Consensus Estimates for CAPM Beta and ERP. This table reports in Panel A, the frequency of analysts estimates for the CAPM beta and equity risk premium that are above and below the consensus (mean) estimates for a given firm year-pair. Data on individual analysts' estimates of CAPM betas and equity risk premiums are hand-collected from sell-side analyst equity research reports.

		$CAPM\ Beta_{a,i,t}$		
		Above Consensus	Below Consensus	Total
$Equity\ Risk\ Premium_{a,i,t}$	Above Consensus	24.2% 815	31.2% 1,049	55.4% 1,864
	Below Consensus	26.6% 895	18.0% 604	44.6% 1,499
	Total	50.8% 1,710	49.2% 1,653	100.0% 3,363

Table 5: Auto-correlation in the Most Conservative and Most Liberal Beta Strategies. This table the results of linear regression models in which the dependent variable is an indicator variable equal to one if the respective beta horizon (e.g., 24 months, 36 months, etc.) corresponds to the most conservative (Panel A) or the most liberal (Panel B) beta strategy. In particular, if an analyst used a strategy to give the most conservative cost of capital (e.g., lowest) or most liberal (e.g., highest), he or she would use the CAPM beta horizon that returned the lowest or highest beta, respectively. The main independent variables of interest are the lagged versions of the dependent variables. The models include year-month fixed effects. The sample period is 2000 through 2003. Data on firms' stock returns is from Datastream. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A: Most Conservative Beta Horizon					
Dependent variable =	I(24 Mo.)$_{i,t}$	I(36 Mo.)$_{i,t}$	I(48 Mo.)$_{i,t}$	I(60 Mo.)$_{i,t}$	I(72 Mo.)$_{i,t}$
	(1)	(2)	(3)	(4)	(5)
I(24 Mo.)$_{i,t-1}$	0.088***				
	(0.003)				
I(36 Mo.)$_{i,t-1}$		-0.056***			
		(0.002)			
I(48 Mo.)$_{i,t-1}$			-0.042***		
			(0.002)		
I(60 Mo.)$_{i,t-1}$				-0.021***	
				(0.001)	
I(72 Mo.)$_{i,t-1}$					0.147***
					(0.005)
Year-Month FE	✓	✓	✓	✓	✓
Observations	131,747	131,747	131,747	131,747	131,747
F Statistic	838.39	1267.44	678.34	227.02	710.81
R^2	0.10	0.05	0.06	0.02	0.05
Panel B: Most Liberal Beta Horizon					
I(24 Mo.)$_{i,t-1}$	0.143***				
	(0.002)				
I(36 Mo.)$_{i,t-1}$		-0.146***			
		(0.001)			
I(48 Mo.)$_{i,t-1}$			-0.109***		
			(0.001)		
I(60 Mo.)$_{i,t-1}$				-0.100***	
				(0.001)	
I(72 Mo.)$_{i,t-1}$					0.299***
					(0.003)
Year-Month FE	✓	✓	✓	✓	✓
Observations	131,747	131,747	131,747	131,747	131,747
F Statistic	3544.16	10033.12	6814.21	6906.20	9660.19
R^2	0.08	0.05	0.05	0.03	0.13

Table 6: ANOVA Variance Decomposition of the Level of Analysts' CAPM Equity Betas. This table reports the results of an analysis of variance (ANOVA) for the level analysts' estimates of equity beta. The independent variables of interest are indicator variables for firm, the brokerage house covering the firm, and for the analyst completing the equity report. The full sample includes only observations in in which there are at least 2 estimates of an equity beta at the firm, brokerage house and analyst level. Data on individual analysts' estimates of equity betas are hand-collected from sell-side analyst equity research reports.

	Sum of Squares (% of Model)	Degrees of Freedom	Adjusted Partial R^2
	(1)	(2)	(3)
Panel A: Full Sample			
Firm Indicators	78%	1,947	0.38
Brokerage Indicators	2%	36	0.01
Analyst Indicators	19%	1,120	0.28
Observations		6,411	
R^2		0.78	
Adjusted R^2		0.57	
Panel B: ≥ 5 Observations per Analyst			
Firm Indicators	86%	1,686	0.38
Brokerage Indicators	2%	28	0.00
Analyst Indicators	13%	395	0.24
Observations		4,475	
R^2		0.78	
Adjusted R^2		0.58	
Panel C: ≥ 5 Observations per Analyst, ≥ 5 Observations per Firm			
Firm Indicators	74%	464	0.34
Brokerage Indicators	4%	28	0.00
Analyst Indicators	22%	353	0.28
Observations		2,180	
R^2		0.75	
Adjusted R^2		0.59	

Table 7: ANOVA Variance Decomposition of Individual Fixed Effects from the Beta Regress. This table reports the results of an analysis of variance (ANOVA) for the individual fixed effects we extract from a linear regression on the level of analysts' CAPM equity betas. The independent variables of interest are indicator variables for the analysts gender, race, whether they have a master's degree, and country of analysts' location. Panel B adds an indicator for young. Young takes a value of 1 if the analyst is under the sample median for age, and 0 otherwise. The full sample includes only observations in in which there are at least 2 estimates of an equity beta at the firm, brokerage house and analyst level. Data on individual analysts' estimates of equity betas are hand-collected from sell-side analyst equity research reports. Personal characteristics of the analysts in our sample is collected from a social networking site and was graciously shared by Marius Guenzel.

	Sum of Squares (% of Model)	Degrees of Freedom	Adjusted Partial R^2
	(1)	(2)	(3)
Panel A			
Gender Indicator	6%	1	0.01
Race Indicators	18%	4	-0.02
Master's Degree Indicator	0%	1	-0.01
Analyst Country Indicators	76%	8	0.02
Observations		154	
R^2		0.09	
Adjusted R^2		-0.00	
Panel B			
Gender Indicator	15%	1	0.00
Race Indicators	16%	3	-0.03
Master's Degree Indicator	0%	1	-0.01
Analyst Country Indicators	69%	8	-0.02
Young Indicator	1%	1	-0.01
Observations		96	
R^2		0.09	
Adjusted R^2		-0.07	

Table 8: Multinomial Probit of Analysts' Choice of CAPM Beta Horizon. This table reports the results of a multinomial logit model in which the categories for the left hand size are indicators that take the value of 1 if the chosen beta horizon is 24 months (base case), 36 months, or 60 months, which represent the three most commonly chosen horizons in our sample . The independent variables of interest are indicator variables for the analysts gender, race, whether they have a master's degree, and country of analysts' location. Moreover, we include indicator variables for the firm's industry. Data on individual analysts' chosen horizon for the CAPM equity betas are collected from sell-side analyst equity research reports using textual analysis.

CAPM Beta Horizon (Base = 24 Months)	36 Months	60 Months	Equality of Distributions Across Horizons
	(1)	(2)	(3)
Binary Characteristics			
I(Gender = Male)	-0.379*	-0.146	1.06
	(0.368)	(0.436)	(0.589)
I(Education = Graduate degree)	-0.078	0.218	0.49
	(0.479)	(0.471)	(0.782)
I(Race = Non-white)	0.012**	-0.055	0.05
	(0.313)	(0.277)	(0.974)
Categorical Characteristics			
I(Region = United States)	Base Category		
I(Region = Europe)	-0.481	-0.276	
	(0.438)	(0.420)	5.25
I(Region = Other)	-0.093	-0.556	(0.263)
	(0.372)	(0.418)	
I(Industry = Manufacturing)	Base Category		
I(Industry = Info/Tech)	0.356	-0.234	
	(0.429)	(0.475)	3.95
I(Industry = Other)	0.425	0.262	(0.413)
	(0.288)	(0.262)	
Constant	-0.566	0.449	
	(0.513)	(0.518)	
Observations	517		
Log pseudolikelihood	-523.17		
χ^2	12.07		
p-Value (χ^2))	0.601		

Table 9: WACC Disagreement and Trading Volume. This table reports the results of linear regression models in which the dependent variables are the monthly trading volume scaled by total common shares outstanding (Panel A) and the monthly trading volume minus the mean of the previous 12 months of trading volume, scaled by total common shares outstanding (Panel B) . The independent variable of interest is the difference between the maximum WACC estimate and the minimum WACC estimate by different analysts covering the same firm in the first quarter of the same year. The scaled trading volume and WACC disagreement variables are measured in the month in which the second analyst forecast is released (e.g., the month in which the disagreement is created). The sample period is 2000-2023. Data on analyst estimates of WACC and TGR is hand-collected from sell-side analyst equity reports. Data on trading volume, market capitalization, and stock returns is from Datastream. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Panel A	$\text{FVOL}_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\text{Max}_{a\in A}(WACC_{i,t})-\text{Min}_{a\in A}(WACC_{i,t})$	1.060***	0.230***	0.238***	0.154***	0.120***	0.134**
	(0.128)	(0.047)	(0.048)	(0.045)	(0.042)	(0.057)
$\text{Max}_{a\in A}(TGR_{i,t})-\text{Min}_{a\in A}(TGR_{i,t})$						0.046
						(0.067)
$\text{FVOL}_{i,t-1}$		1.018***	1.019***	0.784***	0.761***	0.742***
		(0.010)	(0.010)	(0.019)	(0.021)	(0.030)
$\log(\text{Market Capitalization}_{i,t})$					-0.009***	-0.012***
					(0.002)	(0.003)
$\text{Cumulative Return}_{i,t-3,t-1}$					-0.010*	-0.006
					(0.006)	(0.008)
$\text{Cumulative Return}_{i,t-12,t-4}$					0.001	-0.001
					(0.003)	(0.004)
$\text{Return Volatility}_{i,t-3,t-1}$					0.000	0.000
					(0.000)	(0.000)
$\text{Months Between Forecasts}_{i,t0,T}$					-0.002***	-0.002**
					(0.001)	(0.001)
Year-Month FE			✓	✓	✓	✓
Firm FE				✓	✓	✓
Observations	16,186	15,836	15,836	14,431	12,351	6,978
F Statistic	68.89	5542.75	5510.84	833.66	210.69	104.30
R^2	0.01	0.80	0.80	0.88	0.88	0.89
Panel B	$\text{Abnormal FVOL}_{i,t}$					
	(1)	(2)	(3)	(4)	(5)	(6)
$\text{Max}_{a\in A}(WACC_{i,t})-\text{Min}_{a\in A}(WACC_{i,t})$	0.092**	0.164***	0.171***	0.090**	0.083**	0.113**
	(0.040)	(0.039)	(0.039)	(0.038)	(0.037)	(0.049)
$\text{Max}_{a\in A}(TGR_{i,t})-\text{Min}_{a\in A}(TGR_{i,t})$						0.061
						(0.063)
$\text{Abnormal FVOL}_{i,t-1}$		0.424***	0.412***	0.482***	0.508***	0.516***
		(0.021)	(0.021)	(0.023)	(0.024)	(0.033)
$\log(\text{Market Capitalization}_{i,t})$					-0.004***	-0.006**
					(0.002)	(0.002)
$\text{Cumulative Return}_{i,t-3,t-1}$					-0.021***	-0.015*
					(0.006)	(0.009)
$\text{Cumulative Return}_{i,t-12,t-4}$					-0.000	0.000
					(0.002)	(0.003)
$\text{Return Volatility}_{i,t-3,t-1}$					-0.000	0.000
					(0.000)	(0.000)
$\text{Months Between Forecasts}_{i,t0,T}$					-0.002**	-0.002**
					(0.001)	(0.001)
Year-Month FE			✓	✓	✓	✓
Firm FE				✓	✓	✓
Observations	15,849	14,794	14,794	13,498	12,350	6,977
F Statistic	5.41	226.87	210.52	219.11	83.47	45.59
R^2	0.00	0.14	0.16	0.41	0.42	0.44

Appendix A: Variable Definitions

Table A1: Variable Definitions

Subscript a indicates a specific analyst, i indicates a specific firm, and t indicates a year.

Variable	**Definition**
Analysts' $\text{WACC}_{a,i,t}$	The weighted average cost of capital (WACC) used by analysts to evaluate firm cash flows in equity reports.
Analysts' terminal growth $\text{rate}_{a,i,t}$ (TGR)	The terminal growth rate used by equity analysts in their DCF models, measured from the equity reports.
Analysts' CAPM equity $\text{beta}_{a,i,t}$	The equity beta used by analysts when computing their discount rate in equity reports.
Analysts' equity risk $\text{premium}_{a,i,t}$ (ERP)	The equity risk premium used by analysts when computing their discount rate in equity reports.
Analysts' risk-free $\text{rate}_{a,i,t}$ (Rf)	The risk-free rate used by analysts when computing their discount rate in equity reports.

Internet Appendix
for
"Resolving Estimation Ambiguity"

Paul H. Décaire[1] , Denis Sosyura[2], and Michael D. Wittry[3]

This Internet Appendix reports results that are mentioned but not tabulated in the main paper. We report 1 table, as outlined below:

1. Table IA1: Beta Horizon Disagreement and Trading Volume

   Reference in the main paper: "" (Section)

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Table IA1: Beta Horizon Disagreement and Trading Volume. This table reports the results of linear regression models in which the dependent variable is the monthly trading volume scaled by total common shares outstanding. The independent variable of interest is disagreement over a set of econometrican estimate CAPM betas. This CAPM Beta disagreement variable is calculated as the difference between the largest and smallest CAPM Beta in month t when using different forecast horizons (e.g., 24-month, 36-month, 60-month, etc) to estimate each beta. The scaled trading volume and CAPM beta disagreement variables are measured in the month in which the second analyst forecast is released (e.g., the month in which the disagreement is created). The sample period is 2000-2023. Data on trading volume, market capitalization, and stock returns is from Datastream. Robust standard errors, clustered at the firm level, are reported in parentheses. *,**, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Dependent variable =	$\text{FVOL}_{i,t}$				
	(1)	(2)	(3)	(4)	(5)
Max($CAPM\ Beta_{E,i,t}$)− Min($CAPM\ Beta_{E,i,t}$)	0.041***	0.005***	0.006***	0.004***	0.004***
	(0.002)	(0.000)	(0.000)	(0.001)	(0.001)
$\text{FVOL}_{i,t-1}$		0.891***	0.889***	0.640***	0.622***
		(0.003)	(0.003)	(0.008)	(0.010)
log(Market $\text{Capitalization}_{i,t}$)					-0.005***
					(0.001)
Cumulative $\text{Return}_{i,t-3,t-1}$					-0.007***
					(0.001)
Cumulative $\text{Return}_{i,t-12,t-4}$					-0.001
					(0.001)
Return $\text{Volatility}_{i,t-3,t-1}$					-0.000***
					(0.000)
Year-Month FE			✓	✓	✓
Firm FE				✓	✓
Observations	468,354	454,478	454,478	454,432	304,138
F Statistic	293.69	52936.35	53347.07	3300.70	834.22
R^2	0.01	0.80	0.80	0.83	0.83