To Stay or to Migrate? Rural-urban Migration and the One-Child Policy in China

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Abstract: With a dynamic macroeconomic model built to tailor the institutions of China, this paper studies how fertility control and migration policies shaped the rural-urban migration pattern and the economic development of China. Different from the reduced-form approach adopted by most of the existing literature, we develop a model of heterogeneous agents with endogenous fertility and migration decisions. More specifically, the land entitlement of rural households, the different fertility control policies across urban and rural areas, and the household registration system are introduced into the model. This study therefore provides a micro-funded macroeconomic model that enables us to examine how different policies interact with each other to affect the fertility, the migration decision and the subsequent differentials in economic development across urban and rural China. Calibration analysis of the model is performed, and numerical analyses examining the significances of the "pull" and "retain" factors of migration and their effects on output levels, the migration decisions as well as the fertility differentials are performed. Finally, policy experiments on the land entitlement system, migration regulations as well as fertility policies are provided.

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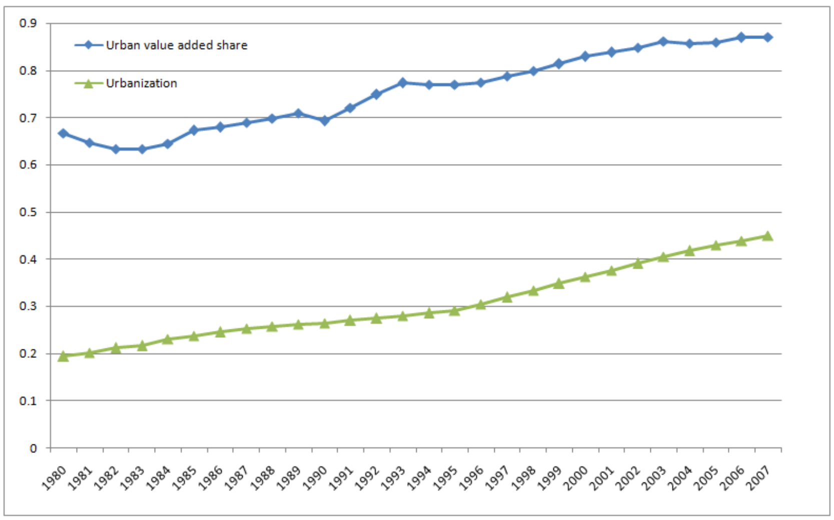
1 Introduction

China has experienced a rapid economic growth since 1980. The per capita real GDP on average grew at 6.0 percent during 1980-2007. It has been even higher since the 1990s, on average the per capita real GDP growing at 7.6 percent over 1992-2007. The rapid growth was accompanied with a phenomenon of fast urbanization, rising urban output shares and large rural-urban migration (see Figure 1).

Figure 1. Urbanization and urban output shares in China.



On the other hand, in the same period of time, China also has implemented the two unique policies in the world: the one-child policy and the household registration (hukou) system. The hukou system serves as a means of internal population movement control. Furthermore, the one-child policy is closely connected to the hukou system, meaning that rural-urban migrants have to follow a stricter fertility policy. Therefore, this paper aims to study how the one-child policy and the hukou system interact with each other to affect the fertility choice, the migration decision and the subsequent differentials in economic development across urban and rural China.

In general, the motivation of rural-urban migration is mainly due to the income gap between rural and urban areas. There are also other non-pecuniary benefits of rural-urban migration, such as better job perspectives, better education for children, medical and retirement benefits, and social insurance. The cost of migration could be the loss of land entitlement if people officially permanently move to cities. In addition, rural-urban migrants will face a much stricter fertility control when they move to cities.

Under China's hukou system, there are three major migration channels: moving from rural to urban areas through attending higher education, working, or making investments in cities.[1] Liao, Wang, Wang and Yip (2015) has shown the importance of education-based migration to the

[1] Migration via higher education refers to *zhaosheng*. Migration through work could be *zhaogong*, *zhaogan* or *nongmingong* since the 1980s. In the 1990s, big cities issue the blue-print hukou to people who invest to the cities. The blue-print hukou is also an urban hukou. However, the rights are different from the regular hukou.

Table 1: Migration by reason.

Reasons of Migration	Total	Others	Study or Training	Job Transfer	Job Assignment	Work or Business	Work related, Subtotal
1985	100.00%	48.05%	11.26%	29.57%	8.04%	3.08%	40.69%
2000	100.00%	50.53%	6.84%	5.32%	3.76%	33.55%	42.63%
Average	100.00%	49.29%	9.05%	17.44%	5.90%	18.32%	41.66%

Note: Others include "to relative and friend", "retired or resigned" (1985 data only), "moved with family", "marriage", "pull down and move" (2000 data only), and "other reasons". Source: *10 Percent Sampling Tabulation on the 1990 Population Census of the People's Republic of China* for the data of 1985; *Tabulation on the 2000 Population Census of the People's Republic of China* for 2000.

development of China. However, Table 1 shows that work-based migration is no less important than education-based migration. Therefore, this paper continue to this line of research and ask the following questions: How did the land-entitlement system and the one-child policy affect rural-urban migration decisions? To what extent did they affect the migration pattern and the development of China?

Most of the existing literature on China's rural-urban migration is based on reduced form approach or partial equilibrium models. However, without a dynamic general equilibrium model, it is difficult to examine the overall impact of rural-urban work-based migration on the Chinese economy, in particular, the distortions and the interactions created by the hukou system and the one-child policy. We therefore construct a dynamic general equilibrium model with endogenous fertility, land reallocation and migration decisions to study the issue of rural-urban work-based migration in China.

Our framework is an infinite-horizon model of one-period lived heterogeneous agents. Agents are heterogeneous in their preference toward quantity of children and their ability level. To highlight the institutional effects in China, there are three features in our model. First, rural farmers' attachment to land is introduced to capture the notion of land entitlement. Second, because the fertility control in rural areas is more lenient, differential above-quota penalties between rural and urban areas are considered. Third, the regulations on movement are included to grasp the main spirit of China's hukou system. The household side of the model is a generalization of Liao, Wang, Wang and Yip (2005), while the production side of the model is based on Harris and Todaro (1970) and Song, Storesletten and Zilibotti (2011). Specifically, we have two manufacturing sectors in the urban area (a state-owned enterprise sector, SOE hereafter, and a private sector) and one agricultural sector in the rural area.

The model is calibration to the data from China. We take the calibrated result as our benchmark model. Based on the benchmark model, we perform quantitative analysis to investigate the impacts of the fertility control and land policies on China's rural-urban migration. Finally, counterfactual experiments on migration policies, fertility control policies and land entitlement policies are conducted.

We find interesting interactions between fertility choices and migration decisions in the counterfactual experiments. In the first scenario, with better land entitlement, urban jobs become less attractive so more rural people (both high and low ability) decide to stay in rural areas. The fertility of high-ability workers staying in rural areas increases. However, because child-rearing cost and the above-quota penalty is proportional to income, low-ability workers staying in rural areas decide to have fewer children. Moreover, more workers stay in rural areas, diluting rural land and hence the income gap between rural and urban areas is larger. The second scenario considers a more relaxed fertility control in urban areas. The policy attracts the migration of high-ability workers. The fertility of high-ability workers working in private sector goes up. In contrast, the fertility of high-ability workers who stay in rural areas (not migrants) declines. Because now more high-ability workers in urban areas, the wage of low-ability workers in private sector increases. This motivates more low-ability migrants. The third scenario discusses the effects of a higher probability of being recruited in the SOE and getting urban hukou. Although the SOE job grants urban hukou immediately, the payment is lower than that in private sector and the above-quota penalty is higher. Migrants will face more effective fertility control if they move to cities. Thus, the policy results in fewer high-ability migrants, the fertility of workers in SOE is lower and fewer low-ability migrants due to a lower wage in private sector.

Previous works on reallocating abundant and over-employed labor from rural agricultural sector to urban manufacturing sector can be traced back to the Lewis (1954). The research focusing on rural-urban migration was pioneered by Todaro (1969) and Harris and Todaro (1970). Since then, economists have attempted to understand the forces driving rural to urban migration and the impacts of rural-urban migration on development process. In China, on one hand, higher wages, better career prospects, better education opportunities, and better social benefits in cities are the "pull" factors that attract rural migrants. On the other hand, social networks, land arrangement, and having elderly household members are plausible "retain" factors for rural workers to stay in their hometown. Yang (1997), Hare (1999) and Zhao (1999) investigated the mechanisms underlying a household's migration decisions or time allocation on farm and non-farm activities with static models. Zhao (1999) further conducted an empirical analysis using household survey data in Sichuan province for 1994 and 1995 and found that shortages of farmland and abundance in household labor were the most important determinants of labor migration decisions. However, these studies are carried out using reduced-form regression approach, which is very different from the structural approach adopted in this paper. Thus, below we focus on the literature that is more model-oriented with calibration method.

Using a theoretical framework with calibration, Bond, Riezman and Wang (2015) studied the effects of reductions in trade and migration barriers on China's growth and urbanization. Garriga, Tang and Wang (2014) studied the structural transformation and the consequent reallocation of labor from rural to urban areas in China. They found that two-thirds of the increases in land and housing prices can be attributed to the urbanization and development processes. Liao, Wang, Wang and Yip (2015) studied the rural-urban migration in China with a focus on education-based migration. They found that the contribution of education-based migration on urban output shares are comparable to that of work-based migration. Tombe and Zhu (2015) study the effects of opening the domestic market to international trade on migration and regional income differences in China.

To our best knowledge, this paper is the first to examine rural-urban migration with land entitlement, the one-child policy, and the hukou system in a dynamic general equilibrium model. The approach and theoretical framework adopted by this paper is generalized from Garriga, Tang and Wang (2014) and Liao, Wang, Wang and Yip (2015). However, different from the existing literature, there are two major advantages of this study. First, we are able to infer decision rules of migration for agents with heterogenous abilities and preferences toward children. Second, with a structural model, we can quantify the contribution of rural-urban migration to the Chinese economy and evaluate the distortions and interactions created by land entitlement, the hukou system and the one-child policy.

The rest of this paper is organized as follows. Section 2 summaries China's *hukou* system, the one-child policy and the land entitlement. Section 3 mentions the model and the theoretical analysis. The calibration strategy and counterfactual experiments are provided in Section 4. Finally, Section 5 concludes this paper.

2 Hukou System, the One-child Policy and the Land Entitlement in China

3 The Model

We construct a model that is tail-mode for the Chinese economy to study the work channel of rural-urban migration. Consider an inifinite horizon model of one-period-lived agents and two geographical regions, urban and rural. In rural areas, production is simply backyard farming, using labor and land. There are two manufacturing sectors in urban areas, namely, a sector of state-owned entreprises (SOE) and a private sector that is constituted by private firms. There are two types of household registration status, urban and rural, corresponding to the nonagricultural and agricultural hukou status in reality. Agents with rural hukou status are entitled with land, whereas agents with urban hukou status enjoy urban benefits.

Agents are heterogeneous in their skill levels and preferences toward children. Children inherit hukou status and skill level from their parents, but the preferences toward children are assumed to be redrawn for every generation. Since all agents live for one period, it is assumed that they give births of n children right before the end of their life. Agents are altruistic in the sense that they enjoy having children in terms of quality and quantity. All agents own one unit of labor time and supply labor inelastically throughout their life.

In this section, we first describe the sectoral production in the economy. We then study the household optimization problems in the two locations. From the value functions of the agents, we delineate how rural agents make migration decisions to the urban sector. Finally, we discuss the evolution of workers and study the competitive equilibrium of the economy.

3.1 Production

Because the model involves migration decisions, the stock of the populations at the beginning of the period and the number of workers actually working in a specific production sector are different. To avoid confusion, we differentiate the stock of workers at the beginning of a period to the number of workers working in the sector during the period with a superscript "+". We dismiss the subscript t for time if there is no confusion.

1. Rural production

Rural workers rely on farming, which takes land as an input, to make a living. The land in rural area is assumed to be constant over time at a level Q and is evenly distributed among rural workers. Denote R^+ as the number of rural workers cultivating the rural farm during a period. The land per rural worker is thus defined as $q \equiv Q/R^+$.A rural worker's income is

$$x = zq = z\frac{Q}{R^+}, \tag{1}$$

where $z > 0$ is the farming technology of rural workers. Total ouput in rural areas is thus

$$X = zqR^+ = zQ. \tag{2}$$

2. Urban production

There are two types of firms in urban area: state-owned entreprises (SOE) and private firms. SOEs are operating with a linear technology, taking relatively high-skilled workers as inputs:

$$Y_S = A_S S^+ \tag{3}$$

where A_S is the technology scaling factor of SOEs, and S^+ is the number of high-skilled workers hired by SOEs.

Unlike SOEs, private firms hire both high- and low-skilled workers in production. Denote P^{H+} and P^{L+} as the quantity of high- and low-skilled workers hired by private firms The production function of private firms thus takes the following CES form:

$$Y_P = A_P \left[\alpha \left(\eta P^{H+}\right)^{\sigma} + (1-\alpha)\left(P^{L+}\right)^{\sigma}\right]^{\frac{1}{\sigma}}, \quad \eta > 1,\, \alpha \in (0,1)\,, \sigma < 1 \tag{4}$$

where A_P is the technology scaling factor of private firms, η is the quality index of high-skilled workers, α is the share paramter and $1/(1-\sigma)$ is the elasticity of substitution between high- and low-skilled workers in production. Depending on the skill level and the workplaces, the wage rates in urban area are:

$$w_S = A_S, \tag{5}$$

$$w_P^H = A_P \left[\alpha \left(\eta P^{H+}\right)^{\sigma} + (1-\alpha)\left(P^{L+}\right)^{\sigma}\right]^{\frac{1}{\sigma}-1} \alpha\eta^{\sigma}\left(P^{H+}\right)^{\sigma-1}, \tag{6}$$

$$w_P^L = A_P \left[\alpha \left(\eta P^{H+}\right)^{\sigma} + (1-\alpha)\left(P^{L+}\right)^{\sigma}\right]^{\frac{1}{\sigma}-1} (1-\alpha)\left(P^{L+}\right)^{\sigma-1}. \tag{7}$$

We further impose the following restriction for further analysis:

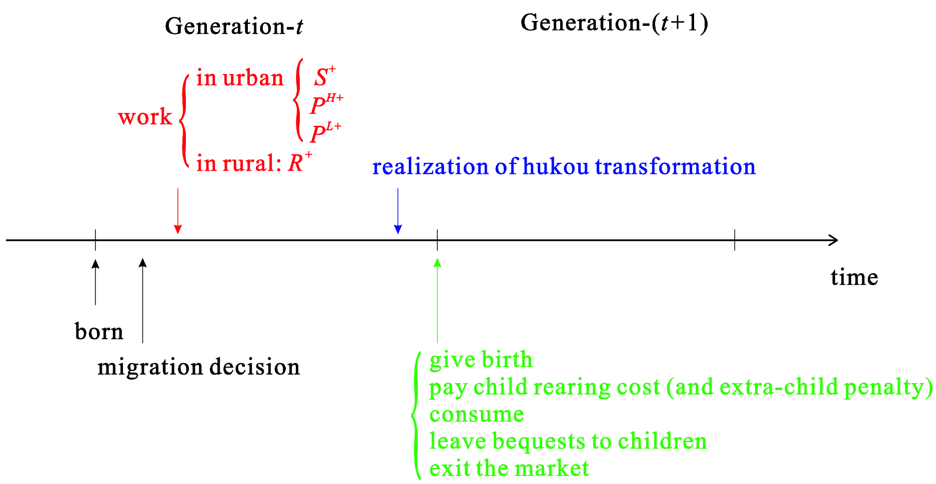
Condition 1 $w_P^H \geq w_P^L$.

Condition 3.1 simply states that high-skilled workers earn at least as large as low-skilled workers in the same sector.

3.2 Households

Figure 2 depicts the timeline of the agent in the model. It is assumed that agents live for one period, have perfect foresight and there is no reverse migration from urban to rural areas. For agents born in rural areas, they choose whether to migrate to cities immediately after they were born. The decision rule goes as follows: If the value of staying in the rural area is higher than the expected value of migrating to cities, then the agnet will choose to stay in the rural area, and vice versa. After the migration decision, agents work in their chosen locations throughout their life. Before the end of their life, they choose the number of children to have subject to the fertility quota based on their hukou status. They also consume, pay child rearing costs, pay the extra-children penalty if they have more children than the quota, decide on the quality of their children, and enjoy urban benefits if they have urban hukou status. After that, they exit the market. To capture the reality, the fertility quota imposed on urban hukou status is smaller than that on rural hukou status. Also, the extra-children penalty is higher for urban hukou status than that for rural hukou status. Agents differ in their preferences over the number of children and the skill levels (although rural production does not require skills).

Figure 2. Timeline.



Below, we first describe the agents' utility by location and hukou status, and then we discuss a rural worker's migration decision.

1. Rural agents

The utility of a rural worker staying in the rural area is:

$$u^R(c, b, n; \beta) = \min[\theta c, (1-\theta) nb] + \beta n^{\varepsilon}, \quad \varepsilon \in (0,1) \tag{8}$$

where $\theta \in (0,1)$ is the altruistic factor (proportional to the net income allocating to total bequests), and $\beta > 0$ measures how the agent values children. Note that the Leontief setting in c and nb is meant to capture the consumption-saving decision of individuals, with nb acting as total saving in offspring. We can think of b is the average quality of a child so that savings become an investment in children. However, given the one-period setting of the model, we do not connect b to the productivity of the child. In that sense, we are thinking b as the cost of establishing the goodness of personal attitude. The parameter $\beta \in \{\underline{\beta}, \bar{\beta}\}$, with $0 < \underline{\beta} < \bar{\beta}$, captures the agent's preference toward children. It is assumed that β is re-drawn for every generation when agents are born. With probably ζ, an agent enjoys less from having children ($\underline{\beta}$) and with probably $1 - \zeta$, he enjoys more from having children ($\bar{\beta}$). An agent thus derives utility from consumption (c), quality of children (b) and number of children (n). It is costly to have children. Assume that the child rearing cost is ϕ_R^0 per child in the countryside. For the fertility control policy in rural China, denote $\bar{n}_R$ as the ferility quota per rural worker imposed by the government. If a rural worker has more than $\bar{n}_R$ children, he has to pay a penalty of $\bar{\phi}_R$ to the government per extra child he has. A rural worker receives an income of zq from rural production. The budget constraint of a rural worker can be written as:

$$c + nb + n\phi_R^0 + I^R \left(n - \bar{n}_R\right) \bar{\phi}_R = zq, \tag{9}$$

where I^R is an indicator function of rural fertility quota such that

$$I^R = \begin{cases} 1, & \text{if } n > \bar{n}_R \\ 0, & \text{otherwise} \end{cases}$$

A rural agent's problem is thus to maximize (8) subject to the budget constraint (9).

2. Urban agents

According to China's household registration system (or the hukou system), a person working in an urban area can either be an urban hukou holder or a nonholder.[2] Workers with urban hukou status can enjoy urban benefits B, but at the same time they are required to pay a urban benefit tax τ. For workers who arrive at the cities without an urban hukou status, they have a probability ρ to convert their rural hukou status into urban after staying in the city for $(1 - \mu)$ of their lifetime. For agents who successfully convert their hukou status, they can enjoy urban benefits B but also pay a tax τ for the remainder of their lifetime μ. Let F denote the case where an agent starts working with "formal urban hukou status" and T denote the case where the agent starts his job in urban areas without a formal urban hukou status. We define two indicator functions I^F and I^T associated with the hukou status of an urban worker below:

$$I^F = \begin{cases} 1, & \text{if the agent holds an urban hukou when starting to work,} \\ 0, & \text{ if the agent does not have an urban hukou when starting to work.} \end{cases}$$

For an agent who do not hold urban hukou status when they start working in urban areas, let I^T be the indicator function to denote whether he has successfully obtained an urban hukou before he exits the market. We have

$$I^T = \begin{cases} 1, & \text{if the agent successfully converts his hukou status to urban when } I^F = 0, \\ 0, & \text{if the agent fails to convert his hukou status.} \end{cases}$$

[2]For rural workers to legally work in cities, they have to get "three certificates and one card." (Add for details and check for details later).

Similar to the case for staying in rural areas, urban workers derive utility from consumption (c), quality of children (b), number of children (n), and urban benefits (B, μB or nothing) depending on their hukou status. An agent working in the urban area (denoted by N) has the following utility function:

$$u^N(c,b,n,B;\beta)|_{I^F,I^T} = \min\left[\theta c, (1-\theta)nb\right] + \beta n^{\varepsilon} + \left[I^F + \left(1-I^F\right)I^T\mu\right]B \tag{10}$$

and the budget constraint specified below:

$$\begin{aligned} & c + nb + n\phi_U^0 + \left[I^F + \left(1-I^F\right)I^T\right]I^U\left(n-\bar{n}_U\right)\bar{\phi}_U \\ & + \left[1-\left[I^F + \left(1-I^F\right)I^T\right]\right]I^R\left(n-\bar{n}_R\right)\bar{\phi}_R \\ = \; & w - \left[I^F + \left(1-I^F\right)I^T\mu\right]\tau \end{aligned} \tag{11}$$

where ϕ_U^0 is the child-rearing cost in urban areas, $\bar{\phi}_U \geq \bar{\phi}_R$ is the urban penalty per child for having extra children, $w \in \{w_S, w_P^H, w_P^L\}$ is the agent's wage income from working in either the SOE sector or the private sector by his skill level, and I^U is an indicator function of urban fertility quota such that

$$I^U = \begin{cases} 1, & \text{if } n > \bar{n}_U, \\ 0, & \text{otherwise.} \end{cases}$$

That is, urban workers face the same child-rearing costs, regardless of whether they have urban hukou status or not. However, the fertility quota that agents have to face depends on their hukou status. As the fertility control policy in urban areas is much more restrictive than that in rural areas, it follows that the quota with urban hukou status is smaller than that with rural hukou status. So we impose

Condition $\bar{n}_R \geq \bar{n}_U$.

3.3 Rural-Urban Migration Decision

For workers born in rural areas, they decide whether to migrate to cities by comparing their value of staying in rural areas to that of migrating to urban areas. Based on their endowed skill levels and preferences toward children, their expected values of migrating to urban areas are different. For high-skilled workers, they have a chance to work in the SOE sector that grants them with urban hukou status immediately. If they fail to obtain a job in the SOEs, they work as high-skilled workers in the private sector with rural hukou status. After staying in cities for a fraction of their lifetime, they have a chance to obtain an urban hukou. As for low-skilled workers, they can only work in the private sector as low-skilled workers, but they also have a chance to convert their hukou status into urban. For those obtain urban hukou successfully, they enjoy urban benefits, pay urban social security tax and are subject to the urban fertility quota.

In the following, we describe rural workers migration decision by first defining the value functions of staying in rural areas, working in the SOE sector, and working in the urban private sector for both high- and low-skilled workers. Then we will delineate the conditions under which a rural worker decides to migrate to cities.[3]

[3] Recall that we do not consider endogenous reverse migration.

3.3.1 Value function of staying in rural areas

The value function for rural agents staying in rural areas, independent of their skill, can be formulated as:

$$\begin{aligned} V^R(\beta) &= \max_{c,b,n} u^R(c,b,n;\beta) \\ &= \max_{c,b,n} \{\min[\theta c, (1-\theta) nb] + \beta n^{\varepsilon}\} \\ s.t.\quad & c + nb + n\phi_R^0 + I^R(n - \bar{n}_R)\bar{\phi}_R = zq. \end{aligned}$$

To solve $V^R(\beta)$, from the Leontief preference in c and nb, we substitute $c = \frac{(1-\theta)}{\theta} nb$ into the budget constaint first. The maximization problem becomes:

$$\begin{aligned} V^R(\beta) &= \max_{b,n}\ (1-\theta) nb + \beta n^{\varepsilon} \\ s.t.\quad & \frac{1}{\theta} nb + n\phi_R^0 + I^R(n - \bar{n}_R)\bar{\phi}_R = zq. \end{aligned}$$

Denote λ as the lagrange multiplier associated with the budget constraint. The first-order conditions for n and b are

$$(1-\theta) b + \varepsilon\beta n^{\varepsilon-1} = \lambda\left(\frac{1}{\theta} b + \phi_R^0 + I^R\bar{\phi}_R\right), \tag{12}$$

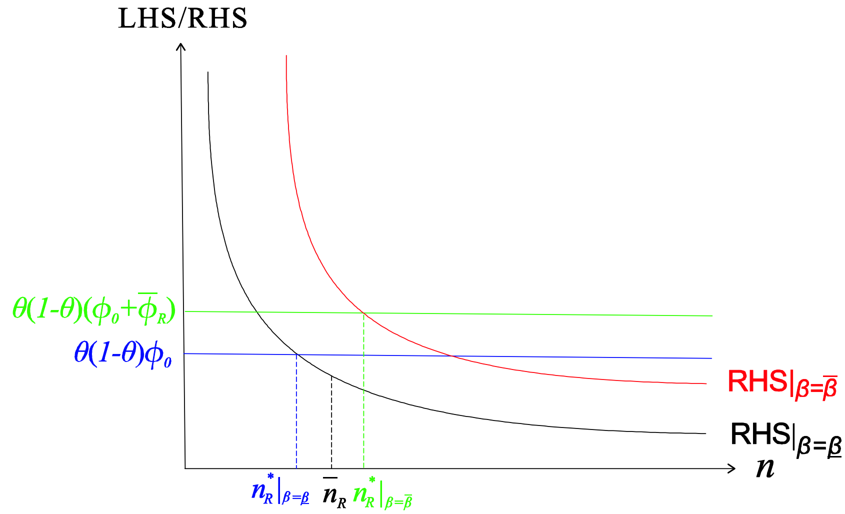
$$(1-\theta) n = \frac{\lambda}{\theta} n. \tag{13}$$

From (13), $\lambda = \theta(1-\theta)$. By substituting $\lambda = \theta(1-\theta)$ back into (12), equation (12) becomes

$$\frac{\varepsilon\beta}{n^{1-\varepsilon}} = \theta(1-\theta)\left[\phi_R^0 + I^R\bar{\phi}_R\right]. \tag{14}$$

The optimal number of children n is determined by equation (14) and is independent of agents' income and wealth. With a particular note, we focus on the case where agents with $\underline{\beta}$ choose $n < \bar{n}_R$, whereas agents with $\bar{\beta}$ choose $n > \bar{n}_R$, paying the penalty for having extra children, and hence $n_R^*|_{\bar{\beta}} > n_R^*|_{\underline{\beta}}$. Equation (14) is plotted in Figure 3.

Figure 3. Determination of fertility for rural workers.



Note that the optimal n_R^* is increasing in β and ε and is decreasing in the cost of having children and the extra-children penalty, ϕ_R^0 and $\bar{\phi}_R$. We can solve n_R^* analytically from (14) as

$$n_R^* = \left[\frac{\varepsilon\beta}{\theta(1-\theta)\left[\phi_R^0 + I^R \bar{\phi}_R\right]} \right]^{\frac{1}{1-\varepsilon}}$$

The optimal quality for children b can be solved from the budget constraint (9) as

$$b^* = \theta \left[\frac{zq}{n_R^*} - \phi_R^0 - I^R \left(1 - \frac{\bar{n}_R}{n_R^*}\right) \bar{\phi}_R \right] \tag{15}$$

With n_R^* and b pinned down by (14) and (15), the value function $V^R(\beta)$ is solved as

$$\begin{aligned} V^R(\bar{\beta}) &= (1-\theta)\theta \left[zq - n_R^*|_{\bar{\beta}} \left[\phi_R^0 + \left(1 - \frac{\bar{n}_R}{n_R^*|_{\bar{\beta}}}\right) \bar{\phi}_R \right] \right] + \beta \left(n_R^*|_{\bar{\beta}}\right)^{\varepsilon} \\ V^R(\underline{\beta}) &= (1-\theta)\theta \left[zq - n_R^*|_{\underline{\beta}} \left(\phi_R^0\right) \right] + \beta \left(n_R^*|_{\underline{\beta}}\right)^{\varepsilon} \end{aligned}$$

3.3.2 Value functions of the SOE and private sector workers

Value function of the SOE worker Only high skilled workers can work in the SOE sector. For rural high-skilled agents who decide to migrate to cities, with probability π they are recruited as the SOE workers and are granted with urban hukou immediately. They enjoy urban benefits and pay urban security tax fully. Otherwise, they end up working in the private sector. An SOE worker has the following value function ($I^F = 1$):

$$V^S(\beta) = \max_{c,b',n} \left\{ \min\left[\theta c, (1-\theta) nb\right] + \beta n^{\varepsilon} + B \right\}$$

$$s.t. \quad c + nb + n\phi_U^0 + I^U (n - \bar{n}_U) \bar{\phi}_U = w_S - \tau$$

Similar to the case when solving n for rural workers, the fertility of the SOE workers is determined by the following equation:

$$\frac{\varepsilon\beta}{n^{1-\varepsilon}} = \theta\left(1-\theta\right)\left[\phi_U^0 + I^U \bar{\phi}_U\right] \tag{16}$$

Again, fertility decision is independent of income, depending only on the child-rearing costs and extra-children penalty in urban areas because of the quasilinear utility function. Once n is pinned down in (16), b_S^* can be solved from the budget constraint as

$$b_S^* = \theta\left[\frac{w_S - \tau}{n_U^*} - \phi_U^0 - I^U\left(1 - \frac{\bar{n}_U}{n_U^*}\right)\bar{\phi}_U\right] \tag{17}$$

An SOE worker's value function $V^S(\beta)$, $\beta = \{\underline{\beta}, \bar{\beta}\}$, is thus

$$\begin{aligned} V^S\left(\bar{\beta}\right) &= \left(1-\theta\right)\theta\left[w_S - \tau - n_U^*|_{\bar{\beta}}\left[\phi_U^0 + \left(1 - \frac{\bar{n}_U}{n_U^*|_{\bar{\beta}}}\right)\bar{\phi}_U\right]\right] + \bar{\beta}\left(n_U^*|_{\bar{\beta}}\right)^{\varepsilon} + B \\ V^S\left(\underline{\beta}\right) &= \left(1-\theta\right)\theta\left[w_S - \tau - n_U^*|_{\underline{\beta}}\phi_U^0\right] + \beta\left(n_U^*|_{\underline{\beta}}\right)^{\varepsilon} + B \end{aligned}$$

Value function of high-skilled workers in the private sector For high-skilled rural migrant workers in the private sector, with probability ρ, they convert their hukou status to urban after staying in urban areas for $(1-\mu)$ of their lifetime ($I^F = 0$, $I^T = 1$). With probability $(1-\rho)$, they fail to convert their hukou status to urban and hold rural hukou status throughout their lifetime ($I^F = 0$, $I^T = 0$). For a high-skilled migrant worker in the private sector who converts his hukou status successfully, his utility maximization problem is:

$$\max u^N\left(c, b, n, B; \beta\right)|_{I^F=0, I^T=1} = \max_{c,n,b}\left\{\min\left[\theta c, \left(1-\theta\right)nb\right] + \beta n^{\varepsilon} + \mu B\right\}$$

$$s.t.\;\; c + nb + n\phi_U^0 + I^U\left(n - \bar{n}_U\right)\bar{\phi}_U = w_P^H - \mu\tau$$

Similarly, for a high-skilled migrant worker in the private sector who fails to convert his hukou into urban, his utility maximization problem is:

$$u^N\left(c, b, n, B; \beta\right)|_{I^F=0, I^T=0} = \max_{c,n,b}\left\{\min\left[\theta c, \left(1-\theta\right)nb\right] + \beta n^{\varepsilon}\right\}$$

$$s.t.\quad c + nb + n\phi_U^0 + I^R\left(n - \bar{n}_R\right)\bar{\phi}_R = w_P^H$$

Denote $V^{P,H}(\beta)$ as the value function of a high-skilled migrant worker in the private sector. Then $V^{P,H}(\beta)$ can be written as:

$$\begin{aligned} V^{P,H}\left(\beta\right) = \rho &\left\{\begin{array}{c} \max\limits_{c,b,n} u^N\left(c, b, n, B; \beta\right)|_{I^F=0, I^T=1} \\ s.t.\; c + nb + n\phi_U^0 + I^U\left(n - \bar{n}_U\right)\bar{\phi}_U = w_P^H - \mu\tau \end{array}\right\} \\ &+ \left(1-\rho\right)\left\{\begin{array}{c} \max\limits_{c,b,n} u^N\left(c, b, n, B; \beta\right)|_{I^F=0, I^T=0} \\ s.t.\;\; c + nb + n\phi_U^0 + I^R\left(n - \bar{n}_R\right)\bar{\phi}_R = w_P^H \end{array}\right\}. \end{aligned}$$

Denote $n_M^*|_{\underline{\beta}}$ and $n_M^*|_{\bar{\beta}}$ as the number of children chosen by the private sector workers without urban hukou status. By substituting the solutions of the maximization problem into the value function, we can write $V^{P,H}(\beta)$ for $\beta = \{\underline{\beta}, \bar{\beta}\}$ as

$$V^{P,H}\left(\bar{\beta}\right) = \rho \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^H - \mu\tau - n_U^*|_{\bar{\beta}} \left[\phi_U^0 + \left(1 - \frac{\bar{n}_U}{n_U^*|_{\bar{\beta}}}\right) \bar{\phi}_U \right] \right] \\ +\bar{\beta} \left(n_U^*|_{\bar{\beta}} \right)^{\varepsilon} + \mu B \end{array} \right\}$$
$$+ (1-\rho) \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^H - n_M^*|_{\bar{\beta}} \left[\phi_U^0 + \left(1 - \frac{\bar{n}_R}{n_M^*|_{\bar{\beta}}}\right) \bar{\phi}_R \right] \right] \\ +\bar{\beta} \left(n_M^*|_{\bar{\beta}} \right)^{\varepsilon} \end{array} \right\}$$

$$V^{P,H}\left(\underline{\beta}\right) = \rho \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^H - \mu\tau - n_U^*|_{\underline{\beta}} \phi_U^0 \right] \\ +\underline{\beta} \left(n_U^*|_{\underline{\beta}} \right)^{\varepsilon} + \mu B \end{array} \right\}$$
$$+ (1-\rho) \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^H - n_M^*|_{\underline{\beta}} \phi_U^0 \right] \\ +\underline{\beta} \left(n_M^*|_{\underline{\beta}} \right)^{\varepsilon} \end{array} \right\}$$

Value function of low-skilled workers in the private sector Denote $V^{P,L}(\beta)$ as the value function of a low-skilled migrant worker in the urban private sector. Then we have

$$V^{P,L}\left(\bar{\beta}\right) = \rho \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^L - \mu\tau - n_U^*|_{\bar{\beta}} \left[\phi_U^0 + \left(1 - \frac{\bar{n}_U}{n_U^*|_{\bar{\beta}}}\right) \bar{\phi}_U \right] \right] \\ +\bar{\beta} \left(n_U^*|_{\bar{\beta}} \right)^{\varepsilon} + \mu B \end{array} \right\}$$
$$+ (1-\rho) \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^L - n_M^*|_{\bar{\beta}} \left[\phi_U^0 + \left(1 - \frac{\bar{n}_R}{n_M^*|_{\bar{\beta}}}\right) \bar{\phi}_R \right] \right] \\ +\bar{\beta} \left(n_M^*|_{\bar{\beta}} \right)^{\varepsilon} \end{array} \right\}$$

$$V^{P,L}\left(\underline{\beta}\right) = \rho \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^L - \mu\tau - n_U^*|_{\underline{\beta}} \phi_U^0 \right] \\ +\underline{\beta} \left(n_U^*|_{\underline{\beta}} \right)^{\varepsilon} + \mu B \end{array} \right\}$$
$$+ (1-\rho) \left\{ \begin{array}{c} (1-\theta)\,\theta \left[w_P^L - n_M^*|_{\underline{\beta}} \phi_U^0 \right] \\ +\underline{\beta} \left(n_M^*|_{\underline{\beta}} \right)^{\varepsilon} \end{array} \right\}.$$

3.3.3 Fertility decisions

As discussed before, due to the quasi-linear specification of the preferences, the fertility decision is recursively determined and is independent of income and wealth. The number of children thus depends only on the costs associated with having children (ϕ_R^0 in rural areas and ϕ_U^0 in urban areas), the penalty per extra child ($\bar{\phi}_R$ and $\bar{\phi}_U$ according to agents' hukou status), the fertility quota in rural and urban areas ($\bar{n}_R$ and $\bar{n}_U$), and the preferences toward children ($\underline{\beta}$ and $\bar{\beta}$). Therefore, as long as the urban agents have the same preference toward children and same child-rearing costs, regardless of which urban sector they work for, they will choose the same number of children.[4] Besides, the difference between $n_M^*|_\beta$ and $n_U^*|_\beta$ is due to the difference in the fertility quota and the extra-children penalty associated with hukou status. Since the extra-children penalty is higher in uban areas, it follows that $n_U^*|_\beta \leq n_M^*|_\beta \leq n_R^*|_\beta$. Hence, below we only compare $n_U^*|_\beta$ and $n_R^*|_\beta$.

Under the assumptions that (i) agents who enjoy less from children are always not affected by the fertility quota, i.e. $n_R^*|_{\underline{\beta}} \leq \bar{n}_R$ and $n_U^*|_{\underline{\beta}} \leq \bar{n}_U$; (ii) agents who enjoy more from children are always affected by the fertility quota, i.e. $n_R^*|_{\bar{\beta}} > \bar{n}_R$ and $n_U^*|_{\bar{\beta}} > \bar{n}_U$; and (iii) $\bar{\phi}_R < \bar{\phi}_U$, we have two possible cases to consider:

1. $\phi_R^0 = \phi_U^0$: Rural and urban agents have the same child-rearing costs, and hence

$$n_U^*|_{\underline{\beta}} = n_R^*|_{\underline{\beta}}$$
$$n_U^*|_{\bar{\beta}} < n_R^*|_{\bar{\beta}}$$
$$n_U^*|_{\underline{\beta}} \leq \bar{n}_U < n_U^*|_{\bar{\beta}}$$
$$n_R^*|_{\underline{\beta}} \leq \bar{n}_R < n_R^*|_{\bar{\beta}}$$

2. $\phi_R^0 < \phi_U^0$: The child-rearing cost is higher in urban areas than in rural areas, and hence

$$n_U^*|_{\underline{\beta}} < n_R^*|_{\underline{\beta}}$$
$$n_U^*|_{\bar{\beta}} < n_R^*|_{\bar{\beta}}$$
$$n_U^*|_{\underline{\beta}} < \bar{n}_U < n_U^*|_{\bar{\beta}}$$
$$n_R^*|_{\underline{\beta}} < \bar{n}_R < n_R^*|_{\bar{\beta}}$$

In our numerical analysis, we focus only on the second case as the child-rearing costs in rural China is lower than that in urban China.

3.3.4 Migration decisions

We are ready to discuss agents' migration decisions. Migration is costly. To simplify the analysis, we assume that migration cost is in the form of utility: migrating from rural to urban areas takes a utility cost $\psi \in \{\psi^L, \psi^H\}$ for low- and high-skilled agents, respectively.[5] That is, rural high- and low-skilled workers may have different utility costs to migrate to cities. For example, high-skilled workers may learn things faster and be more easily to be adapted to urban life than low-skilled workers, and hence $\psi^H < \psi^L$.

Rural agents will migrate to cities only if the expected value of migrating to urban areas is higher than that of staying in the rural. Since high-skilled migrant workers have a probability π

[4] This assmption will be relaxed in the numerical analysis.

[5] We have actually four types of agents, namely, type-$\{L, \underline{\beta}\}$, type-$\{L, \bar{\beta}\}$, type-$\{H, \underline{\beta}\}$ and type-$\{H, \bar{\beta}\}$ agents; in the most general case, we can allow different types of agents to have different migration costs.

to obtain a job in the SOEs, and a probability $1-\pi$ to work in private sectors, high-skilled rural workers will migrate only if the following inequality is satisfied:

$$V^{R}(\beta) \leq \pi V^{S}(\beta) + (1-\pi) V^{P,H}(\beta) - \psi^{H}, \text{ for } \beta \in \{\underline{\beta}, \bar{\beta}\}. \tag{18}$$

For low-skilled rural workers, they can only work in the private sector after migrating to urban areas. Hence, they will migrate to urban areas only when the following inequality is satisfied:

$$V^{R}(\beta) \leq V^{P,L}(\beta) - \psi^{L}, \text{ for } \beta \in \{\underline{\beta}, \bar{\beta}\}. \tag{19}$$

When the above two equations are held with equality, rural agents are indifferent between migrating to urban areas and staying in rural areas. The migration decision depends on the relative magnitudes of rural and urban incomes (zq, which is a function of R^{+}, and w_S, w_H^P, w_L^P), the relative child-rearing costs in urban and rural areas (ϕ_R^0 and ϕ_U^0), the fertility control policies ($\bar{n}_R$, $\bar{n}_U$, $\bar{\phi}_R$ and $\bar{\phi}_U$), the urban benefits B, easiness of obtaining urban hukou and enjoying urban benefits (π, ρ and μ), and the urban social security tax (τ).

Since we have four types of agents, type-$\{L, \underline{\beta}\}$, type-$\{L, \bar{\beta}\}$, type-$\{H, \underline{\beta}\}$ and type-$\{H, \bar{\beta}\}$, we need to consider each type of agents' migration decisions separately. Because of the stricter fertility control in urban area, it is expected that

$$\begin{aligned} V^{S}(\underline{\beta}) &> V^{S}(\bar{\beta}) \\ V^{P,H}(\underline{\beta}) &> V^{P,H}(\bar{\beta}) \end{aligned}$$

Thus, when the wage incomes in urban areas is higher than that in rural areas and $w_P^H > w_P^L$, high-skilled rural agents who enjoy less from children ($\beta = \underline{\beta}$) will be the first to migrate to cities, followed by either low-skilled agents who preferred children less ($\beta = \underline{\beta}$) or high-skilled agents who enjoy more from children ($\beta = \bar{\beta}$), or both. Low-skilled rural workers who enjoy more from children ($\beta = \bar{\beta}$) will be the last group of workers to migrate to cities. Since we have four types of agents and ψ^H may be different from ψ^L, depending on the urban and rural income levels, urban benefits, and probabilities of obtaining SOE jobs and urban hukou status, the model thus has several possible equilibriums. The evolution of workers and the supply of urban ammenities under each of the equilibriums also differ.

We relegate the detailed equilibrium conditions and the definition of the migration equilibrium to Section 2.6. To facilitate the discussion on the supply of urban ammenities and the evolutions of workers, it is convenient that we discuss the equilibrium of interests here. From now on we confine our attention to a specific *migration equilibrium*: type-$\{H, \underline{\beta}\}$ workers always migrate, type-$\{L, \bar{\beta}\}$ always stay in rural areas, and type-$\{H, \bar{\beta}\}$ and type-$\{L, \underline{\beta}\}$ are indifferent between migrating to cities and staying in rural areas. Denote Γ_H as the fraction of high-skilled workers with preference $\beta = \bar{\beta}$ being indifferent between migrating and staying, ending up moving to cities, and $(1-\Gamma_H)$ as the fraction of them ending up staying in their rural hometown. Similarly, denote Γ_L as the fraction of the low-skilled workers with preference $\beta = \underline{\beta}$ indifferent between migrating to cities and staying in rural areas but ending up moving to cities, and $(1-\Gamma_L)$ as the fraction of them ending up staying in rural areas. Table 2 summarizes the migration patterns of high- and low-skilled workers of the equilibrium that we focus on.

Table 2. Migration pattern under the migration equilibrium.

	$\underline{\beta}$ (ζ of workers)	$\bar{\beta}$ ($(1-\zeta)$ of workers)
H	move	Γ_H move $(1-\Gamma_H)$ stay
L	Γ_L move $(1-\Gamma_L)$ stay	stay

Note that if $0 < \Gamma_H < 1$ and $0 < \Gamma_L < 1$, we have the *mixed migration equilibrium* – "mixed" in the sense that both a group of high- and low-skilled workers migrate to cities in the equilibrium. If $\Gamma_H = 1$ (all high-skilled workers migrate to cities) and $0 < \Gamma_L < 1$, we have segregation migration equilibrium: all high-skilled workers migrate, low-skilled workers with preference $\underline{\beta}$ are indifferent between migrating and staying, and low-skilled workers with preference $\bar{\beta}$ always stay.

3.4 Evolution of Workers

As mentioned before, the stocks of the beginning-of-the-period workers and the during-the-period workers in each of the production sector are distinguished by a superscript "+". Before examining evolution of workers, we first write down the beginning-of-the-period and during-the-period population identity equations. In the beginning of a period, we have:

$$U = S + P_F \tag{20}$$

$$R = H + L \tag{21}$$

where U, P_F and R denote total workers with urban hukou status, total private-sector workers with urban hukou status,[6] and total workers with rural hukou status at the beginning of a period respectively. We first describe the total workers and the evolutions of workers in urban areas. The workers devoted to each of the production sector during the period in urban areas are:

$$S^+ = S + \zeta\pi H + (1-\zeta)\Gamma_H \pi H \tag{22}$$

$$\begin{aligned} P^{H+} = {} & \underbrace{P_F^H + \zeta(1-\pi)\rho H + (1-\zeta)\Gamma_H(1-\pi)\rho H}_{\text{urban hukou}} \\ & + \underbrace{\zeta(1-\pi)(1-\rho)H + (1-\zeta)\Gamma_H(1-\pi)(1-\rho)H}_{\text{rural hukou}} \end{aligned} \tag{23}$$

$$P^{L+} = \underbrace{P_F^L + \zeta\rho\Gamma_L L}_{\text{urban hukou}} + \underbrace{\zeta(1-\rho)\Gamma_L L}_{\text{rural hukou}} \tag{24}$$

where P_F^H and P_F^L denote high- and low-skilled workers with urban hukou in the private sector at the beginning of the period. Denote P_I as workers with rural hukou status but working in the urban

[6] The subscript F means "formal urban hukou status." Note that the children of the private sector workers who do not have urban hukou status in the preious period have to start their life in rural areas. Therefore, P_F is the total number of children of the private sector workers who hold urban hukou status before they die.

private sector in the period, we then have

$$P_I = \zeta(1-\pi)(1-\rho)H + (1-\zeta)\Gamma_H(1-\pi)(1-\rho)H + \zeta(1-\rho)\Gamma_L L. \tag{25}$$

Recall that N denotes the total workers working in urban areas, then we have:

$$N^+ = S + \zeta\pi H + (1-\zeta)\Gamma_H\pi H + P_F^H + \zeta(1-\pi)H + (1-\zeta)\Gamma_H(1-\pi)H + P_F^L + \zeta\Gamma_L L. \tag{26}$$

Under the assumptions that children inherit their parents' jobs and hukou status directly and agents only live for one period, the evolutions of workers in the SOE and the private sectors are:

$$S' = S\left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)n_U^*|_{\bar{\beta}}\right] + \zeta\pi H n_U^*|_{\underline{\beta}} + (1-\zeta)\pi\Gamma_H H n_U^*|_{\bar{\beta}} \tag{27}$$

$$P_F^{H'} = P_F^H\left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)n_U^*|_{\bar{\beta}}\right] + \rho\zeta(1-\pi)H n_U^*|_{\underline{\beta}} + \rho(1-\zeta)(1-\pi)\Gamma_H H n_U^*|_{\bar{\beta}} \tag{28}$$

$$P_F^{L'} = P_F^L\left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)n_U^*|_{\bar{\beta}}\right] + \rho\zeta\Gamma_L L n_U^*|_{\underline{\beta}} \tag{29}$$

The evolution equation for U can be written accordingly:

$$\begin{aligned} U' &= S' + P_F^{H'} + P_F^{L'} \\ &= S\left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)n_U^*|_{\bar{\beta}}\right] + \zeta\pi H n_U^*|_{\underline{\beta}} + (1-\zeta)\pi\Gamma_H H n_U^*|_{\bar{\beta}} \\ &\quad + P_F^H\left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)n_U^*|_{\bar{\beta}}\right] + \rho\zeta(1-\pi)H n_U^*|_{\underline{\beta}} + \rho(1-\zeta)(1-\pi)\Gamma_H H n_U^*|_{\bar{\beta}} \\ &\quad + P_F^L\left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)n_U^*|_{\bar{\beta}}\right] + \rho\zeta\Gamma_L L n_U^*|_{\underline{\beta}} \end{aligned} \tag{30}$$

In a similar manner, we can write down the during-the-period workers in rural areas:

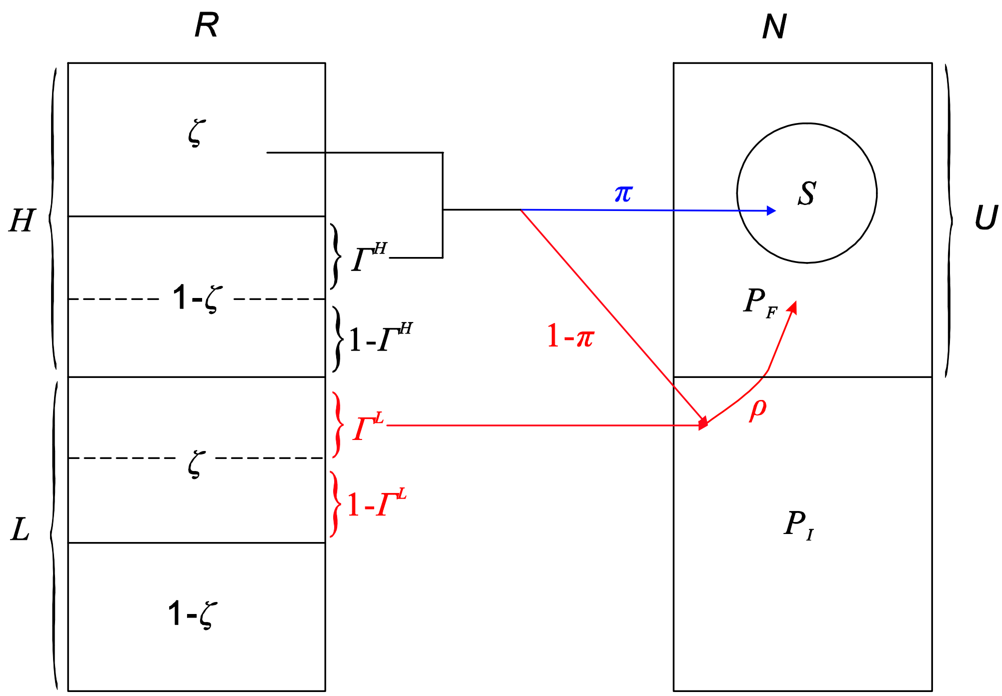
$$R^+ = \underbrace{(1-\zeta)(1-\Gamma_H)H}_{H^+} + \underbrace{[\zeta(1-\Gamma_L) + (1-\zeta)]L}_{L^+}. \tag{31}$$

And the evolution equation for workers in rural areas is given by:

$$\begin{aligned} R' &= \underbrace{H\left\{(1-\rho)(1-\pi)\zeta n_R^*|_{\underline{\beta}} + (1-\rho)(1-\pi)(1-\zeta)\Gamma_H n_R^*|_{\bar{\beta}} + (1-\zeta)(1-\Gamma_H)n_R^*|_{\bar{\beta}}\right\}}_{H'} \\ &\quad \underbrace{+L\left\{\Gamma_L(1-\rho)\zeta n_R^*|_{\underline{\beta}} + \zeta(1-\Gamma_L)n_R^*|_{\underline{\beta}} + (1-\zeta)n_R^*|_{\bar{\beta}}\right\}}_{L'}. \end{aligned} \tag{32}$$

Figure 4 provides a summary chart of the population flow in the model.

Figure 4. Population flow chart.



3.5 Urban amenities

Urban amenities, such as urban infrastructure, parks, museums, hospitals, and etc., are assumed to be financed by urban security taxes and the bequests left for children. To simplify the analysis, we assume that urban amenities are non-rival and non-excludable public goods. Total urban security tax collected by the government is:

$$T = \left(S^{+} + P_F^H + P_F^L\right)\tau + \left[\zeta\left(1-\pi\right)\rho H + \left(1-\zeta\right)\Gamma_H\left(1-\pi\right)\rho H + \zeta\rho\Gamma_L L\right]\mu\tau$$

Using the social security taxes collected, the government provides urban amenities with the following technology:

$$B = B_0 T, \tag{33}$$

where B_0 is the government's technology scaling factor in public goods provision. Since B is non-rival and non-excludable, the technology parameter B_0 is responsible to adjust so as to assure (33) to hold in equilibrium.

3.6 Equilibrium

In equilibrium, all urban labor markets clear under the factor prices $\left\{w_S, w_P^H, w_P^L\right\}$ given by (5), (6) and (7):

$$S^d = S^+,\ P^{H,d} = P^{H,+},\ P^{L,d} = P^{L,+} \tag{34}$$

where S^d, $P^{H,d}$ and $P^{L,d}$ are the labor demands in specific sectors. Finally, rural labor market clears under the rural farming income given by (1):

$$R^{dt} = R^+ \tag{35}$$

where R^d is the demand for rural labor.

We define the competitive equilibrium of the model below.

Definition. *A* ***dynamic competitive migration equilibrium*** *(DCME) of the model consists of migration decisions, rural farming income zq, and urban wage rates $\{w_S, w_P^H, w_P^L\}$, such that*

(i) *(Optimization) given rural farming income zq and urban wage rates $\{w_S, w_P^H, w_P^L\}$, based on their hukou status, agents choose numbers of children according to (14) and (16); furthermore, rural high- and low-skilled agents make migration decisions by comparing their values of staying in rural versus their values of moving to cities given by (18) and (19), respectively;*

(ii) *(Market clearing) rural farming income satisfy (1), urban wage rates $\{w_S, w_P^H, w_P^L\}$ satisfy (5), (6) and (7), and labor markets clear according to (34) and (35);*

(iii) *(Urban infrastructure) the infrastructure in urban areas is supplied according to (**??**);*

(iv) *(Workers laws of motion) given the initial population $\{H^0, L^0, S^0, P^{H,0}, P^{L,0}\}$, high- and low-skilled workers in rural, urban SOE and urban private sectors evolve according to (27)-(30) and (32), with workers actually devoted to the production of a specific sector given by (22)-(24) and (31).*

We next define the steady-state equilibrium for the remainder of our analysis.

Definition. *A* ***steady-state migration equilibrium*** *of the model is a dynamic competitive migration equilibrium with the growth rate of population variables being constant as follows:*

$$\frac{X'}{X} = \textit{constant},$$

where $X = S, R, P_F^H$ and P_F^L.

In the steady state, from (27)-(29), we get

$$\frac{S'}{S} = \left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)\, n_U^*|_{\bar{\beta}}\right] + \left[\zeta \pi n_U^*|_{\underline{\beta}} + (1-\zeta)\, \pi \Gamma_H n_U^*|_{\bar{\beta}}\right] \frac{H}{S} \tag{36}$$

$$\frac{P_F^{H\prime}}{P_F^H} = \left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)\, n_U^*|_{\bar{\beta}}\right] + \left[\rho\zeta\,(1-\pi)\, n_U^*|_{\underline{\beta}} + \rho\,(1-\zeta)\,(1-\pi)\,\Gamma_H n_U^*|_{\bar{\beta}}\right] \frac{H}{P_F^H} \tag{37}$$

$$\frac{P_F^{L\prime}}{P_F^L} = \left[\zeta n_U^*|_{\underline{\beta}} + (1-\zeta)\, n_U^*|_{\bar{\beta}}\right] + \left(\rho\zeta\Gamma_L n_U^*|_{\underline{\beta}}\right) \frac{L}{P_F^L}. \tag{38}$$

Thus, in the steady state, we have a constant ratio of SOE workers to high-skilled workers (S/H), as well as constant ratios of different skill-type workers in the urban private sector (P_F^H/H and P_F^L/L). Next, from (32), we have constant growth rates of different skill-type workers. These together with the fact that the growth rate of R is constant, we get a constant ratio of high-skilled to low-skilled workers (H/L). This in turn implies that the ratio of high-skilled to low-skilled workers in the urban private sector is constant (P_F^H/P_F^L). Finally, from (20), (25), and (26), we can show that both the growth rates of U, P_I and N, as well as their ratios to high-skilled workers are constant. So we conclude

Propostion. *The model exhibits common growth rate property in the steady state:*

$$\frac{X'}{X} = g,$$

where $X = S, U, N, R, H, L, P_I, P_F^H$ and P_F^L.

With common growth of H and L, then (32) implies that the following restriction on the parameters:

$$\frac{n_R^*|_{\bar{\beta}}}{n_R^*|_{\underline{\beta}}} = \frac{\zeta}{1-\zeta}\frac{\rho\Gamma_L - [1-(1-\rho)(1-\pi)]}{\Gamma_H[1-(1-\rho)(1-\pi)]} > 1.$$

4 Quantitative Analysis

In this section, we discuss how we calibrate the model in detail. Based on the parameters obtained from the calibration, we then perform counterfactual analysis to study the effects of the relaxation of the fertility constraints on the migration and urbanization patterns, the effects of changes in the land entitlement policy on economic development, and how various regulations of the hukou system affect fertility and migration decisions.

4.1 Calibration

The period under examination is 1980-2007. A model period is set as 45 years since agents only live for one period. Although in the model, agents give birth to their offspring in the end of their life, in calibration we set the length of raising up a child to be 20 years. All child-rearing related costs are thus adjusted according to the model period and length of child rearing. There are 27 parameters to be calibrated, including (1) preference parameters: altruistic factor of saving in offspring θ, preference toward children $\underline{\beta}$ and $\bar{\beta}$, the utility concavity in the quantity of children ε, and utility costs migration for high- and low-skilled workers, ψ_H and ψ_L; (2) proportion of agents less preferring children, ζ; (3) child-related expenses: ϕ_U^0, ϕ_R^0, $\bar{\phi}_U$ and $\bar{\phi}_R$; (4) fertility constraints in urban and in rural areas, $\bar{n}_U$ and $\bar{n}_R$; (5) urban social security tax and benefits, μ, B and τ; (6) production technologies, A_S, A_P, a, r, η, z and B_0; (7) probabilities of obtaining urban hukou and position in the SOE sector, ρ and π; (8) fractions of type-$\{H, \bar{\beta}\}$ and type-$\{L, \underline{\beta}\}$ workers migrating to urban areas, Γ_H and Γ_L. Note that in the data, we do observe that workers with different educational attainment have different fertility rates, both in urban and in rural areas. To allow the model to fit the data better, we thus allow child-rearing costs (ϕ_U^0 and ϕ_R^0) to depend on agents' income levels. Moreover, we allow differentiations in extra-children penalty schemes for high-skilled and SOE workers, i.e. $\bar{\phi}_U$ and $\bar{\phi}_R$ are different both across sectors and regions. The reason is based on the observation that rural high-skilled workers usually were located to rural areas by the government and held positions either in the public sector. We do not have good data of workers by hukou status. From official estimate, we only know that roughly 80% of urban workers are of urban legal status. We thus assume that 90% of urban informal residents work as low-skilled workers in the private sector. Based on this assumption, we can compute ratios of workers such as N^+/U and P^{H+}/P^{L+}. We will provide sensitivity tests to test this assumption later.

Our model is a unisex model. Following the tradition in the endogenous fertility literature, whenever we use the total fertility rate (TFR) data or fertility constraint set by the Chinese government per couple, we divide the numbers by two. According to Ebenstein (2010), the average fertility quota per couple in urban and in rural areas are one child and 1.6 children, we thus set $\bar{n}_U$ and $\bar{n}_R$ to 0.5 and 0.8, respectively. Regarding the extra children penalty, $\bar{\phi}_U$ and $\bar{\phi}_R$, according to Ebenstein (2010) and several other reports, there were several forms of penalty if workers have more children than allowed. For a worker in the public sector, he or she could even lose her job if violating the fertility control policy. Therefore, it is inaccurate to impute extra children penalties based on fine payments only. However, due to limited information, penalties in the form of fine can still serve as a proxy for the true penalty. We thus compute the fine payment as a percentage of income for rural low-skilled workers and urban private sector using the data provided by Ebenstein (2010). As mentioned above, we then calibrate the extra children penalty for the rural high-skilled workers, SOE workers and urban private sector high-skilled workers by matching the average fertility rates in these three groups to that of data.

Since we have more parameters to pin down than model equations, we have to preset some parameter values. The concavity in the utility function for number of children ε is set at 0.1, which is roughly equal to the number used in Gobbi (2013). We choose to set $\underline{\beta} = 0.3$, $\bar{\beta} = 0.6$, $\pi = 0.2$ and $\Gamma_L = 0.6$. For the length of period that agents can enjoy urban benefits if they successfully transform their hukou status from rural to urban (μ), since the average duration to obtain urban hukou via the blue print channel takes 2 to 5 years, we thus compute μ using the following formula: $\mu = 1 - (2+5)/(45 \cdot 2) = 0.9222$. The annual successful rate of hukou conversion was controlled by the government, and was roughly equal to 2 percent throughout 1994 to 2007 (14 years). We thus compute $\rho = 0.02 \cdot 14 = 0.28$. As for the altruistic parameter of saving in offspring θ, we set it to 0.5, meaning that parents equally value their own consumption and children's quality. To gauge the fraction of people who less preferred children ζ, we resort to surveys on family size preferences and mean number of children preferred in China. Note that the number of children preferred is affected by child rearing costs, the fertility control policy and the prevailing social norm. We thus choose to use "cleaner" survey results – we use the survey results of the number of children wanted in rural China in the 1980s only. With the categorization criteria that people who preferred to have no children or one child only are classified as "children-hating" type and people who preferred to have two children or more are classified as "children-loving" type, we compute the fraction of people who preferred to have children less (ζ) from Hermalia and Liu (1990) and Scharping (2003).

These two sources of surveys both indicate that there were roughly 15 percent of people preferred to have either no children or only one child. We thus choose to set $\zeta = 0.15$. Later we will perform sensitivity tests on the assumptions without empirical evidence or support in the literature to see the robustness of our results.

We then turn our focus to calibrate the values of parameters for production. Essentially we rely on income ratios and worker ratios to pin down these production function parameters. The data on income ratios and workers ratios in urban sector are obtained and computed from Urban Household Survey, 1986-1997. The high-to-low skilled workers ratio in the private sector is obtained from China Economic Census 2004. Other data, such as rural to urban per capita income ratios, rural and urban consumption rates, population by residence, rural land per rural worker, and high- and low-skilled worker ratios in rural areas are obtained from China Statistical Yearbook or China Rural Statistical Yearbook. We first normalize rural per capita income to one. Since the land operated per rural worker was roughly 2.28 mu, the agricultural technology z is computed as 0.4370 by using equation (1). As for the calibration for the parameters of urban production functions, we set $\sigma = 0.83$, which corresponds to an elasticity of substitution (EIS) between high- and low-skilled workers to be 6. This value of the EIS between high- and low-skilled workers falls within the range of the EIS estimates for East Asian countries. As for the CES production share of high-skilled workers α, we set it at the value of 0.5. Then, urban technology scaling factors, A_S and A_P, are jointly solved to match urban to rural per capita income ratio (y_U/y_R) and the wage ratio of the SOE and private sector workers who held a high school degree or above. The calibrated A_S and A_P are equal to 1.8678 and 3.6158, respectively. This result shows that the private sector is much more efficient in using production inputs than the SOE sector. The quality index of high-skilled workers, η, is calibrated from the wage ratios of high- and low-skilled workers in the private sector, and is equal to 1.3843.

We are left with five parameters uncalibrated: urban social security tax τ, the fraction of type-$\{H, \bar{\beta}\}$ agents migrating to cities Γ_H, urban benefits B, and migration disutility for type-$\{H, \bar{\beta}\}$ and type-$\{L, \underline{\beta}\}$ agents, ψ_H and ψ_L. Urban security tax τ is solved to match urban consumption as a percentage of disposable income and is equal to 0.5981; the fraction of type-$\{H, \bar{\beta}\}$ agents migrating to cities Γ_H is computed to match private sector high-to-low skilled worker ratios (P^{H+}/P^{L+} ratio in the model) and is equal to 0.0971; urban benefits B is computed using k/y ratio and average income per capita. Finally, ψ_H and ψ_L are solved from the migration indifference equations, (18) and (19), and are equal to 0.5148 and 0.1923, respectively. This result indicates that high-skilled workers who preferred to have more children actually suffer more from migrating to cities.

The calibration results are summarized in Table 2. In Table 3 we further report important ratios computed based on the calibration results. Our results indicate that private sector has much better technology than the SOE sector and the rural agricultural sector; the fraction of low-skilled workers migrating to cities only account for less than 10 percent of all rural low-skilled workers. Moreover, despite the higher income in urban areas, urban workers actually bear relatively higher child-rearing costs than rural workers.

Based on the calibrated parameters, we now proceed to counterfactual analysis.

4.2 Counterfactual Experiments

Based on the benchmark model, we are now ready to conduct counterfactual analysis. We are particularly interested in learning how changes in land entitlement, how relaxations in fertility control policies in urban and in rural areas, and how changes in the hukou regulation affect the migration patterns, the fertility rates and the urban-rural income ratios. To proceed on the analysis, we first compute the equilibrium of the benchmark model based on the calibrated parameters. Then we change the values of interested parameters one by one and compute the new equilibrium values. We then compute the percentage point changes relative to the benchmark model. The results are reported in Table 4.

First we consider an improvement in land entitlement. When the land entitlement is better (or quality of land is improved), one would expect that with the same amount of land, a rural worker expects to have a higher income if he or she stays in rural areas. Urban jobs thus become less attractive, and more type-$(H, \bar{\beta})$ and type-$(L, \underline{\beta})$ agents decide to migrate to urban areas, resulting in a smaller Γ_H and Γ_L. As Γ_H goes down, there will be more type-$(H, \bar{\beta})$ staying in rural areas, boosting up the fertility of rural high-skilled workers. When Γ_L is smaller, there are more type-$(L, \underline{\beta})$ agents staying in rural areas (a smaller Γ_L), lowering down the fertility rate of rural low-skilled workers. The same rationale applies to explain the lower fertility rates for the SOE workers and the private sector high-skilled workers. As child-rearing costs and extra-children penalty are assumed to be proportional to income, when there are fewer low-skilled workers in urban areas, the income of low-skilled private sector workers increases. The fertility rate of low-skilled workers in urban areas thus declines. Furthermore, because of the increase in the fertility of high-skilled workers, rural land is diluted. Together with the higher income of low-skilled workers in urban areas, the urban-to-rural income ratios increase as a result.

We then turn our focus to study the relaxations in fertility control policies. When the fertility control policy in urban areas is relaxed so that urban workers are allowed to have more children ($\bar{n}_U$ increases), type-$(H, \bar{\beta})$ rural agents will "sacrifice" less in number of children wanted if they decide to migrate to cities. Thus, when $\bar{n}_U$ is higher, the direct effect leads to a higher Γ_H. As the majority of migrant type-$(H, \bar{\beta})$ agents ends up in the private sector, the fertility rate of private sector high-skilled workers goes up. Since there are more private sector high-skilled workers, the wage rate of low-skilled workers in the private sector increases, resulting in a higher Γ_L. On the contrary, when the fertility control policy in rural areas is relaxed, it is expected that rural workers preferring children more will give more births. This then dilutes the land allocated to rural farmers, leading to a higher Γ_H and Γ_L. Since there are more low-skilled workers moving to urban areas, it is not surprising to find that the urban-to-rural income ratio decreases relative to that in the benchmark model when the fertility control policy in rural areas is loosened.

Finally, we consider changes in the regulation in the hukou system as well as the job recruiting in the SOE sector. When it is relatively easy to obtain urban hukou status (a higher ρ or a higher π). Although the SOE job grants migrant workers with urban hukou immediately, it pays much lower than the private sector. Besides, agents also face a higher extra-child penalty. This creates an implicitly more stringent "effective" fertility constraint for high-skilled workers if they migrate to cities. Therefore, the fraction of type-$(H, \bar{\beta})$ agents migrating to cities is smaller (Γ_H decreases), the fertility rates among the SOE workers and the private sector high-skilled workers are lower. Since there are fewer high-skilled workers in urban areas, the wage of low-skilled workers falls, and resulting in a smaller fraction of type-$(L, \underline{\beta})$ agents migrating to cities.

5 Conclusion and Extensions

Previously, in Liao, Wang, Wang and Yip (2015), we attempt to understand the contribution of rural-urban migration on China's development via the higher education channel. Along this line of research, this paper is our follow-up attempt in understanding the rural-urban migration pattern in China, with a focus on work-based migration. We confine ourselves to study the interactions between land reallocation, the endogenous fertility decisions as well as the endogenous migration decisions under the Chinese hukou system and the one-child policy. More specifically, we construct a dynamic general equilibrium model with endogenous fertility, land reallocation and migration decisions tailored to fit the Chinese institutions. The model is then calibrated to fit the data from China, and quantitative analysis to investigate the impacts of the fertility control and land policies on China's rural-urban migration is provided.

Different from the existing literature, our model is more closed to the reality in the way that the reallocation of land is incorporated in the model. With this new channel, we find interesting interactions between fertility choices and migration decisions in the counterfactual experiments. Because of the higher child-rearing costs in urban areas, a higher fertility quota in urban areas only leads to very minor increases in urban fertility rates. However, less strict fertility control policies in urban areas attract more rural workers, resulting in a higher migration rate and a lower birth rate in rural areas when workers who love children more migrate to cities. When the fertility quota in rural areas is loosened, more rural workers will migrate to cities due to the land dilution effect. As the dilution of land drives rural workers who prefer to have more children away from their hometown, the rural fertility rates drop as a result.

To our best knowledge, this paper is the first to examine rural-urban migration with land entitlement, the one-child policy, and the hukou system in a dynamic general equilibrium model. The ready extension of the current work includes: (1) a more careful calibration to break the period under examination into several sub-periods to study policy changes in the hukou system and fertility control policy; (2) a more detailed counterfactual analysis on fertility control policies as well as hukou regulation to study their true effects on migration pattern, fertility rates and income levels; (3) as the direct policy changes, the land dilution effect, and the changes in income levels due to substitution in inputs in the production, a further decomposition analysis of all the policy experiments performed is called for. These tasks are lying on our desks for completion now.

Table 2: Summary of calibration

Parameters	Values	Target	Value of target	Explanation/Data source
Production				
z	0.4370			Computed under the normalization such that rural per capita income $zq = 1$
q	2.2884			Average land operated per person (unit: mu). Data source: China Rural Statistics Yearbook
A_S	1.8678	$\frac{y_U}{y_R}$	1.9641	A_S and A_P are jointly solved to match $\frac{y_U}{y_R}$ and $\frac{w_S}{w_P^H}$ ratios.
A_P	3.6158	$\frac{w_S}{w_P^H}$	0.73	
σ	0.8333			Elasticity of substitution between high and low skilled workers = 6.
α	0.5			Preset
η	1.3843	$\frac{w_P^H}{w_P^L}$	1.4438	Match $\frac{w_P^H}{w_P^L}$. $\frac{w_P^H}{w_P^L}$ is computed from urban household survey, 1986-1997.
Preferences				
ζ	0.15			1980s fraction of rural families preferring number of children1. Computed from Scharping (2003) and Hermalia and Liu (1990).
$\underline{\beta}$	0.3			Preset
$\overline{\beta}$	0.6			Preset
ε	0.1			Preset. Roughly close to Gobbi (2013).
θ	0.5			Conusmption equally important to total assests left for children.
β	0.1069			Computed using annual $\frac{k}{y}$ ratio=3.5 and average income per capita in the economy.
ψ_H	0.5148			Solved from $(H,\overline{\beta})$ rural workers' migration indifference equation.
ψ_L	0.1923			Solved from $(L,\underline{\beta})$ rural workers' migration indifference equation.
Probabilities/Duration of time				
π	0.2			Preset
μ	0.9222			Match average duration to obtain urban hukou via the blue print channel (2-5 years).
ρ	0.28			Annual successful rate of hukou conversion = 2% (for 14 years)
τ	0.5981	Urban consumption rate	0.8288	Match urban consumption as a percentage of disposable income.
Fraction of workers migrate				
Γ_H	0.0971	$\frac{p^{H+}}{p^{L+}}$	0.5611	Match $\frac{p^{H+}}{p^{L+}}$ ratio under the assumption that 90% of urban informal residents work as low-skilled workers in the private sector.
Γ_L	0.6			Preset
Fertility related				
$\bar{n}_R$	0.8			Average rural fertility constraint divided by 2. Computed from Ebenstein (2010).
$\bar{n}_U$	0.5			Average urban fertility constraint divided by 2. Computed from Ebenstein (2010).
$\tilde{\phi}_U^0$	0.1865	n_U^L	1.0388	Match fertility of urban workers with educational attainment below senior high school (1989 data, taken from Scharping (2003)). Child rearing cost to income ratio.
$\tilde{\phi}_R^0$	0.3128	n_R^L	1.3487	Match fertility of rural workers with educational attainment below senior high school (1989 data, taken from Scharping (2003)). Child rearing cost to income ratio.
$\bar{\phi}_R$	1.5928			Average penalty to annual income ratio for having an extra child in rural areas. Computed from the data provided by Ebenstein (2010).
$\bar{\phi}_U$	1.6654			Average penalty to annual income ratio for having an extra child in urban areas. Computed from the data provided by Ebenstein (2010).
Extra penalty for high-skilled workers to have extra children				
ϕ_H	3.7584	n_R^H		Match fertility of rural workers with educational attainment of senior high school and above (1989 data, taken from Scharping (2003)).
ϕ_{SOE}	1.7719	n_{SOE}		Match fertility of SOE workers (1989 data, taken from Scharping (2003)).
ϕ_P^H	0.11409	n_P^H		Match fertility of urban workers with educational attainment of senior high school and above (1989 data, taken from Scharping (2003)).

Table 3: Calibration Results

Ratios	Values	Explanation
$\frac{A_P}{A_S}$	1.9358	Private sector TFP relative to SOE sector TFP.
$\frac{A_P}{z}$	8.2742	Private sector TFP relative to rural agricultural sector TFP.
$\zeta\Gamma_L$	0.09	Fraction of rural low-skilled workers moving to cities.
$\frac{\tilde{\phi}_U^0 w_S}{\tilde{\phi}_R^0 w_R}$	1.1135	Urban SOE child rearing cost to rural child rearing cost ratio.
$\frac{\tilde{\phi}_U^0 w_P^H}{\tilde{\phi}_R^0 w_R}$	1.5253	Urban private sector high-skilled workers' child rearing cost to rural child rearing cost ratio.
$\frac{\tilde{\phi}_U^0 w_P^L}{\tilde{\phi}_R^0 w_R}$	1.0565	Urban private sector low-skilled workers' child rearing cost to rural child rearing cost ratio.

Table 4: Percentage points change relative to the benchmark model

							Average fertility rates					Income ratio
	Γ_H	Γ_L	$\frac{P_F^H}{H}$	$\frac{P_F^L}{L}$	$\frac{H}{L}$	$\frac{S}{H}$	Rural H	Rural L	SOE	Private H	Private L	$\frac{y_U}{y_R}$
z increase by 5%	-16.71%	-78.42%	-15.73%	-79.76%	-76.14%	-22.08%	0.50%	-5.09%	-0.03%	-0.35%	-0.31%	0.18%
$\bar{n}_R$ relaxes by 5%	0.42%	1.15%	1.90%	1.44%	0.63%	2.12%	-0.08%	0.00%	0.00%	0.10%	-0.04%	-0.05%
$\bar{n}_U$ relaxes by 5%	0.04%	0.10%	0.16%	0.12%	0.05%	0.17%	-0.01%	0.00%	0.00%	0.01%	0.00%	0.00%
π increase by 5%	-3.74%	-6.07%	-11.02%	-7.52%	-2.04%	-6.16%	0.58%	0.01%	-0.01%	-0.59%	0.20%	0.17%
ρ increase by 5%	-7.25%	-18.85%	-14.30%	-21.27%	-13.42%	-25.42%	1.58%	0.09%	-0.04%	-0.71%	-0.15%	1.34%

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