Free-riders and Underdogs: Participation in Corporate Voting *

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Abstract

Voting outcomes can differ from underlying preferences due to strategic selection into voting. We discuss one explanation for such selection effects: lower participation of shareholders with popular preferences (free-rider effect) relative to those with unpopular preferences (underdog effect). We develop a rational choice model where the voting participation decision depends on the probability of being pivotal and the costs and benefits of voting. Our model yields an algorithm that uncovers unobserved shareholder preferences. Empirically, we find that strategic selection into voting is relevant: the realized support for a proposal on average differs by 21% from its popularity in the shareholder base.

Keywords: voting participation; corporate governance; shareholder proposals; shareholder preferences; heterogeneity of ownership; institutional ownership

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1 Introduction

Voting is the main avenue to aggregate shareholder preferences. Alas, not all shareholders need to vote. In the US, mutual funds have an obligation to vote and report their votes. These funds on average constitute 20% of shareholders. Netting out these mandatory votes, we document average discretionary participation rates in US corporate voting of 73% and considerable variation across proposals. Selection into voting participation creates a wedge between the underlying shareholder preferences and the observed voting patterns. How large is this wedge? Would mandatory voting change voting support or even sway the outcome?

We address these questions from a benefit to cost trade-off perspective, where the benefit of voting is weighted by the likelihood that the voter is pivotal. To capture this trade-off, we propose a rational choice model. The model introduces ownership heterogeneity to the class of pivotal voter models in the political science literature (especially Myatt (2015)). Specifically, we juxtapose *regular* voters (such as the mutual funds in the US) to *discretionary* voters that choose whether to vote (such as hedge funds or wealth managers in the US). Our description of equilibria via parameter regions and associated participation rates allow us to identify preference parameters from observed voting outcomes. Based on this structure, we estimate model parameters using US voting data between 2003 and 2011.

We illustrate how the participation decision can change voting outcomes. In particular, the model unravels the tension between the groups of regular and discretionary voters: discretionary voters who agree with the majority of regular voters turn out to vote less, *the intergroup free-riding effect*; those who disagree turn out more, *the intergroup underdog effect.* In the US data, selection on average augments the observed voting support for the minority by 21% compared to the support in the entire shareholder base (i.e., the population). This leads to a measurable but rather small 3.7% probability of overturning the population preference. We compute counterfactual values for this selection-implied probability of a non-representative outcome as a function of the costs of voting. On a spectrum between the full participation cost level and the cost level with no discretionary participation, we locate the US case at a low level, close to the cost level that implies full participation. The selection-driven probability of a non-representative outcome (misalignment) has a reverse-U shape, with a peak of 35% at a medium cost level with the highest occurrence of an equilibrium with partial participation (i.e., mixing) on both sides. In such 'mixed-mixed' equilibrium the average outcome is a tie and the ultimate outcome is decided with a coin-flip (so there is a 50% chance that the favourite won't win).

Our model follows the long tradition of pivotal voter models in the political economy literature. Similar to those, we consider a voting contest with two options, where voting is costly and voters have a preferred alternative (i.e., they are partisan). In the corporate context, such partisanship is analogous to disagreement between shareholders, which is a widespread phenomenon among participants in firms; see, for example, Aghion and Bolton (1989), Bolton, Li, Ravina, and Rosenthal (2018) and Li, Maug, and Schwartz-Ziv (2019).[1] Our model is closest to Myatt (2015), who introduces aggregate uncertainty about the popularity of the proposal amongst voters. The substantive innovation of our model is that we account for heterogeneity in shareholder characteristics and distinguish between regular (or committed) and discretionary (or intermittent) voters. We can interpret the first group as either investment funds, which are legally required to vote, or blockholders such as families, who hold such a large fraction of shares that voting is always beneficial to them. The second group can be thought of as dispersed shareholders (such as hedge funds or private wealth managers) who choose whether they will vote or not. Discretionary voters are homogeneous other than their preferred alternative.

We solve for the equilibrium participation choices of discretionary voters who are either *against* or *for* the proposal, who we refer to as *types* of voters. We obtain six equilibria for non-overlapping parameter regions available in closed form. These parameters are the fraction of shares owned by regular voters, their preferences (as captured by the popularity of against/for amongst them), the benefit to cost ratio per discretionary voter, and the mean and standard deviation of their preferences (as captured by the popularity of against/for amongst them). For each equilibrium we have a different set of participation rates for each type (against/for), which are given in closed-form for all equilibria. For example, in the 'mixed-mixed' equilibrium both types are indifferent between voting and not, which results to incomplete turnout on both sides.

The model unravels the effects of ownership structure and preferences on discretionary partici-

[1] Disagreement may arise due to differences in beliefs [agreeing to disagree (Harrison and Kreps (1978)) and overconfidence (Scheinkman and Xiong (2003))] or conflicts of interest [portfolio allocation (Cohen and Schmidt (2009)), business ties (Davis and Kim (2007), Cvijanović, Dasgupta, and Zachariadis (2016)), and proxy advisors (Li (2016))], or differences in incentives [flow motives/reputational concerns (Chevalier and Ellison (1999))], among others.

pation and ultimate voting outcome. Let us start with the type of discretionary voters that support the favorite alternative amongst regular voters (that is, they agree). Those agreeing voters turn out less across all equilibria the strongest the support of their preferred alternative is amongst regular voters. Hence, they *free-ride* on the regular voters, an effect comparable to that in the context of takeover bids (Grossman and Hart (1980)). In contrast, disagreeing discretionary shareholders turn out more across all equilibria the weaker their support by regular voters, an *underdog effect*.[2] The combined effect on the voting outcome depends on the equilibrium. For example, the equilibrium with mixed participation on both sides exhibits a full underdog effect: disagreeing voters turn out more to overcome their ex ante disadvantageous position, to the extent that the equilibrium outcome is on average a tie. In contrast, the equilibrium with full turnout by disagreeing voters and partial turnout by agreeing ones exhibits a partial underdog effect where the outcome is on average the favorite of the regular voters.

The model guides the design of an algorithm to estimate the unobservable parameters (such as the benefit to cost ratio per voter) using observable ones (such as the fraction of shares owned by regular voters). In particular, we perform a first stage generalized method of moments (GMM) estimation across all equilibria. We use as inputs the first and second moments of discretionary support for each type across all proposals, as well as the observed fraction of shares owned by regular voters and their voting behaviour (which we assume is known ex ante). Our estimated parameters are the benefit to cost ratio per voter, the equilibrium participation rates for each type, and the mean and standard deviation of the popularity of the proposal amongst discretionary voters.

We apply the algorithm to a sample of US proposals. In the US, investment funds have a fiduciary duty to vote and report their vote in the N-PX form. We use these shareholders as an empirical approximation of the regular voters in the model. To this end, we match aggregate voting records from ISS for non-standard proposals in Russell 3000 firms between 2003 and 2011 to ownership data from 13F form filings, and the individual number of votes and their direction from N-PX forms. The algorithm is able to assign 95% of the proposals to an equilibrium region and yields parameter estimates within those. The algorithm performs significantly better in predicting voting outcomes out of sample compared to a linear ordinary least squares (OLS) model based on the previous literature, in particular Malenko and Shen (2016). Its parameter estimates in terms

[2]In the political science literature, where there is a single group of only discretionary voters, the underdog effect refers to the higher participation rate of supporters of the option that is (ex ante) less popular among the electorate.

of the benefit to cost ratio per voter are comparable to the previous literature's estimates of share price returns to shareholder proposals, Cuñat, Gine, and Guadalupe (2012).

Our estimates reveal that in the US data the most frequent equilibrium is one in which the supporters of the majority turn out partially (i.e., they play a mixed strategy), while those of the minority turn out fully. Intuitively, supporters of the majority free-ride on each other and especially their supporters among the regular voters, while supporters of the minority have a small chance to overturn the result to their favour and participate over-proportionally. Essentially, they turn out to lose with a tiny probability of winning. Their presence leads to the discrepancy of 21% (in absolute terms) between actual voting support and the hypothetical support under full participation and overturns the decision in 3.7% of the observations. To put the 21% difference between the full participation benchmark and actual support in perspective, it is useful to compare the actual support to the other 'extreme' benchmark, the voting support for the majority preference under no discretionary participation (i.e., only regular voters). From the no-discretionary participation benchmark, the observed support differs by 31%. In summary, the observed support for the majority's favourite outcome is 21% away from its popularity within the entire population and 31% away from its popularity within the regular voters.

We then use the US estimates to compute counterfactuals. The counterfactuals not only speak to current policy debates, but also allow us to draw the shape of selection effects as a function of different parameters. One parameter that is especially interesting to policy makers is the cost of voting. While the cost of voting is low in the US, the EU aims to reduce its cost of voting towards the US level, most notably with its recent roll-out of the Shareholder Rights Directive. We illustrate how equilibrium outcomes descend with increasing voting costs from the full participation benchmark to the only regular voters benchmark. We locate the full participation benchmark (where the population favourite option always wins) at a counterfactual voting cost of 1/4 of the US level. At 20 times the US level, voting costs reach a level that implies virtually full intergroup free-riding, with less than 1% of the proposals likely to have any discretionary participation in support for the majority preference of regular voters. After that point, the only meaningful equilibrium is the one with no discretionary participation for the majority preference of regular voters and partial participation against it (i.e., by the "underdog"). At around 30 times the US level, the population favourite option wins with probability 92%, reflecting the 7% difference in its popularity between

US regular and discretionary voters. Between the full participation level and the no free-riding level of voting costs, the probability of a minority win takes a reverse-U shape in the cost of voting, with a peak at 35% at a voting cost of 3 times the US level. Selection is strongest here because we have the highest occurrence of an equilibrium with partial participation (i.e., mixing) on both sides. In such 'mixed-mixed' equilibrium the average outcome is a tie and the ultimate outcome is decided with a coin-flip. As a consequence, there is a 50% chance that the favourite loses. We argue that the US is an exceptionally low cost of voting regime and hence (ceteris paribus) the intermediate benefit to cost ratio case is more likely to occur in the rest of the world.

We document large variations in our estimates of selection effects. The probability of underdog wins is highest for governance related shareholder proposals, and in general larger for shareholder proposals than management proposals. The benefit to cost ratio is highest for management proposals on takeover defense; smallest for board and governance related shareholder proposals.

We report a range of robustness checks. Those in large verify the qualitative results of the main section. Among our alternative estimations are those for subsamples where our assumptions are more likely to hold. Most importantly, our model focuses on the participation decision and thus does not consider how voters arrive at their partisan preferences nor their knowledge about other voters' preferences. This situation is more realistic later in the voting season, after investors have observed many votes on similar ballots.

To maximize the transparency of the structural estimation, we deliberately keep the model as simple as possible. Our estimation algorithm performs well compared to reduced form despite only using the moments of two variables as input. However, when interpreting our estimates, it is important to be aware of the assumptions and limitations of the model. We discuss these in detail in Section 7.1.

Related Literature. Our theoretical work builds on the extensive literature on participation in political elections. In this literature, we follow the stream on pivotal voter models that describes the participation decision as a function of the costs and benefits of voting and the probability that rational voters will be pivotal. An early contribution is Downs (1957), where the focus is on explaining the observed participation rates in political elections. These rates are high compared to model predictions when voting is costly, the so-called voters' paradox. Palfrey and Rosenthal

(1983) uses a game theoretic approach for this class of models (see Feddersen (2004), Geys (2006) for overviews of the literature). Our own work is closer to recent pivotal voter models, which introduce aggregate uncertainty, Krishna and Morgan (2012) and Evren (2012), and is even closer to Myatt (2015). We contribute to this literature in that we introduce heterogeneity in the ownership structure and hence illustrate a way to extend the vast literature on political elections to the corporate context.

A separate stream of the political voting participation literature focuses on other reasons to explain participation rates. In Feddersen and Sandroni (2006), voters are motivated to vote by ethical considerations, and in Feddersen and Pesendorfer (1996), some voters may abstain to allow more informed voters to assist in information aggregation.

The current theoretical literature on corporate voting focuses on how dispersed private information is aggregated in a framework with a common value and zero cost of voting. In this context, Maug and Yilmaz (2002) consider how two-class voting can resolve conflicts of interest; Maug and Rydqvist (2009) study the effect of majority rules on strategic voting; Bond and Eraslan (2010) endogenize the proposal; and Levit and Malenko (2011) study the advisory role of voting on non-binding proposals. The aforementioned papers do not allow for abstention and assume homogeneity in ownership (similar to the seminal work of Feddersen and Pesendorfer (1997)). In a contemporaneous paper, Bar-Isaac and Shapiro (2017) consider blockholders and dispersed shareholders and show that the former may not vote all their shares to assist in information revelation in the voting process. We are the first to examine a cost to benefit analysis of corporate voting, with a pivotal voter model in which voting is costly, ownership is heterogeneous, and there is disagreement among shareholders.

Voting is part of the voice mechanism in corporate governance. The theoretical literature has focused on another important channel: the sale of shares in the open market, that is, the exit mechanism, and its interplay with voice.[3] Specifically, Admati and Pfleiderer (2009) and Edmans (2009) offer the first treatments of the 'Wall Street Walk' phenomenon; Edmans and Manso (2011) study exit with multiple blockholders; and Dasgupta and Piacentino (2015) examine the effect of career concerns on exit.[4]

[3] In early years, the interplay between exit and voice was highlighted by Hirschman (1970) and first studied theoretically by Kahn and Winton (1998) and Maug (1998).

[4] See Edmans and Holderness (2017) for a comprehensive overview.

The empirical literature on voter participation in political elections documents how participation varies with institutional differences [e.g., compulsory voting (Blais (2006))] and voter characteristics [e.g., age (Wolfinger and Rosenstone (1980) and Blais (2000)), altruism (Blais (2000)), and education (Abramson, Aldrich, Paolino, and Rohde (1992))] and candidates [e.g., viability (Abramson, Aldrich, Paolino, and Rohde (1992))]. We document how voting participation in corporate contests varies with proposal and sponsor types.

The only empirical paper on corporate voting participation is by Van der Elst (2011), who empirically shows that block ownership significantly affects corporate voting participation. We build on this result and contribute to the empirical literature, first, by theoretically showing how ownership matters and second, by isolating the effects of ownership and describing how to back out importance-related information from participation rates. We also document the role of participation in voting outcomes.

We generally contribute to the empirical literature on voting. This literature shows that voting outcomes affect the decision making of firms (Thomas and Cotter (2007), Guercio, Seery, and Woidtke (2008), Cai, Garnier, and Walkling (2009), Levit and Malenko (2011), Becht, Polo, and Rossi (2016)). Much of the literature has focused on the voting patterns of US investment funds, which since 2003 have had a fiduciary duty to vote and have to disclose their votes (Brickley, Lease, and Smith Jr. (1994), Matvos and Ostrovsky (2010), Iliev and Lowry (2014), Malenko and Shen (2016), Cvijanović, Dasgupta, and Zachariadis (2016), Appel, Gormley, and Keim (2016), and Schwartz-Ziv and Wermers (2016)). For example, Iliev and Lowry (2014) empirically analyze mutual funds' voting behaviour and posit that these funds base their voting strategies on the costs and benefits of "active" voting, while Matvos and Ostrovsky (2010) highlight systematic differences in the voting behavior of more management-friendly funds. Bach and Metzger (2015) indicate that an abnormal share of shareholder proposals were instituted by management, with a very small difference in the results for 2003 and 2016. We contribute to this literature by showing how the differences in preferences between blockholders, institutional investors, and other investors can affect the participation decision.

Another stream of the literature aims to estimate the importance of voting. This literature has used the stock market reaction to the passing of proposals (Cuñat, Gine, and Guadalupe (2012) and Bach and Metzger (2015)); the market for votes in the US equity loan market (Christoffersen, Geczy, Musto, and Reed (2007) and Aggarwal, Saffi, and Sturgess (2015)); and the discrepancy between stock and option prices around voting contests (Kalay, Karakaş, and Pant (2014)). We develop another method to estimate the importance of proposals that does not rely on a discontinuity analysis of only close elections but rather uses all proposals.

2 Empirical Regularities

We begin our analysis with some simple facts about voting participation in US firms.

2.1 Data

We use aggregate voting data from the ISS Voting Results database. The data provide for Russell 3000 firms from 2003-2013: the *voting direction* for all proposals (total votes for, votes against, and empty votes casts, or abstentions), the *voting outcome* (Pass/Fail), the appropriate base for calculating the voting outcome (for plus against, for plus against plus abstain, or outstanding), the majority rule (simple or super-majority), and the recommendation of management and the ISS. As in the previous literature (e.g., Cvijanović, Dasgupta, and Zachariadis (2016)), we exclude director elections, as many of these are under plurality voting standards where abstentions have a different interpretation than a "No" vote (Matvos and Ostrovsky (2010), Cai, Garner, and Walkling (2013)); ratification proposals, as they are routine and noncontroversial (Bethel and Gillan (2002)); and say-on-pay frequency proposals, where the outcome is not binary. Finally, we exclude the very few (approximately 2%) super-majority voting contests since our model only addresses simple majority elections.

We combine the aggregate voting results with the ISS Mutual Fund Voting database, which provides the number of votes per voting direction (for, against, and abstentions) of individual investment funds for each proposal. The source for this database is the mandatory N-PX filings that funds have to report. We aggregate fund level voting information at the corresponding fund-family level.

We obtain data on institutional ownership from the quarterly 13F filings, which are collected by Thomson Reuters. Institutions that report 13F filings include investment funds, which also disclose their votes on the N-PX forms, as well as hedge funds and other asset managers. We complement this data with the fraction of shares owned and the type of owner (institutional or private), which we hand collect from the proxy statements. In the proxy filing, which contains the voting invitation, the firms must report the ownership of blocks greater than 5%.

2.2 Summary Statistics

Table 2 presents the summary statistics for the data sample used in the paper. Panel A provides the number of observations per year. Our sample includes 8,568 meetings with 18,520 nonstandard proposals. There are on average two such proposals per meeting. Both the number of meetings and proposals per meeting increase over time, with 1.7 proposals per meeting in 2003 and 2.5 proposals per meeting in 2011. Panel B presents the characteristics of the firms in our sample. Our firms are comparable to the samples used by other papers on shareholder meetings (e.g., Cvijanović, Dasgupta, and Zachariadis (2016)), with an average book asset size of $18 billion, leverage of 23%, and a market to book ratio of 1.9.

Panel C presents the summary statistics for share ownership at the meeting level. At the time of the meeting, there are on average 272 million shares outstanding, of which on average 68% are owned by institutional investors. Among these, 20% report their votes (N-PX shares). The blocks over 5% reported in the proxy statement account for 25% of the shares owned. Most of these blocks belong to institutional shareholders, in total, amounting to 23% of the shares. Private shareholders that own blocks over 5% account for only 2% of all shares. Finally, the directors own on average 1.6% of the shares.

[Insert Table 2 about here]

2.3 Regular and discretionary voting

In the US, certain shareholders must vote their shares, while others can choose whether to vote or not. In particular, investment funds have a fiduciary duty to vote on behalf of their clients (SEC Final Rule IA-2106). This duty is enforced for mutual funds and other registered investment management companies, which are required to disclose their votes on the N-PX forms. As Table 2, Panel C shows, these shareholders hold a significant fraction, 20%, of shares but not typically the majority. Examples for other shareholders that do have discretion over voting participation are Hedge Funds, Private Wealth Managers and Family Offices.

A similar duty to vote on behalf of assets managed is going to become part of EU regulations with the 2019 Shareholder Rights Directive. However, there are other examples for regular voters. One example are family voting trusts that combine the voting power of family members in family firms. Similarly, shareholders can delegate voting to independent voting trusts that always vote (these exist, for example, in the UK and Netherlands).

To represent the participation decision accurately, we calculate discretionary participation rates excluding the N-PX shareholders' votes. To that end, we calculate the fraction owned by the regular voters (henceforth γ) as the fraction of N-PX voters from the 13F filings. To calculate the number of votes by discretionary voters, we subtract the votes of regular (N-PX) voters from the aggregates in each category (for, against, and abstentions). These "NonN-PX" votes can come from other institutional investors (such as hedge funds and pension funds) as well as individuals (such as insiders, directors, and dispersed shareholders). We then calculate discretionary participation as the number of these NonN-PX votes out of the total number of "NonN-PX" shares. Total participation is calculated as all votes cast as a percentage of the shares outstanding or the sum of the discretionary and regular voting participation (by definition 100%). The shareholders can also formally cast abstention votes. The number of abstentions is very small: 1.6% of all votes and 2.4% of all N-PX votes. In our results going forward, we include the official abstention votes to obtain participation rates. Due to the low incidence of such votes, our results are not qualitatively changed when we deduct these votes.

2.4 Voting Participation: Stylized Facts

To set the stage for our empirical analysis, in Table 3, we show the basic summary statistics for voting support (as a fraction of the valid base) and participation. The base can be either the number of shares outstanding or the number of shares that voted and depends on state laws and the company charters (Bach and Metzger (2015)).

[Insert Table 3 about here]

Panel A shows that voting participation is non-trivial but also not full on average. Total participation averages 77% of shares outstanding, and discretionary participation 73%. These percentages are substantial compared to the participation in political elections, such as the 55%

participation in the 2016 US presidential election.

Panel B reports the voting direction and participation by type of proposal (see Appendix B for details on the definition of each type). The proposals that receive the most support are related to mergers (73%) and payouts (70%). CSR proposals receive the least support (10%). These averages already reveal that participation is not always related to the level of support: discretionary voting participation is at its lowest for mergers, at 70%, but at its highest for payout proposals, at 76%.

Panel C shows the voting direction and participation by the type of sponsor. Management proposals receive the highest support rates (60%), and shareholders are most likely to follow the ISS's recommendation with these proposals (76%). This result highlights that shareholders self-select themselves into firms and thus on average support management.

Among the shareholder proposals, support is highest for coalitions (30%) and proxy advisors (37%) and lowest for corporations (9%). In contrast, proposals made by social groups receive the highest discretionary participation rates (75%). The lowest discretionary participation rates are made by employee proposals (47%). The shareholders are most likely to follow the ISS's recommendations for proposals made by social groups (71%) and least likely to do so for proposals made by coalitions (50%), unions (51%), and, not surprisingly, proxy advisors (52%).

Our results highlight that participation rates contain information in addition to support rates: shareholders seem to consider proposals made by social groups important enough to vote for (or against) even if they are unlikely to succeed.

3 Model

In this section, we present a rational-choice model of voting participation. This model is an extension of the one proposed in Myatt (2015), where the important variation is that we allow for heterogeneity between shareholders in their voting discretion. We solve the model with the intent to unravel new effects that are unique to corporate voting and link these effects to certain parameters, which we then estimate using US voting data.

Setup. Consider a corporate proposal, where shareholders choose between two options R and L. The firm has $n+1$ voting shares. A fraction $\gamma \in [0, 1)$ of the n shares belongs to regular voters, while a fraction $1-\gamma$ of n belongs to discretionary voters with a single voting share and so a single vote each, with the last voting share belonging also to a discretionary voter. Hence, for $\gamma = 0$, we are in the model of Myatt (2015).[5]

The $\gamma \in [0, 1)$ regular voters always vote. Their voting preference is captured by a constant $q \in (1/2, 1)$, which is the fraction of this group who vote for R. Thus, regular voters can be thought of either as i) two subgroups (blockholders) with sizes q and $1 - q$ supporting R and L, respectively, or alternatively as ii) coalitions of dispersed shareholders who always participate and vote in proportion q, $1 - q$ for R and L, respectively. The choice of R as the most popular option (i.e., the favorite) among regular voters (i.e., $q > 1/2$) is without a loss of generality.

Among discretionary voters, option R has ex ante popularity $p \in (0, 1)$. The crux of the model is that p is unknown (in contrast to q), distributed according to density f in $(0, 1)$, with a mean of $\overline{p}$. Discretionary voters will vote with a probability of a, which is distributed according to density g in $(0, 1]$ and has a mean of $\overline{a}$ (i.e., a is an 'availability' shock); p and a are independent random variables, while q is known. The shareholders are partisan: they vote according to their preferences (types) regardless of the others' voting preferences and participation. Hence, if all shareholders were available and always voted, then the expected votes for R would be $q\gamma + \overline{p}(1 - \gamma)$.

Discretionary voters decide to vote or not based on an instrumental benefit $v > 0$, which they receive only if their preferred option wins, and a cost $c > 0$, which they face when they vote, regardless of the outcome. We assume that R and L supporters have the same v and c, respectively, and are risk-neutral. Hence, the discretionary voters are homogeneous except for their voting preferences. This assumption primarily assists our identification of several parameters in the data. The benefit v can be thought of as the subjectively perceived payoff in \$/shares accruing to a discretionary voter when her preferred option wins. The benefit is a combination of any (forecasted) short-run stock market reaction and any (unpriced) long-run benefit (including portfolio concerns and "altruistic" motives). The opportunity cost c captures the time and effort spent to cast a vote. Once (confidential) voting is done, the outcome is decided by the simple majority of the votes cast and in case of a tie, a fair coin toss is the tie-breaker. All the information above is common knowledge.

The only choice variable (strategy) is whether a discretionary voter votes. Hence, the model

[5] The number of voting shares n can be thought of as the market capitalization of the firm divided by the average holdings in that firm; for example, in a firm with \$10M market capitalization and \$10K average holding, we have $n = 1000$. For comparison, the average number of non-NPX institutions in our sample is 532 (see Table 4).

is silent on how shareholders choose how to vote, but considers whether she chooses to voice her opinion by voting or not. We look for symmetric strategies across types R or L of discretionary voters and the solution is determined by the Bayesian Nash Equilibrium.

The main difference between our model and previous models of corporate elections [e.g., Maug and Rydqvist (2009), Levit and Malenko (2011), and Bar-Isaac and Shapiro (2017)] is that we assume that voters are partisan, and although there is aggregate uncertainty, there is no dispersed private information. The partisan assumption can be microfounded in one of the following ways: i) investors have common values (i.e., care about the stock price) but "agree to disagree" (i.e., have heterogeneous priors and do not learn), as in asset pricing models (e.g., Harrison and Kreps (1978)); ii) investors have private values. For example, investors may have different incentives [e.g., flow motives, cf. Chevalier and Ellison (1999)], conflicts of interest [e.g. portfolio allocation, cf. Cohen and Schmidt (2009)], or different ideologies [cf. Bolton, Li, Ravina, and Rosenthal (2018)]; and iii) investors may have common values but are of extreme types and/or receive 'extreme' information (so they do not learn from the voting contest), cf. Feddersen and Pesendorfer (1997) and Yilmaz (2000). In reality, shareholders probably have both common and private values. However, to the best of our knowledge, there is no canonical model of voter turnout (in either political economics or financial economics) that incorporates both values, as well as costly participation.[6]

Our main goal is to have a tractable model for which voting participation is endogenous and the parameters, which capture shareholder preferences, can be estimated using the data. To this effect, the partisan assumption implies that our model's predictions are stronger when disagreement is more likely to occur and/or private-information-driven voting is less prominent. A detailed discussion of all the assumptions of the model and how we deal with them in the data appears in Section 7.1. We now proceed to the solution of the model.

Primitives. For discretionary participation to be possible, we rule out the case where either type of regular voter can decide the outcome unilaterally. Since $q > 1/2$, we need only assume the following:

A1: $\gamma < 1/(2q)$.

[6]Models that incorporate private and common values usually make some simplifying assumptions; most notably Krishna and Morgan (2011) assume that ideology (private) trumps competence (common).

Consider a focal discretionary voter (shareholder) of type $i \in \{R, L\}$. Let b_R and b_L be the votes of nonfocal discretionary voters for each option. Then, the total votes for R are $b_R + q\gamma n$, and for L, they are $b_L + (1-q)\gamma n$. The focal shareholder is pivotal if either i) her type is losing by one vote, she pushes the score to a tie and the coin toss is favorable (with a probability of 1/2), or ii) if there is already a tie, the coin toss is against her type and with her vote, she gives a clear majority to her type; that is,

$$\Pr[\text{Pivotal}|R] = \frac{\Pr[b_R + q\gamma n = b_L + (1-q)\gamma n] + \Pr[b_R + q\gamma n - 1 = b_L + (1-q)\gamma n]}{2},$$

$$\Pr[\text{Pivotal}|L] = \frac{\Pr[b_R + q\gamma n = b_L + (1-q)\gamma n] + \Pr[b_R + q\gamma n + 1 = b_L + (1-q)\gamma n]}{2}.$$

The shareholder votes if $v \Pr[\text{Pivotal}|i] > c$ or $\Pr[\text{Pivotal}|i] > c/v$ and does not vote otherwise, for $i \in \{R, L\}$. Hence, for any participation to be possible, we also assume that the cost should not be higher than the benefit:[7]

A2: $v \geq c$.

Now, if

$$\Pr[\text{Pivotal}|i] = \frac{c}{v}, \tag{1}$$

for either type $i \in \{L, R\}$, then that type is indifferent between voting or not, which follows a mixed strategy, and we have partial participation for i.

Large Elections. As Myatt (2015) notes, the pivotal probabilities are cumbersome to calculate unless n is large. Let t_R and t_L denote discretionary voter participation rates, depending on the shareholders' type. Below, we present the pivotal probabilities as approximated for large elections and the case where a is equal to $\overline{a}$ (i.e., g is degenerate). The proof appears in Appendix A.

Lemma 1. *Assuming $g(a) = \delta(a - \overline{a})$, that is, the Dirac function, and A1, then the pivotal*

[7] We can strengthen this requirement to $v \geq 2c$ as a "benevolent dictator" would enforce but this would not change significantly our subsequent calculations.

probabilities for R and L in large elections are approximately:

$$\Pr[\text{Pivotal}|R] \approx \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}(t_R+t_L)}f\left(p^*\right)p^*, \tag{2}$$

$$\Pr[\text{Pivotal}|L] \approx \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})(t_R+t_L)}f\left(p^*\right)(1-p^*), \tag{3}$$

where

$$p^* \equiv \frac{t_L}{t_R+t_L} - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}(t_R+t_L)}. \tag{4}$$

The value p^* is the average probability of support for R among the discretionary voters, for which the total average support for R and L are equal; that is,

$$\overline{a}(1-\gamma)p^*t_R + \gamma q = \overline{a}(1-\gamma)(1-p^*)t_L + \gamma(1-q). \tag{5}$$

Although (2) and (3) are approximations, we use them as equalities in what follows. In that sense, we are looking at *approximate equilibria*, as defined in Myatt (2015, p. 10). Note that since p^* is a probability, it should be in $(0,1)$, and hence, we can see from (4) that t_L cannot be zero. Hence:

Corollary 1. *There is no equilibrium where discretionary voters of type L do not vote; that is, $t_L \neq 0$.*

Hence, ruling out trivial equilibria (with $t_L = 0$ where R wins), there are 6 possible equilibria to compute $t_L \in \{(0,1),1\}, t_R \in \{0,(0,1),1\}$. To illustrate, we go over the steps of deriving the equilibrium with incomplete participation by both types. We then present all possible equilibria, with the detailed derivations provided in the Internet Appendix.

Equilibrium with Incomplete Participation. Given the expressions for the pivotal probabilities, we now seek to determine whether an equilibrium exists with incomplete participation for both L and R [i.e., t_L, $t_R \in (0,1)$]. From (1), we know that since the cost to benefit ratio is the same for both types, the pivotal probabilities should also be the same for both types. Hence, using

(2) and (3), in equilibrium, we must have:

$$p^* \;=\; \overline{p}, \tag{6}$$

which given (5) means that at equilibrium, the total average supports for L and R are equalized. Hence, the expected outcome is a tie. In other words, the advantage of the favorite of the whole population of voters is overcome by higher participation rates of L voters. From (6) the pivotal probabilities in equilibrium are:

$$\Pr[\text{Pivotal}|R] = \Pr[\text{Pivotal}|L] \;=\; \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(t_R+t_L)}f\left(p^*\right).$$

Moreover, the pivotal probability for type R is equal to her cost to benefit ratio (1) in the equilibrium with incomplete participation; hence,

$$t_R + t_L \;=\; \frac{1}{(1-\gamma)n\overline{a}}f(\overline{p})\frac{v}{c}. \tag{7}$$

Furthermore, according to the definition of p^* (4) and the fact that it is equal to $\overline{p}$ (6), after some simple algebra, we have

$$t_L \;=\; (t_R+t_L)\overline{p} + \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}}. \tag{8}$$

Using (7) with (8), we derive the equilibrium t_L and t_R,

$$t_L \;=\; \frac{1}{n}\frac{1}{\overline{a}}\frac{1}{(1-\gamma)}f(\overline{p})\overline{p}\frac{v}{c} + \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}}, \tag{9}$$

$$t_R \;=\; \frac{1}{n}\frac{1}{\overline{a}}\frac{1}{(1-\gamma)}f(\overline{p})(1-\overline{p})\frac{v}{c} - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}}, \text{ and} \tag{10}$$

$$\overline{t} \;=\; \frac{2\overline{p}(1-\overline{p})}{n(1-\gamma)}\frac{v}{c}f(\overline{p}) + \frac{(2q-1)\gamma}{1-\gamma}(1-2\overline{p}), \tag{11}$$

$$\overline{t}_{\text{total}} \;=\; \frac{2\overline{p}(1-\overline{p})}{n}\frac{v}{c}f(\overline{p}) + 2\gamma\left(\overline{p}(1-q)+q(1-\overline{p})\right), \tag{12}$$

where in the third and fourth line, we used the definitions of average participation for discretionary voters and average total participation (i.e., including regular voters):

$$\begin{aligned}\bar{t} &\equiv \bar{a}\left(\bar{p}t_R + (1-\bar{p})t_L\right), \\ \bar{t}_{\text{total}} &\equiv (1-\gamma)\bar{t} + \gamma.\end{aligned}$$

Incomplete participation means that $(t_L, t_R) \in (0,1)$. These restrictions lead to a set of necessary and sufficient conditions in terms of the parameters of the model, in particular n, γ and v/c or equivalently, $v/(cn)$. The full set of parameter regions that imply and are implied by incomplete participation are presented in Proposition 2 in Appendix A. Below, we present the part of the result for large n, which is the case where our approximations work well and is also empirically relevant.

Proposition 1 (Equilibrium mm)**.** *Assume that* $q \in (1/2, 1)$, $\bar{p} \in (0,1)$, $f(\bar{p}) > 0$, $\bar{a} \in (0,1]$, $g(a) = \delta(a-\bar{a})$. *If*

$$n \in N_{mm} \equiv \left(\frac{f(\bar{p})\left(\bar{a}(1-\bar{p}) + 2q - 1\right)}{\bar{a}(2q-1)}, \infty\right), \tag{13}$$

$$\gamma \in \Gamma_{mm} \equiv \left(\frac{f(\bar{p})(1-\bar{p})}{n(2q-1)}, \frac{\bar{a}(1-\bar{p})}{\bar{a}(1-\bar{p}) + 2q - 1}\right), \tag{14}$$

$$\frac{v}{cn} \in V_{mm} \equiv \left(\frac{\gamma(2q-1)}{f(\bar{p})(1-\bar{p})}, \min\left\{\frac{\left(\bar{a} - \gamma\left(\bar{a} - 1 + 2q\right)\right)}{f(\bar{p})\bar{p}}, \frac{\left(\bar{a} - \gamma\left(\bar{a} + 1 - 2q\right)\right)}{f(\bar{p})(1-\bar{p})}\right\}\right). \tag{15}$$

Then, there exists an incomplete participation equilibrium for both types; that is, t_L, $t_R \in (0,1)$, *which is given by equations (9) and (10). Furthermore, the average participation of discretionary voters* $\bar{t}$ *and the average total participation* $\bar{t}_{total}$ *are given by (11) and (12), respectively. Finally, in such equilibrium, the probability that either L of R is pivotal is equal to the common cost to benefit ratio* c/v *(1), and the total expected votes for L and R are equal (5); that is, the expected outcome is a tie.*

Our approximations work well for large n; therefore, we assume that there are many voters and that the restriction imposed by $n \in M_{mm}$ is innocuous. Then, the incomplete participation equilibrium exists for a set of points (region) in the two-dimensional space $(\gamma, v/(cn))$; that is, the space of a fraction of regular voters (henceforth, the regular block size) and the benefit to cost ratio per voter. To differentiate across the equilibria in Section 3.2, we label them by the corresponding strategy of type L and R, respectively. Hence, for the incomplete participation equilibrium, the label is mm, signifying a *mixed* strategy for both types of discretionary voters. Conditional on a large n, the infimum of Γ_{mm} is essentially zero. Hence, we cover all plausible scenarios in the data. The interval Γ_{mm} depends on parameters q, $\overline{p}$, $f(\overline{p})$, and $\overline{a}$. The interval V_{mm} depends on all the above plus γ. Geometrically, this means that the region $\Gamma_{mm} \times V_{mm}$ is not rectangular. The region is depicted in Figure 1 below.

In Proposition 1, the lower bound of γ guarantees that v/c is higher than one (see assumption A1), and hence, some participation is possible. The upper bound of γ guarantees that the lower bound of $v/(cn)$ does not exceed the upper bound, and hence, an equilibrium with incomplete participation exists. A large n guarantees that the lower bound of γ does not exceed the upper bound. The lower bound of $v/(cn)$ guarantees that the participation of the agreeing shareholders t_R is positive. The upper bound of $v/(cn)$ guarantees that neither of the types participates fully. In summary, for $(\gamma, v/(cn)) \in \Gamma_{mm} \times V_{mm}$, participation is strictly between zero and one for both types.

3.1 Comparative Statics

The main selection effects —the underdog and free-riding effects— are visible in the formulas for the rates (9)–(12). All rates are the sum of two terms: an intragroup one, also present in Myatt (2015) (i.e., the instance of our model where $\gamma = 0$), capturing the interactions among discretionary voters and an intergroup term, unique to our setup, capturing interactions between regular and discretionary voters, which is a key feature of corporate voting.

We focus on the intergroup terms first. Note that L is the least populous option, that is, the underdog, among regular voters given our innocuous assumption that $q > 1/2$. Then, the stronger the favoritism is for R among the regular voters, the more the supporters of the underdog L among discretionary voters will participate [i.e., $\partial t_L/\partial q > 0$ in (9)]. This process reflects the *intergroup* (in contrast to intragroup) underdog effect. In turn, the stronger the favoritism is for R among regular voters, the less the supporters of that favorite R among the discretionary voters will participate [i.e., $\partial t_R/\partial q < 0$ in (10)], that is, the (intergroup) free-rider effect. Both of these effects are combined in discretionary participation and in total participation. Which effect dominates depends on which one is the underdog/favorite among the discretionary voters. If the discretionary voters also on average prefer R, there is *agreement* between the discretionary and regular voters. Then, discretionary and total participation decrease as the free-riding effect dominates the underdog effect [i.e., $\partial\bar{t}, \bar{t}_{\text{total}}/\partial q < 0$ in (11) and (12) if $\overline{p} > 1/2$]. In contrast, discretionary and total participation increase if there is *disagreement* and the underdog effect dominates [i.e., $\partial\bar{t}, \bar{t}_{\text{total}}/\partial q > 0$ in (11) and (12) if $\overline{p} < 1/2$].

Now, we turn to the intragroup terms (present also in the incomplete participation equilibrium of Myatt (2015, Proposition 1)). More average $\overline{p}$ (support for R among the discretionary voters) increases t_L and decreases t_R; this is the intergroup underdog effect, which is standard in political elections. Moreover, $\bar{t}$ and $\bar{t}_{\text{total}}$ increase with the expected contestedness of the vote among discretionary voters; that is, $-|\overline{p} - 1/2|$, capturing the fact that (ignoring regular voters) close elections command more participation. All rates decrease with the size of the electorate n since in a larger pool of shareholders, each voter has a smaller probability of being pivotal; increase with the concentration of the ex ante beliefs arounds the mean $f(\overline{p})$ because shareholders are more likely to vote if they are more certain about each other's preferences; and increase with the benefit to cost ratio, which represents the 'importance', of election v/c. As expected, the average availability shock $\overline{a}$ decreases both t_L and t_R and does not affect $\bar{t}$ and $\bar{t}_{\text{total}}$. Finally, the first parts of all rates increase with γ, capturing the fact that more regular voters translates to fewer discretionary voters.

The parameters of our model also affect the region $\Gamma_{mm} \times V_{mm}$ of the incomplete participation equilibrium. For example, for a large regular block size γ (or support for R among regular voters q), the length of the interval of permissible values for the benefit to cost per voter diminishes. Intuitively, for high γ, high values of $v/(cn)$ (which were permissible for lower values of γ) lead to full participation for voters of L, while low values of $v/(cn)$ (which were permissible for lower values of γ) will lead to no participation for voters of R. Geometrically, this result means that the region $\Gamma_{mm} \times V_{mm}$ will become narrower as γ (or q) increases. This result is depicted in Figure 1 below.

[Insert Figure 1 about here]

3.2 Equilibria

Table 1 reports for all possible equilibria the pair (t_L, t_R) (in columns 2 and 3, respectively) and the corresponding regions in the space $(\gamma, v/(cn))$ (in columns 4 and 5, respectively), referring the

Table 1: List of all possible equilibria.

Eqm.	Prop.	t_L	t_R	$\gamma \in$	$v/(cn) \in$	Avg. outcome
mm	1	$\in (0,1)$, (9)	$\in (0,1)$, (10)	Γ_{mm}, (14)	V_{mm}, (15)	Tie
$1m$	3	=1	$\in (0,1)$, (IA.4)	Γ_{1m}, (IA.2)	V_{1m}, (IA.3)	Tie/Right
10	4	=1	=0	Γ_{10}, (IA.6)	V_{10}, (IA.7)	Right
11	5	=1	=1	Γ_{11}, (IA.9)	V_{11}, (IA.10)	Left/Tie/Right
$m1$	6	$\in (0,1)$, (IA.14)	=1	Γ_{m1}, (IA.12)	V_{m1}, (IA.13)	Left
$m0$	7	$\in (0,1)$, (IA.18)	=0	Γ_{m0}, (IA.16)	V_{m0}, (IA.17)	Right
00	8	=0	=0	Γ_{00}, (IA.19)	V_{00}, (IA.20)	Right

reader to the explicit formulas in the relevant propositions and equations in the Internet Appendix. The name of the equilibrium (column 1) denotes the participation by L and R, where m stands for mixed, 1 for full, and 0 for no discretionary participation.[8] For equilibria where only one side uses a mixed strategy (i.e., $1m$, $m1$, and $m0$), we need an additional assumption regarding the distribution of p to obtain closed form expressions for the participation rate of the type that uses the mixed strategy. In particular, we assume that $p \sim \mathcal{U}[l,h]$, where $(l,h) \subseteq (0,1)$.[9]

Assuming that $g(a) = \delta(a - \overline{a})$ and majority is decided among voting participants (as we have done throughout), we define the voting outcome with discretionary participation as:

$$O_{\text{disc}}(p) \equiv \overbrace{\gamma q + \overline{a}(1-\gamma)pt_R}^{\text{total support for } R} - \overbrace{[\gamma(1-q) + \overline{a}(1-\gamma)(1-p)t_L]}^{\text{total support for } L} = \overbrace{\gamma(2q-1)}^{\text{regular voters}} + \overbrace{\overline{a}(1-\gamma)\left[t_R p - t_L(1-p)\right]}^{\text{discretionary voters}}.$$

$O_{\text{disc}}(p)$ is the difference between total support for R and L when the ex ante support for R among discretionary voters is p. If $O_{\text{disc}}(p) > 0$, then R wins; if $O_{\text{disc}}(p) = 0$, then there is a tie; and if $O_{\text{disc}}(p) < 0$, then L wins. From the definition of p^* (see 5), we know that $O_{\text{disc}}(p^*) = 0$. Now, if we compute $O_{\text{disc}}(\overline{p})$, we can infer the average outcome of a voting contest. In particular, we know that for all parameters $\{q, l, h, \overline{a}, n, \gamma, v/(cn)\}$, where equilibrium mm exists, we have that $p^* = \overline{p}$ and so the outcome in this equilibrium, as we mentioned, is on average a tie. However, this is not true for the other equilibria listed in Table 1. For example, for equilibrium $1m$, there are parameter values in the permissible for this equilibrium region where we have a tie on average and others where R wins (on average). Hence, in Table 1, for each equilibrium in the list, we mention the possible average outcomes (column 7, with the added assumption that $\overline{a} = 1$).

[8]For all equilibria, we posit that n is large enough so that the equilibrium requirements are met, which is sensible since our approximations work well for large n, and empirically, the number of voters is rarely small.

[9]This assumption implies that $\overline{p} = (h+l)/2$ and $f(p) = 1/(h-l)$ for all $p \in [l,h]$, and zero otherwise.

A few notes about these equilibria. First, the $\Gamma \times V$ regions are non-overlapping; hence, given the parameter values, the equilibrium prediction is unique (if an equilibrium exists at all for those parameters). Second, outside of the parameter regions of the equilibria in Table 1, there are no equilibria where the two types use symmetric strategies (pure or mixed). Third, aggregate uncertainty regarding p is essential for sustaining equilibria with some participation (i.e., given our assumption regarding f, it is necessary that $h > l$; otherwise, the regions $\Gamma \times V$ are empty). The intuition is that if voters were certain of the ex ante preferences, then their perceived probability of being pivotal would almost always be zero, and hence, their incentives to participate are diminished. Fourth, in Myatt (2015, Proposition 2), i.e., for $\gamma = 0$, we have only equilibria mm, $1m$ (or $m1$ depending on the assumption regarding who is the underdog/favorite) and 11. Hence, the inclusion of regular voters (i.e., $\gamma > 0$) not only enhances the space where certain equilibria exist but also results in a richer set of strategies for the discretionary voters (e.g., equilibrium $m0$).

The existence of equilibria for large n and general $(\gamma,\, v/\,(cn))$ depends on the values of q, l, h, and $\overline{a}$ (see Propositions 1 and 3-8). We summarize these findings here: case i) all equilibria exist [for different regions of $(\gamma,\, v/\,(cn))$] if $\overline{p} < 1/2$, $l < 1/2$ and $h < (\overline{a} + 2q - 1)/(2\overline{a})$; case ii) equilibrium 11 does not exist (all others do) if $\overline{p} < 1/2$, $l < 1/2$ and $h > (\overline{a} + 2q - 1)/(2\overline{a})$; case iii) equilibrium $m1$ does not exist (all others do) if $1/2 < \overline{p} < 1/2 + (\overline{a} + 2q - 1)/(2\overline{a})$, $l < 1/2$ and $h < (\overline{a} + 2q - 1)/(2\overline{a})$; and case iv) Equilibria 11 and $m1$ do not exist (all others do) if $\overline{p} > 1/2$, and either $l > 1/2$ or $\{l < 1/2$ and $h > (\overline{a} + 2q - 1)/(2\overline{a})\}$.

Each outcome can be consistent with several equilibria. Hence, observing the average outcome of a voting contest does not lead to a unique equilibrium prediction. For example, L is the (possible) average outcome under Equilibria 11 and $m1$. Note that equilibrium 11 ($t_L = t_R = 1$, Proposition 5) exists only when $l < 1/2$ and $h < (\overline{a} + 2q - 1)/(2\overline{a})$. Moreover, the region in which L wins (i.e., $O_{\text{disc}}(\overline{p}) < 0$) exists only when $\overline{p} = (h + l)/2 < 1/2$. In turn, equilibrium $m1$ ($t_L \in (0, 1)$, $t_R = 1$, and Proposition 6) exists only when $\overline{p} < 1/2$ [for $\overline{p} > 1/2$, the corresponding region is taken over by equilibrium $1m$ ($t_L = 1\, t_R \in (0, 1)$, Proposition 3), see Figure 1]. Similarly, the average outcome of R is consistent with equilibria $1m$, 10, 11, and $m0$, while the average outcome of T(ie) is consistent with equilibria mm, $1m$ and 11.

The comparative statics for the equilibria exhibit interesting patterns. In Figure 1 we depict the possible equilibria in the space $(\gamma, v/\,(cn))$ for two sets of parameters $\{q, l, h, \overline{a}\}$; the first corresponds to agreement between the regular and discretionary voters [case i) mentioned above], and the second leads to disagreement [case iv) mentioned above].[10] The maximum value of γ in both graphs is $1/2q$ (see Assumption 1). The upper value of $v/(cn)$ is given by the equilibria regions in agreement, while in disagreement, we truncate it to 100 (i.e., the blue region theoretically extends to infinity). As we noted above, the region of equilibrium mm is not orthogonal, and its 'width' reduces for higher γ (see (14)). Moreover, the region is larger when there is disagreement, capturing the greater likelihood of a tie in this case.

In equilibrium $1m$, the effects are given by t_R [see (IA.4)] since t_L is at a 'corner': there is a free-riding effect (i.e., $\partial t_R/\partial q < 0$), which survives in average discretionary participation $\bar{t}$ for any $\overline{p}$. However, note that when there is disagreement (i.e., $\overline{p} < 1/2$), then equilibrium $1m$ occupies a smaller region [i.e., Γ_{1m} in (IA.2) is smaller; see also Figure 1], which is consistent with the observation we made above that less agreement should result in less of the free-riding effect. Equilibrium $1m$ does not exist for $\gamma = 0$ when there is disagreement. The intuition is that for $\gamma = 0$ and (what we call) disagreement supporters of R are the (intergroup) underdogs (since there are no regular voters), and hence, there cannot be an equilibrium where the supporters of the underdog participate incompletely while those of the favorite (L in this case) participate fully.

In equilibria 10 and 11, the turnout rates for both types are in the 'corner', so any effect stems from the regions. As expected, equilibrium 10 (where the intergroup underdog participates fully and the intergroup favorite does not participate) is more prevalent (and for smaller values of γ) when we have agreement (see Proposition 4), otherwise some discretionary supporters of R would want to participate. The full participation equilibrium 11, as mentioned, is present when there is disagreement (for relatively high values of benefit to cost per voter ratio (see (IA.10) and Figure 1)) and when there is some agreement (so that $\overline{p}$ is not that much higher than $1/2$; see case iii) above and Proposition 5).

Now, in equilibria $m1$ and $m0$, all the action comes from t_L (see (IA.14) and (IA.18), respectively). In both equilibria, the underdog effect is present (i.e., $\partial t_L/\partial q > 0$). However, note that equilibrium $m1$ exists only when there is disagreement (see Proposition 6) because if there is agreement, the supporters of R do not have an incentive to turn out completely, while those of the obvious underdog do. Equilibrium $m0$, again, does not exist for $\gamma = 0$ or for (what we call)

[10]Note that the y axis is in logarithmic scale.

agreement or disagreement. In both cases, if there is participation by L, then certainly, R has an incentive to participate; otherwise, R will lose the voting contest (for zero or a fraction that is close to zero of regular voters).

In summary, underdog and free-riding effects are weakly present in all equilibria. That is also the case for the intragroup underdog effect, since across all equilibria, t_R (weakly) decreases in $\overline{p}$, while t_L increases. Finally, the rest of the effects manifest as well; that is, all rates (weakly) increase in the benefit to cost per voter ratio $v/(cn)$, decrease in number of voters n, and in dispersion $h-l$. However, note that although all these effects are monotonic (for a given equilibrium), they are not linear. Moreover, there are 'kinks' as we transition from one equilibrium to another. All the aforementioned observations guide our estimation process, which we discuss in the next section.

4 Estimation

In this section, we elaborate how we can use the model to estimate unobservable parameters in the data.

4.1 Identification

In ex post voting data (discussed in Section 2.1) for each proposal we observe: γ (i.e., the fraction of regular voters) and q (i.e., the fraction of regular voters in support of their favorite option R); $dSuL$, the discretionary support of type L voters among those who vote; and $dSuR$, the discretionary support of type R voters among those who vote.[11] In the data, we standardize as R the direction (for or against) that is most popular (on average) among the regular voters for a proposal type. Table 4 reports univariate statistics for these input variables.

We estimate the following unobserved parameters: $v/(cn)$ (i.e., the benefit to cost ratio per voter); $\overline{p}$ (i.e., the average fraction of discretionary voters in support of R); and std(p) (the standard deviation of the fraction of discretionary voters in support of R).[12] Assume throughout that the average availability is $\overline{a} = 1$.

The unit of estimation is a quantile of γ times quantile of n (proxied with the market capitalization) per proposal-type. For each bin in this triple-sort, we compute the average $\overline{\gamma}$; the average

[11]Note that $dSuL$ and $dSuR$ correspond to $t_L(1-p)$ and $t_R p$ in the model.
[12]Given our assumption on $p \sim \mathcal{U}[l,h]$ we have $\overline{p} = (h+l)/2$ and std$(p) = (h-l)/\sqrt{12}$.

preference for the favorite among regular voters $\overline{q}$;[13] and the averages $\overline{dSuR^2}$, $\overline{dSuL}$, $\overline{dSuR^2}$, $\overline{dSuL^2}$. The bins are necessary because the observed $dSuR$, $dSuL$ are in a firm×year×proposal-type dimension. To compute meaningful averages for a given proposal-type, we therefore have to 'fix' the firm×year parameters of the model: γ and n. Hence, our *identifying assumption* is that within each bin (i.e., a quantile of γ, quantile of n, and proposal-type) unobserved $\{v/(cn), \overline{p}, \text{std}(p)\}$ are constant and the averages $\overline{\gamma}, \overline{q}$ are representative. Therefore, we postulate that variation in (discretionary support for R) p across proposals is the (only) variation that allows us to identify the bin-specific parameters.

4.2 Algorithm

The algorithm performs an exhaustive search for *every* bin. We consider a dense grid of points in the permissible space of the unobserved parameters $\{v/(cn), \overline{p}, \text{std}(p)\}$: $v/(cn)$ is positive, while from the above, for $t_L, t_R \leq 1$, we have $\overline{p} \in \left[\overline{dSuR}, 1 - \overline{dSuL}\right]$ and $\text{std}(p) \in \left[\max\{\text{std}(dSuR), \text{std}(dSuL)\}, 1/\sqrt{12}\right]$, where $\text{std}(dSuR) \equiv \overline{dSuR^2} - \overline{dSuR}^2$ and $\text{std}(dSuL) \equiv \overline{dSuL^2} - \overline{dSuL}^2$.[14] Given a point in the grid, the algorithm performs the following (sub)steps for *each* possible equilibrium:

i) calculates the interval Γ and asks if $\overline{\gamma}$ belongs in it; if it does, then the calculations continue for that equilibrium. Otherwise, we proceed to the following equilibrium;

ii) if $\overline{\gamma} \in \Gamma$, then the algorithm calculates the interval V and asks if the $v/(cn)$ under consideration belongs in it; if it does, then calculations continue for that equilibrium. Otherwise we proceed to the following equilibrium;

iii) If $v/(cn) \in V$, then the algorithm calculates t_L and t_R and creates estimates for

$$\overline{dSuL}_{\text{est}} = t_L(1-\overline{p}), \qquad \overline{dSuR}_{\text{est}} = t_R\overline{p},$$
$$\overline{dSuL^2}_{\text{est}} = (t_L\text{std}(p))^2 + (t_L(1-\overline{p}))^2, \qquad \overline{dSuR^2}_{\text{est}} = (t_R\text{std}(p))^2 + (t_R\overline{p})^2.$$

[13] Our results stay similar if we use medians

[14] Note that $1/\sqrt{12}$ is the standard deviation of a uniform in $[0, 1]$.

iv) Finally, the algorithm calculates the error:

$$\begin{aligned}\text{Estimation Error} &= \left(\overline{dSuL}_{est} - \overline{dSuL}\right)^2 + \left(\overline{dSuR}_{est} - \overline{dSuR}\right)^2 \\ &+ \left(\overline{dSuL^2}_{est} - \overline{dSuL^2}\right)^2 + \left(\overline{dSuR^2}_{est} - \overline{dSuR^2}\right)^2.\end{aligned}$$

In the third and final step, the algorithm picks for every bin the point in the grid and the associated equilibrium that minimizes the above estimation error. Hence, since we take an identity weighting matrix for our errors we perform a one-stage GMM (see Hansen (1982), Hansen and Singleton (1982)), referred to as 'Baseline' henceforth.[15]

A few observations about the estimation. First, in terms of $v/(nc)$, the algorithm returns a point estimate only if in the equilibrium with the minimum estimation error, both rates are not 'corners' [i.e., there is a rate strictly in $(0, 1)$, which is true for equilibria mm, $m1$, $m0$, and $1m$]. In turn, for equilibria 11 and 10 we obtain a set estimate $V = \left[v/(nc)_{\text{lower}}, v/(nc)_{\text{upper}}\right]$ because in those equilibria all $v/(nc) \in V$ will lead to the same estimated values (as the particular value of $v/(cn)$ does not affect the rates t_L, t_R). Second, given the possibility for set estimates we essentially use four moments to estimate in principle four parameters, hence our system is exactly identified. Third, the uniqueness of the identified parameters should be noted. Recall that for fixed parameters $\{\gamma, q, v/(cn), \overline{p}, \text{std}(p)\}$, the model predicts a unique equilibrium. In addition, the algorithm picks the parameter values that minimize the estimation error using exhaustive search. Hence, we can be certain that no other parameter values in the grid and associated equilibrium would result in lower estimation error, given the data. Fourth, we perform, as mentioned, the estimation for each quantile of γ, each quantile of n, and each proposal-type. We face the following tradeoff choosing the size of each bin: more observations within a bin make our computed averages more accurate but also reduce the 'representativeness' of the computed $\overline{\gamma}$ and $\overline{q}$. We checked that this tradeoff does not affect our results qualitatively with robustness checks with respect to the bin size.

[15] As we know the one-stage estimates are consistent but not efficient. As part of robustness we also perform the two-stage (efficient) GMM estimation (referred to as 'GMM' henceforth) and results are qualitatively similar. However, as noted in Parker and Julliard (2005, bottom of p. 193) and references therein: "*...GMM with a pre-specified weighting matrix has superior small-sample* [as our bins are] *properties...*".

4.2.1 Delta Method

This subsection describes how we compute standard errors for the estimates. The t-statistics of the errors, described in Appendix B, are calculated using the variance-covariance matrix from one-stage GMM. In particular, we compute our standard errors using the Delta Method approach (see Wooldridge (2010, pp. 44-45)).

Let the estimated parameters $\theta \equiv [v/(nc)_{\text{lower}}, v/(nc)_{\text{upper}}, \overline{p}, \text{std}(p)]$, given the moments $m \equiv [\overline{dSuL}, \overline{dSuR}, \overline{dSuL^2}, \overline{dSuR^2}]$. First, we use the data to numerically compute the sensitivity of these estimates to changes in the moments, i.e., $\partial\theta_i/\partial m_j$, for $i, j \in \{1, 2, 3, 4\}$. Second, we estimate the variance-covariance matrix, let S, of the four errors that we base our estimation on, i.e.,

$$\overline{dSuL}_{est} - dSuL,\ \overline{dSuR}_{est} - dSuR,\ \overline{dSuL^2}_{est} - dSuL^2,\ \overline{dSuR^2}_{est} - dSuR^2.$$

Then, the variance of our error in estimating parameter θ_i is

$$\Delta_i \times S \times \Delta_i^T,$$

where vector $\Delta_i \equiv [\partial\theta_i/\partial m_1, \partial\theta_i/\partial m_2, \partial\theta_i/\partial m_3, \partial\theta_i/\partial m_4]$, for $i = \{1, 2, 3, 4\}$.

4.2.2 Probabilities of Misalignment

This subsection describes how we compute the probability of a misalignment ("swing") between the actual pass/fail decision and the counterfactual pass/fail decision under full participation. Intuitively, holding the parameters of each bin constant, we simulate the probability of a misalignment in the voting outcome *per bin*. First, given our estimated $\overline{p}, \text{std}(p)$ we simulate proposals $p \sim \mathcal{U}[l, h]$. Second, for each p given our estimated t_L, t_R for this bin we compute:

i) The estimated outcome index under discretionary participation:

$$O_{\text{disc}}(p) \equiv \overbrace{\overline{\gamma}\overline{q} + (1-\overline{\gamma})\, t_R p}^{\text{total support for} R} - \overbrace{\overline{\gamma}\,(1-\overline{q}) + (1-\overline{\gamma})\, t_L(1-p)}^{\text{total support for} L}.$$

ii) The estimated outcome index under full participation is:

$$O_{\text{full}}(p) \equiv \overbrace{\overline{\gamma}\overline{q} + (1-\overline{\gamma})\,p}^{\text{total support for} R} - \overbrace{\overline{\gamma}\,(1-\overline{q}) + (1-\overline{\gamma})\,(1-p)}^{\text{total support for} L}.$$

The sign of $O_{\text{disc}}, O_{\text{full}}$ determines whether the proposal passes or fails. Hence, a measure of the difference in the decision between discretionary and full participation is the indicator

$$\mathbb{I}\left(O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right).$$

Finally, we average for all p and this gives as a per bin estimate of $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$.

Two observations for this measure. First, O_{disc} for every p is computed using the estimated t_R, t_L and the bin $\overline{\gamma}$, $\overline{q}$, and not the actual $dSuR$, $dSuL$. This is because we do not want our swing measure to be 'polluted' by estimation error. Second, the only difference between O_{disc} and O_{full} are the rates t_R and t_L, which capture the very selection effect we want to quantify with this measure.

Moreover, we can perform exactly the same exercise as above but instead of using full participation as our benchmark, use the case of no discretionary, only regular participation. To this end define:

$$O_{\text{only-reg}} \equiv \overbrace{\overline{\gamma}\overline{q}}^{\text{total support for} R} - \overbrace{\overline{\gamma}\,(1-\overline{q})}^{\text{total support for} L}.$$

Note that $O_{\text{only-reg}}$ does not depend on p, which is only relevant for discretionary voters, and given our assumption that $q > 1/2$ is always positive, i.e., R wins if only regulars vote (which is trivial since we defined R to be the favourite amongst regular voters.).

Given the above definitions the outcome index under full participation can be written as:

$$\begin{aligned} O_{\text{full}}(p) &= \underbrace{\underbrace{\overline{\gamma}\overline{q} - \overline{\gamma}\,(1-\overline{q})}_{O_{\text{only-reg}}} + \underbrace{(1-\overline{\gamma})\,t_R p - (1-\overline{\gamma})\,t_L(1-p)}_{O_{\text{only-disc}}(p)}}_{O_{\text{disc}}(p)} \\ &\quad + \underbrace{(1-\overline{\gamma})\,(1-t_R)\,p - (1-\overline{\gamma})\,(1-t_L)\,(1-p)}_{O_{\text{no-part}}(p)}, \end{aligned}$$

where we decomposed O_{full} in three parts: $O_{\text{only-reg}}$ due to regular voters, from above; $O_{\text{only-disc}}(p)$

due to discretionary voters who participate; and $O_{\text{no-part}}(p)$ the part due to discretionary voters who do not participate in equilibrium. As also noted above the sum of the first two parts is the outcome index in equilibrium $O_{\text{disc}}(p)$.

5 Results

This section reports the main empirical findings. We begin with the estimation results and then set them in context. To be more precise, we provide a more in-depth discussion of the selection effects in relation to their associated equilibrium, and a discussion of the benefit to cost ratio across different proposals. We then present counterfactuals where we vary the cost fo voting c to illustrate selection effects in a spectrum between full and no discretionary participation. The section closes with a comparison of the model fit relative to the previous literature.

5.1 Parameter estimates

Panel A in Table 5 reports the point estimates of v/cn, $\overline{p}$ and $std(p)$. The estimates of v/cn, the benefit to cost ratio per voter, is 2.07 in our baseline estimation. The 95% confidence interval ranges from 2.02 to 2.10. The estimated distribution of p, the fraction for R among discretionary voters, has 0.83 mean and 0.24 standard deviation. The 95% confidence intervals are 0.81 to 0.84 for the mean and 0.23 to 0.24 for the standard deviation.[16] All the two-step and baseline confidence intervals overlap. In terms of model fit, Panel A provides the proposal-weighted model mean absolute error (m.a.e.) for each of the moments. The m.a.e. equals 2.3% for the first moment of $dSuL$ and 1.6% for the one of $dSuR$. These numbers compare to a proposal-weighted mean $dSuL$ of 15% and $dSuR$ of 56%. The second moment m.a.e. are higher, with an average of 4.4% for $dSuL$ and 1.3% for $dSuR$.

To set these numbers in context, we now compare the estimates to the previous literature, starting with v/cn. Out of v, c, and n, the literature on v, the benefits of winning a vote, has been the most extensive. Although we do not disentangle the three parameters in our estimation, we can approximate a range for the magnitude of v using rather primitive assumptions on c and n. For an assumed average share holding size of \$1.5 million (the average holding size of insider

[16] The magnitudes of estimates and confidence intervals from a two-step (efficient) GMM are comparable and listed next to our one-stage, baseline estimates.

holdings convicted by the SEC (Ahern (2015))) and a cost of \$1, the "return" is 1.3% . This result compares to an average return of 1.6% for the passing of governance-related proposals by Cuñat, Gine, and Guadalupe (2012). Assuming a higher cost would linearly translate into lower returns.

Next, we set our estimates of $\overline{p}$, the mean discretionary shareholder popularity of R, in relation to other parameters (reported in Panel B). We start by comparing $\overline{p}$ to $\overline{q}$ (i.e., the bin average of q, the regular voter support for their favourite option R). Regular and discretionary voters preferences differ on average by 7.0%. Direction-wise, this difference is almost entirely attributable to a greater support of the regular voters towards their preferred option R: the signed (as opposed to absolute) difference is 6.2%. This compares in magnitude to a standard deviation of q, the support of regular voters for R, of 7% and a mean of 89% (remember that q is defined to be above 50%).

Our $\overline{p}$ estimates per bin enable us to quantify selection effects by comparing discretionary voting support for the average proposal to the counterfactual benchmark equilibria under zero and full discretionary participation. To this end, we contrast the estimated $O_{\text{disc}}(\overline{p})$ to $O_{\text{only-reg}}$ and $O_{\text{full}}(\overline{p})$, as defined in Section 4.2.2. Any differences are due to the endogenous participation choice of discretionary voters, captured by the rates t_L and t_R. We find, that the participation decision augments the difference between regular and discretionary voters significantly. Discretionary voters who participate increase (in absolute terms) the support for R by 31.1% relative to what it is with only regular voters. This is intuitive as regular and discretionary voters by an large agree (the average $q - \overline{p}$ is 6.2%). Moreover, it would be increased by a further 21.4% if we were to force all discretionary voters to vote. The substantial support for R that is being 'lost' by the non-participating discretionary voters is due to the free-riding and underdog effects: the discretionary supporters of R (i.e., the majority of regular voters) free-ride on regular voters with their preference and participate less than the "underdogs" L, the opponents of the majority of regular voters.

How often can selection into voting overturn the voting decision? We document estimates for the probability of a voting decision that does not equal the majority preference of the full shareholder base (misalignment probability). This probability represents the fraction of such a misalignment within Monte-Carlo simulations within each bin under the parameter estimates for the distribution of p. On average, this probability is 3.7% in our sample of proposals. In other words, free-riding results in a probability of 3.7% for proponents of the majority in losing the vote, or: for the "underdog" to win the vote despite being the minority in the entire shareholder base.

5.2 Selection Effects and Equilibria

The participation decision leads to a pronounced difference of 21% between the observed voting support and the hypothetical one under full participation. The probability that these 21% overturning the majority's preferred decision, however, is only 3.7%. What drives this discrepancy in magnitudes? To explain the apparent disconnect, we document the estimates by equilibrium in Panel C. Equilibria differ in terms of the expected outcomes and the relationships between the parameters and participation rates (see Table 1). Thus, the identity of the equilibrium affects how parameters can affect the observed participation, the actual outcome, or even the equilibrium itself. Graphically, this means that the region in Figure 1 affects the consequences of moving in any direction.

First, Panel C reports the number of proposals that correspond to each equilibrium in Table 1. The vast majority of such proposals correspond to the $1m$ equilibrium, where the intergroup "underdogs" (discretionary voters against the average direction of regular voters) participates fully and the "free-riders" (discretionary voters for the average direction of regular voters) participate partially. In other words, discretionary voters against the regular voters are more likely to participate. In contrast, only 80 proposals correspond to the $m1$ equilibrium where the side supporting the regular voters participates fully and the the side against regular voters participates partially. The incomplete participation equilibrium mm is the most appropriate for 41 of the proposals. Recall that in this equilibrium, the expected outcome is a tie. We do not find any proposals that correspond to equilibrium 10 or 11 (full participation only against regular voters, full participation on both sides).

The pronounced participation of "underdogs" in the $1m$ equilibrium increases the observed voting support for the underdog compared to the average popularity in the total population. Indeed, over the entire sample the average participation rate for L is almost 100%, compared to an average participation rate of 66.7% for R. Such free-riding on the R side results into the high actual support for the minority option compared to the counterfactual under full participation. In contrast, in the mm and $m1$ equilibria, the R side receives more support under discretionary participation than under mandatory participation.

The probability of a decision misalignment between discretionary and mandatory participation also differs across equilibria. Recall that the probability of any average outcome in the mm equilibrium is 50% (because we have on average a tie and a coin toss to decided the ultimate outcome). Indeed, the probability of overturning the majority of the underlying shareholder population is 18.8% for the mm equilibrium. That probability is much lower, with 3.7%, for the $1m$ equilibrium. For the $m1$ equilibrium, the probability is 24.5%. Because most of our proposals correspond to the $1m$ equilibrium, the sample average probability of overturning is so small. In the next section, we consider how changes to the cost of voting can affect equilibrium incidence, participation, and outcomes.

5.3 Heterogeneity in Preferences

The algorithm produces estimates of otherwise unobserved shareholder preferences: the popularity of a proposal among the entire shareholder population (as discussed in the previous section) and the benefit to cost ratio per voter. In Table 6, we show these estimates by proposal type. Among shareholder proposals, CSR proposals have the highest benefit to cost ratio per voter. There is variation in terms of selection bias among the different proposal types. The likelihood of a minority-win is 13% for governance shareholder proposals. The absolute difference between the voted outcome and the preference of the entire shareholder is largest for CSR proposals (28%); the difference between the voted outcome and the hypothetical outcome under no discretionary participation is highest for payout proposals (30%). Among the management-sponsored proposals, takeover-defense, payout and board-related proposals have the highest benefit to cost ratios. The distance between the voted outcome and the preference of the entire shareholder base across the management proposals is smallest for CSR proposals (9%) and highest for say-on-pay and other compensation proposals (22%). The variation is smaller for the discrepancy between the actual outcome and the hypothetical one under no discretionary turnout.

The discrepancy between the shareholder proposals and the management proposals indicates that not only the proposal type matters but also the sponsor. Table 6, Panel B shows the outcomes of the algorithm using bins of sponsor types instead of proposal types. In this specification, management counts as one sponsor type; hence, this estimation is coarser for management proposals than our baseline results, where we estimate management proposals by proposal type. Nevertheless, the results are similar to the proposal-type estimation. Table 6 Panel B shows that management proposals have the highest benefit to cost ratio per voter, followed by proposals made by social groups and corporations. The proposals with the lowest benefit to cost ratio per voter are made by proxy advisors and employees. The likelihood of an outcome different from the preference of the majority of the shareholder base is highest for pension funds sponsored proposals (7%). In terms of the absolute distance between the voted outcome and the preference of the entire shareholder base, the distance is smallest for coalition proposals (9%) and management proposals (20%) and highest for employee-sponsored proposals (38%). The selection effects are different when we compare actual outcome to the no-discretionary-turnout outcome: there, the highest distance occurs for proposals sponsored by management, the smallest by those from employees.

In Panel C of Table 6, we report the outcomes of the algorithm using bins of years instead of proposal types. The most common equilibrium is $1m$, with full participation by "underdog" voters and partial participation of "free-riding" voters. Table 6 shows an increasing trend in the benefit to cost ratio per voters. This trend may reflect a declining cost of voting, which is consistent with the increasing popularity of online voting, or a trend in the average number of shares held. Despite the trend in the benefit to cost ratio, selection-related estimates do not exhibit any meaningful time trends.

5.4 Model comparison

To help understand the fit of the model, in this section we compare the precision of out-of-sample predictions to models similar to the previous literature. To be more precise, we use an estimation model based on Table 3 of Malenko and Shen (2016) on our data up to 2010 to forecast 2011 voting results. Malenko and Shen (2016) shows that a negative ISS recommendation causes a significant decline in say-on-pay voting support, with a high R^2 of 0.63. We use their estimation model in reduced form, adding their instrument of the ISS recommendation –whether firm performance has been below an ISS specific threshold– as an additional explanatory variable. To ensure that this model is directly comparable to ours, we also add the information we use in our baseline estimation as explanatory variables: γ, q, $\overline{p} \times (1 - \overline{p})$, market cap. We use this model to predict total participation, $dSuL$, $dSuR$, and the outcome index $O_{\text{disc}}(p)$.

We provide the estimates in Table 8 Panel A. Similar to Malenko and Shen (2016), NegRec predicts the outcome significantly. We obtain estimates for 13,520 proposals. For total participation as dependent variable, we obtain a R^2 of 0.65. For discretionary participation, the R^2 equals 0.59.

To compare the errors, we compute the squared residuals and average them per bin. Because the OLS residuals average zero by construction, for each observation we determine whether our algorithm performs better than the OLS models. We repeat this exercise with an out-of-sample forecast for the last year (2011) of our sample, where we predict the model parameters using data up to 2010.

Note that the applicable sample differs across the two sets of models. As Panel A, columns 1-4 show, the number of observations (proposals) with valid data for the reduced form models is 11,602 across all years for total participation and outcome, and 10,851 (10,907) for $dSuL$ and $dSuR$, respectively. This compares to 15,666 proposal observations for which the baseline model obtains parameter outputs that are consistent with any equilibrium boundaries and do not imply trivial benefit to cost ratios (i.e., benefits smaller than cost) out of a total of 16,493 proposals. Intuitively, any trivial proposal that may have already been excluded by the company because it was deemed immaterial.

To compare the prediction accuracy of our baseline model to the reduced form models, we calculate model parameters with data up to 2010. We then use these parameters (v/cn and the distribution of p) to predict participation, $dSuL$, $dSuR$, and outcome for 2011 proposals, using 2011 data for any other input needed (proposal type, γ, and q). For the reduced form model, we obtain regression coefficients with data up to 2010 and use them with 2011 data to obtain predictions. For both sets of predictions, we calculate the mean squared errors (MSE) of the 2011 predictions and test whether our baseline estimation yields smaller errors than the reduced form model.

Panel C reports the MSEs of both models and their difference. The baseline model produces smaller MSEs than the reduced form model. Indeed, the reduced form model MSEs are around twice the magnitude of the baseline MSEs. We use a Diebold-Mariano (Diebold and Mariano (1995)) test to show that the difference in MSEs is significant; for this test we treat the predictions as a time series with the meeting order as time stamp. The differences in MSE are the highest for total participation and $dSuL$. For this discretionary support against R, the OLS obtains MSEs twice as high as for the support towards R, while the difference among our estimates are more comparable. This discrepancy is consistent with a prevalence of corner equilibrium outcomes on the L side.

6 Counterfactuals

An appealing feature of our approach is that we can use the estimated model to evaluate the effects of counterfactual experiments on participation and voting decisions, and on the outcomes they induce. This approach can be useful for practitioners who wish to optimize their campaign budget as well as regulators who wish to alter voting procedures. Here we consider variations of the cost of voting c, a variable of interest for policy makers. The cost in our model enters through the benefit to cost ratio per voter v/cn. Hence an increase in the cost, keeping v and n constant leads to a decrease in the v/cn or an increase to cn/v. Clearly, our counterfactuals are equivalent if one was to think of changing these ratios directly (by for example changing v and keeping c and n constant).

Reducing the cost of voting is an objective of regulators around the world, for example see the 2017 version of the Shareholder Rights Directive in the European Union.[17] Notable examples of cumbersome and thus costly voting procedures includes pre-registration requirements (e.g., in Switzerland), Power of Attorney requirements (e.g., in Sweden), the non-availability of electronic voting outside the US and Europe (see Eckbo, Paone, and Urheim (2010), Eckbo, Paone, and Urheim (2011), and of Institutional Investors (2011)). In addition, the concentrated nature of shareholders meetings (in spring in the US, on 2 days annually for the entire Japanese population of public firms) provides challenges especially for small, but diversified asset managers.

In contrast, costs of voting in the US are likely to be small. Most firms in the US support electronic voting and do not require cumbersome paperwork to prove ownership or pre-register for voting. The US low cost of voting is reflected in the high participation rate among L voters we estimate with the US data. The dominance of the $1m$ equilibrium means that shareholders face a low cost of voting compared to the potential benefit of winning (given the relatively small probability of being pivotal).

In Table 7, we report $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$ (Panel A), i.e., the probabilities of a non-representative outcome, and associated equilibrium incidence (Panel B) for different multiples of cost of voting, keeping all other parameters constant. Technically, we compute these for multiples of the estimated cn/v (i.e., the inverse of the benefit to cost ratio per voter). We begin with a c of $0.25, 0.5$, and 0.75 times the US level and proceed with multiples of 2–5, $10, 15, 20, 25$, and 30 the US level. For comparison, we also report the actual estimates (multiple of 1).

[17] Available online at http://bit.ly/2rtcZA5.

The probability of a non-representative outcome is very low at voting costs below the US level. The probability is zero at a cost of 0.25 times the US level and $2{\cdot}10^{-4}\%$ at 0.5 times the US level (Panel A). This is because, as Panel B shows, for cost levels of 0.25 times the current US level all proposals with any associated equilibrium are of the full participation (i.e., 11) type (and 60 out of 62 proposals for 0.5 times the US level).

For voting costs above the US level, the probability of a non-representative outcome peaks at 35%, for costs 3 times the US level. Higher levels of costs than that are associated with a lower probability of a non-representative outcome. This is because costs of 3 times the US level leads to the highest incidence of the mm equilibrium, 4286 out of 15,665 proposal (see Panel B). Recall that in that equilibrium, both sides use a mixed strategy and the average outcome is a tie. The ultimate outcome is decided by a coin flip, so conditional on an mm equilibrium the favourite loses with 50% probability. This associated randomness produces voting outcomes that are non-representative of the full population in 44% of all proposals. At the 3 times cost level, we also start seeing more incidences of the 10 and $m0$ equilibria instead of the $1m$ equilibrium, so that discretionary support for R falls to even zero. In other words, we obtain maximum free-riding on regular voters from the discretionary voters who agree with their favourite outcome.

Note that even with cost of voting as high as 30 times the US level the "underdog" will participate fully in some proposals. Hence, the predominance of proposals where the "underdog" participates fully (the $1m$ equilibrium) in the US sample is not unusual. Indeed, the probability of changing the voting outcome in their favour in the $1m$ equilibrium is as high as 24% with costs of voting 2 times the US level and 30% with costs of voting 3 times the US level. In the 10 and $m0$ equilibria, for 3 times the US level, the "underdogs" have an even higher probability of changing the voting outcome, with averages of 34% and 46%, respectively.

With increasing cost of voting, participation moves from full to none. That is, we move from the 11 equilibrium with full participation to the $m0$ equilibrium where almost only regular voters and a few underdogs participate. The representativeness of such high-cost elections depend on how much discretionary voters agree with regular voters. Regular voters that perfectly agree with discretionary voters can represent all shareholders even if voting is prohibitively costly for discretionary voters. Such perfect alignment of preferences would support regulations that force institutional investors to vote. However, our estimations above show that this is not always the case. The average distance between q and $\overline{p}$ ranges from 4% in board-related shareholder proposals to 19% in takeover defense-related shareholder proposals.

To study the importance of disagreement between regular and discretionary voters, we report the counterfactual misalignment probabilities for these two types of proposals. As expected, disagreement yields a higher probability of non-representative outcomes for high costs of voting. Starting from a level of 7 times the US cost of voting, the high-disagreement defence proposals yield a higher probability that the minority wins than the low-disagreement board proposals. However, the low-disagreement board proposals face a higher probability of misalignment = at lower costs levels. That is, disagreement does not necessarily translate into a higher probability of non-representative outcomes at medium levels of costs of voting, as L voters may be sufficiently dissuaded from participating.

7 Discussion and Robustness

7.1 Model Assumptions

Our model is stylized and meant to illustrate how the participation decision can change voting outcomes. The good fit of our estimates compared to that of a reduced form model suggests that even with very few input parameters within a simple model, the algorithm performs well. To facilitate the interpretation of the estimations, however, we point out here the limitations imposed by the model assumptions.

Most important, our model focuses on the decision whether to participate in voting and abstracts from all other decisions. In particular, the model abstracts from how the potential voters obtained their votes, how they chose their preferred side, and how they arrived at their common knowledge of the model parameters (i.e., the fraction and preferences of the regular voters and the distribution of preferences among the discretionary voters). In reality, however, these decisions are likely to be endogenous with the participation decision as well as depend on factors that are not considered in our model.

In particular, how do shareholders obtain information needed to make their decisions in practice? Much of the relevant information is fairly easy to access today due to disclosure regulations: the ownership structure, which is disclosed in the invitation to vote, voting manifestos, commonly published online by institutional investors, and recommendations given by proxy advisory companies such as the ISS and Glass Lewis (Iliev and Lowry (2014), Malenko and Shen (2016)). Alas, high quality information can still be costly to acquire: for example, products such as Proxy Insights provide aggregate voting predictions using the past voting behaviour of institutional investors by subscription. Hence, it would be natural to assume that instead of being partisan, shareholders need to acquire potentially costly information about the potential benefits of the proposals as well as the preferences of the other shareholders. While such costs are sunk at the stage of the participation decision we focus on, the heterogeneity of such costs or the distribution of information (both the parameters and noise) are likely to have an effect on our results.[18]

For example, what would happen if the voting cost c was heterogeneous among discretionary voters? According to Myatt (2015, Section 5), heterogeneous costs imply that in the incomplete participation equilibrium mm, the underdog effects may not be able to cause a tie (this is referred to as the "partial underdog compensation effect"). We suspect a similar qualitative result in our setting with both regular and discretionary voters. As a consequence, when there are heterogeneous costs, the selection effects can be considerable even without resulting in a $1m$ equilibrium, which would infer a subtly different channel for our findings, where most proposals are in a $1m$ equilibrium. We plan to explore this issue further both theoretically and empirically in future work.

Another relevant practical matter is equity lending. The literature has documented significant equity lending (see Christoffersen, Geczy, Musto, and Reed (2007) and Aggarwal, Saffi, and Sturgess (2015)) around shareholder meetings. In the presence of such a market, discretionary voters may decide to lend their shares instead of voting, effectively increasing the opportunity cost of voting. If voting constitutes a significant factor in the equity lending market, the demand, supply, and interest should be endogenous to shareholder preferences and the voting decision. Given that the expected benefits of voting are greater for shareholders with a preference for the underdog, these should be willing to pay more to borrow shares. This will allow them to acquire more votes, which will lead to closer votes than that occurring in a world without equity lending. Closer voting outcomes can lead to more incidences of the partial equilibrium outcome, which is rare in our data set.[19] In the

[18]There are institutional reasons to believe that such heterogeneity exists. For example, Bach and Metzger (2015) argue that management has access to inside information about the votes already cast and therefore can run a targeted soliciting campaign. Moreover, shareholder pressure groups, such as Shareaction or Peta, advocate for their cause in highly visible advertising campaigns. Indeed, this may be one channel through which the underdog side achieves the full participation we document.

[19]A more problematic bias may arise if external reasons unrelated to voting increase the demand or supply of equity

next section, we compare the top and bottom quintile of firms in terms of the demand and supply of equity lending and investigate the effect on our estimation results.

We also make procedural assumptions that are standard in the literature (see Heard and Sherman (1987), McGurn (1989), and Monks and Minow (2003) for further details about the mechanics of proxy voting). First, we assume that voting occurs simultaneously. In practice, there might be instances of early access (e.g., provided by vote solicitors such as Broadridge, see Bach and Metzger (2015)). However, most voters and brokers submit their votes at the deadline to prevent such access and to avoid having to change their votes should they change their opinion. Second, we assume that regular voters never abstain from voting, which is consistent with the data that shows virtually no abstentions by regular voters, with fewer than 1% of empty votes cast within our sample. Third, our model assumes that the vote is for a single issue/proposal, i.e., abstracts from bundling, which occurs in reality. To address this potential issue, we conduct a robustness check by splitting meetings into different degrees of proposal bundling.

Finally, shareholder preferences and types are, in reality, much more heterogeneous than we assume in the model. We assume that the cost and benefit of voting are constant and symmetric across types and that discretionary voters all own the same number of shares. We make these assumptions to keep the model as simple as possible and to avoid overfitting the empirical estimation. Our model also allows for a set up in which the discretionary voters have a different number of votes as long as these differences are small.

7.2 Alternative Estimation Methods

In this section, we show how robust our estimations are to alternative estimation methods. In Panel A of Table 9, we report the MSE for variations of our algorithm, compared to the one of the Malenko-Shen model, as described in Section 5.4. For comparison, we report the MSEs for both the baseline and the comparison model (and both for the entire and the outsample) on top of the table. In Panel B, we report the equilibrium incidence using the different variations. For all variations, MSEs are significantly smaller than the comparison model, and the $1m$ equilibrium remains dominant.

[Insert Table 9 about here]

lending.

Two-step GMM Our baseline estimations use a non-weighted GMM because the two-step optimization does not usually perform well for small numbers of observations as in our bins. Here we show that a two-step GMM procedure yield similar MSEs. The equilibrium instance distribution is also similar, with more mm estimates (205 compared to 41 in the baseline) but still a large dominance of the $1m$ equilibrium.

Alternative bins Our baseline estimations use bins per proposal type$\times\gamma$ decile$\times n$ decile. In Table 9, we report the performance and equilibrium incidence for bins using quintiles of γ and size instead. Using quintiles, our model performs similar to the OLS model. The coarser equilibrium estimates do not identify any equilibria other than the mm and $1m$ ones, and only 58 of the proposals have the mm equilibrium. We also compute alternative n deciles using the actual number of non-NPX institutions. Note that we do not observe the number of non-institutional investors and hence cannot compute the actual value of n.

Medians instead of means Our baseline estimations use the means and standard deviations per bin. In Table 9, we report the performance and equilibrium incidence using medians instead of means. Using medians does not change the model performance or equilibrium allocation significantly.

Excluding the $1m$ equilibrium How robust is our allocation of proposals to equilibria, especially the most popular $1m$ equilibrium? If allocating proposals to other equilibria increases errors only marginally but changes our results significantly, we should consider those alternative results more seriously. In Table 9, we report MSEs and equilibrium allocations if we do not allow the $1m$ equilibrium. Doing so increases our MSEs massively. It doubles the MSE for $dSuL$, triples it for $O_{\text{disc}}(p)$, inflates the one for $dSuR$ by 5 and the one for turnout t 14 times. (Note that the resulting errors are still significantly below the reduced form MSEs.) Out of 15,545 proposals that our baseline estimation allocates to be in the $1m$, 7,313 are now in the mm equilibrium, 3,045 in the 10, 5,228 in the 11 equilibrium. Note that the baseline estimation does not allocate any proposals to the latter two corner equilibria. The number of proposals in the $m1$ equilibrium remains 80 like in the baseline estimation.

ISS recommendations Proxy advisors have a large influence on the voting direction of regular voters (Iliev and Lowry (2014), Malenko and Shen (2016)). To reflect this influence, we compute alternative estimates for bins of ISS recommendation×proposal type×γ decile×size decile. Errors are similar and the dominance of the $1m$ equilibrium is unchanged. We report the average estimates in Table D1.

Equity lending In the presence of an equity lending market, discretionary voters may decide to lend their shares instead of voting, effectively increasing the opportunity costs of voting. We compare the firms above and below the median in terms of the demand and supply of equity lending and investigate the effect on our estimation results. Currently, the US equity lending market operates over-the-counter, and data are available from Markit for the period of 2001-2016.[20] The Markit database covers over 90% of that market and contains firm-quarter level information on the supply of lendable shares for the majority of stocks listed in public exchanges. Following Campello and Saffi (2015), we define equity lending supply as the difference between the value of a firm's lendable shares and the number of lendable shares currently on loan divided by the firm's market capitalization. This calculation gives us a precise measure of the net lendable supply. We define equity lending demand as the value of shares actually borrowed divided by the firm's market capitalization. We then compare the firms above and below the median in terms of the demand and supply of equity lending and investigate the effect on our estimation results. As we can see in Table 9, Panel A, our algorithm performs well across the different subsamples in terms of the supply and demand of equity lending. Across the equity lending supply and demand subsamples, the $1m$ equilibrium remains dominant, ranging from 96% for the low equity lending supply subsample to 99% for the high equity lending supply subsample.

7.3 Sample Splits

In this section, we examine our model's performance and the robustness of the estimates for the various subsamples. We report proposal type averages estimates in Table 10.

[Insert Table 10 about here]

[20] We are most grateful to Pedro Saffi for providing us with part of this dataset.

Number of proposals First, we split the sample at the median number of proposals being voted on in a meeting. Since our model focuses on one proposal, the results for meetings that have fewer proposals should be more representative. Neither the MSEs (see Panel A, Table 9) nor the equilibrium allocation differs significantly between the two subsamples. The incidence of the $1m$ equilibrium is dominant for both subsamples (Panel B, Table 9).

Ownership We repeat the analysis by splitting the sample at the annual median percentage of shares owned by hedge fund activists, the total ownership by institutions (including below 5%), and the Herfindahl concentration index of institutional ownership. Neither the errors nor the equilibrium incidence differ significantly across the subsamples.

Importance of information An important ingredient of the voting process that is not included in our model is information aggregation of dispersed private information. Hence, the algorithm should be more suitable when information aggregation is less important. This should be the case later in the proxy season, after shareholders have already observed the voting preferences in many firms and guidance on the respective proposal types. We split the sample by the date of the meeting (before or after the median month in any given year) as well as by whether ISS has issued recommendations in both directions for the same proposal type in the same season yet ("Early/Late meeting/ISS"). For a firm-specific measure, we also split the sample by the annual median of the standard deviation of analysts forecasts. Neither the MSEs nor the equilibrium incidences differ significantly from the base algorithm.

Equity lending In Panels D and E, we finalize the analysis by splitting our sample into stocks with high (low) equity lending demand (Panel D) and high (low) equity lending supply (Panel E). As described above, we define equity lending supply as the difference between the value of a company's lendable shares and the number of lendable shares currently on loan, divided by its market capitalization. We define equity lending demand as the value of shares actually borrowed divided by its market capitalization. We then compare the firms above and below the median in terms of the demand and supply of equity lending. The different subsamples produce similar MSEs and equilibrium allocations.

7.4 Holding size

The algorithm estimates the benefit to cost ratio per voter $v/(cn)$. In this section, we illustrate the benefit to cost ratio distribution as a function of the average holding size, which allows us to calculate the number of voters n.

[Insert Figure 2 about here]

Figure 2 depicts the estimated range of the benefit to cost ratio for an average share block worth from $100,000 up to $3 million. We exclude estimations where the benefit to cost ratio is lower than 1 (i.e., irrelevant proposals). We plot the "return" of a proposal by dividing the benefit to cost ratio estimate by the assumed block size. For this analysis, we hold the cost estimate constant: the magnitude shown can be interpreted as the return on a proposal if we assume a cost of $1. Assuming a higher cost would linearly translate into lower returns.

The graph shows that an average share holding size over $500,000 can yield realistic estimates of the benefit to cost ratios (below 20%). For an assumed average share holding size of $1.5 million (the average holding size of insider holdings convicted by the SEC (Ahern (2015)), the "return" is 1.3% . This result compares to an average return of 1.6% for the passing of governance-related proposals by Cuñat, Gine, and Guadalupe (2012). Hence, using the average holding size, following Ahern (2015), yields estimates that are roughly comparable to those of the previous literature.

8 Conclusion

In this paper, we show how shareholders decide whether to vote. In a rational choice model where participation depends on the cost and benefits of voting and the probability that one's vote matters, we illustrate how shareholders conduct a cost to benefit analysis of voting based on their preferences and beliefs and the ownership structure. The model shows that the voting outcome can differ from the preferred outcome of the shareholders. This is due to the lower participation of shareholders with popular preferences (free-rider effect) relative to those with unpopular preferences (underdog effect).

Our model yields an algorithm that uncovers unobserved shareholder preferences such as a proposal's popularity among the entire shareholder base and its perceived benefits. Despite the stylized assumptions of the model, the algorithm performs strikingly well, producing significantly smaller estimation errors than a comparable reduced form model. Using aggregate voting data from the US, we find that strategic selection into voting is relevant. On average, voting outcomes differ by 22% from the preference of the entire shareholder base. Even though we estimate the underlying population preferences of discretionary (such as hedge funds) voters to be only 7% different from the preferences of regular voters (such as mutual funds), the actual voting support differs by 31% from the counterfactual equilibrium where no discretionary voters were to vote. This discrepancy stems from discretionary shareholders that agree with regular voters and free-ride on them. Such free-riding results into an average 3.7% probability that the actual voting decision differs from the preferred option of the total shareholder base. This probability of selection-driven outcome misalignment is most pronounced for governance-related shareholder proposals. Finally, the cost of voting is arguably small in the US. Taking multiples of it we produce counterfactuals for the misalignment probability: the result is a reverse-U shape relationship, which peaks at probability 35% for cost three times the US level.

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APPENDIX

A. Equilibrium With Incomplete Participation

Proof of Lemma 1. Define $u_R \equiv apt_R$, $u_L \equiv a(1-p)t_L$, the actual voting probabilities for R and L among discretionary voters, and the probability of absentee votes $u_0 \equiv 1 - u_R - u_L$. Vector $u \equiv (u_R, u_L, u_0)$ lives in simplex Λ and assumes that the beliefs regarding u are represented by density $h(\cdot|i)$, for $i \in \{R, L\}$. Then, by adapting Lemma 1 proposed by Myatt (2015) for our purposes, we know that the probability of a tie with x votes of discretionary voters for R is:

$$
\begin{aligned}
\Pr\left[b_R = x, b_L = x + (2q-1)\gamma n \middle| h(\cdot|i)\right] &= \\
\int_\Lambda \frac{((1-\gamma)n)!\, {u_R}^x {u_L}^{x+(2q-1)\gamma n} {u_0}^{(1-2q\gamma)n-2x}}{x!(x+(2q-1)\gamma n)!((1-2q\gamma)n-2x)!} h(u|i)du &\approx \\
h\left(\frac{x}{(1-\gamma)n}, \frac{x}{(1-\gamma)n} + \frac{(2q-1)\gamma}{(1-\gamma)}, \frac{1-2q\gamma}{1-\gamma} - 2\frac{x}{(1-\gamma)n}|i\right) \frac{\Gamma\left((1-\gamma)n+1\right)}{\Gamma\left((1-\gamma)n+3\right)} &\approx \\
\frac{1}{(1-\gamma)n} \frac{h\left(\frac{x}{(1-\gamma)n}, \frac{x}{(1-\gamma)n} + \frac{(2q-1)\gamma}{(1-\gamma)}, \frac{1-2q\gamma}{1-\gamma} - 2\frac{x}{(1-\gamma)n}|i\right)}{(1-\gamma)n}&,
\end{aligned}
$$

for $i \in \{R, L\}$.[21] By summing the x, the overall probability of a tie is:

$$
\sum_{x=0}^{n/2-q\gamma n} \Pr\left[b_R = x, b_L = x + (2q-1)\gamma n \middle| h(\cdot|i)\right].
$$

Given the above approximation, when n is large enough, the sum can be approximated by the integral

$$
\frac{1}{(1-\gamma)n} \int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y + (2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma) - 2y|i)dy.
$$

[21] These approximations depend on the observation that when n is large, most of the distributution will be concentrated at its mode.

Now, by using Lemma 2 of Myatt (2015), the probability of a tie and a near tie are equal for a large n and hence for $i \in \{R, L\}$:

$$\Pr[\text{Pivotal}|i] \approx \frac{1}{(1-\gamma)n} \int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|i)dy. \tag{16}$$

Below, we revert the above expressions from vector u to vector (p, a) so that we can transition from density h to densities f and g. Recall that $u_R = apt_R$, $u_L = a(1-p)t_L$; hence, the Jacobian $\partial(u_R, u_L)/\partial(p, a)$ has a determinant equal to $at_R t_L$. Moreover, note that each shareholder updates her beliefs based on her own availability and so

$$h(x, y, 1-x-y|i) = \frac{f\left(p(x,y)|i\right) g\left(a(x,y)|\text{available}\right)}{a(x,y)t_L t_R},$$

for any $x, y \in (0,1)$ and $i \in \{R, L\}$, where

$$g\left(a|\text{available}\right) = \frac{g(a)a}{\bar{a}},\ f(p|L) = f(p)\frac{1-p}{1-\overline{p}},\ f(p|R) = f(p)\frac{p}{\overline{p}}.$$

Hence, for $u_R = y$ and $u_L = y + (2q-1)\gamma/(1-\gamma)$ from (16), after some simple algebra we have

$$\begin{aligned}
p(a) &\equiv \frac{t_L}{t_R+t_L} - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{a(t_R+t_L)}, \\
a(y) &\equiv \frac{y(t_R+t_L)}{t_R t_L} + \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L}.
\end{aligned}$$

By substituting all the above in the integrand of (16), we have

$$\begin{aligned}
h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|R) &= \frac{f\left(p(a(y)\right) g\left(a(y)\right) a(y) p\left(a(y)\right)}{a(y)t_R t_L \bar{a}\overline{p}} \\
&= p\left(a(y)\right) f\left(p(a(y)\right) g\left(a(y)\right) \frac{1}{t_R t_L \bar{a}\overline{p}}
\end{aligned}$$

and similarly,

$$h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|L) = (1-p\left(a(y)\right)) f\left(p(a(y)\right) g\left(a(y)\right) \frac{1}{t_R t_L \bar{a}(1-\overline{p})}.$$

Now, to calculate the integral in (16), we change the variable from y to $a = a(y)$. We have $da = dy(t_R + t_L)/(t_R t_L)$ and

$$\begin{aligned} a(0) &= \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L}, \\ a\left((1/2 - q\gamma)/(1-\gamma)\right) &= \frac{t_R\left(1/2 - \gamma(1-q)\right) + t_L\left(1/2 - \gamma q\right)}{(1-\gamma)t_R t_L}. \end{aligned}$$

Then, we have that:

$$\begin{aligned} \Pr[\text{Pivotal}|R] &\approx \frac{1}{(1-\gamma)n}\int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma) - 2y|R)dy.dy \\ &= \frac{1}{(1-\gamma)n}\frac{1}{\bar{a}\bar{p}(t_R+t_L)}\int_{a(0)}^{a((1/2-q\gamma)/(1-\gamma))} f(p(a))p(a)g(a)da. \end{aligned} \tag{17}$$

Since we are seeking to develop a simple formula that we can use with the data, we assume that a follows a degenerate distribution around its mean; that is, $g(a) = \delta(a - \bar{a})$, where δ is the Dirac function. Then, to have a strictly positive probability of being pivotal, we need:

$$a(0) < \bar{a} < a\left((1/2 - q\gamma)/(1-\gamma)\right) \iff \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L} < \bar{a} < \frac{t_R\left(1/2-\gamma(1-q)\right) + t_L\left(1/2-\gamma q\right)}{(1-\gamma)t_R t_L}. \tag{18}$$

The above is a restriction on equilibrium t_R, t_L which we need to verify after the equilibria are derived.[22] Given $g(a) = \delta(a - \bar{a})$ and A1, we know that (17) becomes (2), and similarly, we reach (3) for $\Pr[\text{Pivotal}|L]$, where $p^* = p(\bar{a})$ is given by (4). ■

A.1 Bounds on the Parameters

Now, we will derive the necessary and sufficient conditions, that is, the parameter regions, with which we can obtain a valid equilibrium with *incomplete participation* for both sides.[23] Since we have obtained equilibrium t_L, t_R from (9) and (10), we can check (18) and express it in terms of

[22] Note that (18) is satisfied for all $t_R, t_L \in (0, 1)$ for $\gamma = 0$ (i.e., the Myatt (2015) setup).

[23] In an ongoing work, we look for equilibria with either complete participation for at least one type or no participation for the supporters of R.

the primitives of the model.[24] First, (18) requires that

$$\bar{a} > \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L} \iff t_L > \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\bar{a}},$$

which is clearly satisfied by t_L in (9). Second, the assumption requires that

$$\bar{a} < \frac{t_R\,(1/2-\gamma(1-q)) + t_L\,(1/2-\gamma q)}{(1-\gamma)t_R t_L}.$$

Plugging in the expressions for t_L, t_R (9) and (10), and after some algebra, the above is equivalent to:

$$\frac{v}{c} \quad < \quad \frac{n\,(1-2\gamma\,(\overline{p}(1-q)+q(1-\overline{p})))}{2f(\overline{p})(1-\overline{p})\overline{p}}. \tag{19}$$

By definition, what we need for incomplete participation is $(t_L, t_R) \in (0,1)$. According to (9), it is evident that $t_L > 0$ for all parameter values. The condition $t_L < 1$ is equivalent to:

$$\gamma \quad < \quad \frac{\bar{a}}{2q-1+\bar{a}}, \text{ and} \tag{20}$$

$$\frac{v}{c} \quad < \quad \frac{n\,(\bar{a}-\gamma\,(\bar{a}-1+2q))}{f(\overline{p})\overline{p}}. \tag{21}$$

Given our assumption that $q > 1/2$, the condition of (20) takes precedence over A1 with respect to the upper permissible value of γ. Now, the condition $t_R > 0$ is equivalent to:

$$\frac{v}{c} \quad > \quad \frac{n\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}. \tag{22}$$

For (22) to be the relevant lower bound of v/c given A2, we define a lower bound on the regular block size and the number of voting shares,

$$\gamma \quad > \quad \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}, \text{ and} \tag{23}$$

$$n \quad > \quad \frac{2qf(\overline{p})(1-\overline{p})}{2q-1}, \tag{24}$$

[24] Note, that for the computed t_L and t_R in (9) and (10), the pivotal probabilities in Lemma 1 (2) and (3) are well defined.

; otherwise, $v/c \geq 1$ from A2. Finally, the condition $t_R < 1$ is equivalent to:

$$\frac{v}{c} < \frac{n\left(\bar{a} - \gamma\left(\bar{a} + 1 - 2q\right)\right)}{f(\overline{p})(1 - \overline{p})}. \tag{25}$$

Hence, for the benefit to cost ratio v/c, we have three possible upper bounds, (19), (21), and (25). We can show that (19) is never the relevant bound, but (21) and (25) can be, depending on the parameter values. Therefore, the upper bound of v/c is

$$\frac{v}{c} < \min\left\{\frac{n\left(\bar{a} - \gamma\left(\bar{a} - 1 + 2q\right)\right)}{f(\overline{p})\overline{p}}, \frac{n\left(\bar{a} - \gamma\left(\bar{a} + 1 - 2q\right)\right)}{f(\overline{p})(1 - \overline{p})}\right\}. \tag{26}$$

For v/c, we also have a lower bound, which is either (22), if γ satisfies (23), or one. In either case, we need to make sure that the lower bound is lower than the upper bound of v/c. Let us look at each case in turn:

1.
$$\gamma > \frac{f(\overline{p})(1 - \overline{p})}{n(2q - 1)},$$

   Then, to ensure the bound of (26) is higher than the bound of (22), we have another restriction on γ,

$$\gamma < \frac{\bar{a}(1 - \overline{p})}{\bar{a}(1 - \overline{p}) + 2q - 1}. \tag{27}$$

   From the two possible upper bounds of γ in (20) and (27), we can show that for $q > 1/2$, the relevant bound is condition (27). Finally, we need to make sure that the lower bound of γ in (23) is lower than the upper bound of (27). This puts a lower bound on the number of voting shares:

$$n > \frac{f(\overline{p})\left(\bar{a}(1 - \overline{p}) + 2q - 1\right)}{\bar{a}(2q - 1)},$$

   which supersedes the other lower bound of (24).

2.

$$\gamma < \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}, \tag{28}$$

Then, to ensure the upper bound of (26) is higher than the lower bound of A2, we add the following two restrictions on γ,

$$\begin{aligned} n(\bar{a}-(2q-1))\gamma &< \bar{a}n - f(\overline{p})(1-\overline{p}), \\ n(\bar{a}+(2q-1))\gamma &< \bar{a}n - f(\overline{p})\overline{p}. \end{aligned}$$

For these to be satisfied, we need

$$n > \frac{f(\overline{p})\overline{p}}{\bar{a}}. \tag{29}$$

Observe also that the second restriction above implies the one in (20), so this latter bound can be ignored in what follows. Then, we have the following subcases:

(a) $n > \frac{f(\overline{p})(1-\overline{p})}{\bar{a}}$ and $\bar{a} < 2q-1$Then, the only other restrictions on γ (i.e., other than (28)) can be written as

$$\gamma < \frac{\bar{a}n - f(\overline{p})\overline{p}}{(\bar{a}-1+2q)\,n}. \tag{30}$$

(b) $n > \frac{f(\overline{p})(1-\overline{p})}{\bar{a}}$ and $\bar{a} > 2q-1$Then, the added restriction on γ can be written as

$$\gamma < \min\left\{\frac{\bar{a}n - f(\overline{p})(1-\overline{p})}{(\bar{a}+1-2q)\,n}, \frac{\bar{a}n - f(\overline{p})\overline{p}}{(\bar{a}-1+2q)\,n}\right\}. \tag{31}$$

(c) $n < \frac{f(\overline{p})(1-\overline{p})}{\bar{a}}$ and $\bar{a} > 2q-1$Then, there is no equilibrium with incomplete participation.

(d) $n < \frac{f(\overline{p})(1-\overline{p})}{\bar{a}}$ and $\bar{a} < 2q-1$Then, provided that $\overline{p} < 1/2$, we need

$$\frac{\bar{a}n - f(\overline{p})(1-\overline{p})}{(\bar{a}+1-2q)\,n} < \gamma < \frac{\bar{a}n - f(\overline{p})\overline{p}}{(\bar{a}-1+2q)\,n}, \text{ for} \tag{32}$$

$$\frac{f(\overline{p})\,(\bar{a}(1-\overline{p})+2q-1)}{\bar{a}(2q-1)} < n < \frac{f(\overline{p})(1-\overline{p})}{\bar{a}}. \tag{33}$$

The above is summarized in the proposition below.

Proposition 2. *Assume that* $q \in (1/2, 1)$, $\overline{p} \in (0,1)$, $f(\overline{p}) > 0$, $\bar{a} \in (0,1]$, $g(a) = \delta(a - \bar{a})$ *and*

$$\frac{v}{c} < \min\left\{\frac{n\,(\bar{a} - \gamma\,(\bar{a} - 1 + 2q))}{f(\overline{p})\overline{p}}, \frac{n\,(\bar{a} - \gamma\,(\bar{a} + 1 - 2q))}{f(\overline{p})(1-\overline{p})}\right\}.$$

In addition, consider the following disjoint parameter regions:

(a) Small regular block size:

$$\begin{aligned} \frac{v}{c} &\geq 1, \\ n &> \frac{f(\overline{p})\overline{p}}{\bar{a}}, \text{ and} \end{aligned}$$

(i) Many voters, high availability:

$$n > \frac{f(\overline{p})(1-\overline{p})}{\bar{a}},\ \bar{a} > 2q - 1, \text{ and } 0 \leq \gamma < \min\left\{\frac{\bar{a}n - f(\overline{p})(1-\overline{p})}{(\bar{a} + 1 - 2q)\,n}, \frac{\bar{a}n - f(\overline{p})\overline{p}}{(\bar{a} - 1 + 2q)\,n}, \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}\right\}, \text{ or}$$

(ii) Many voters, low availability:

$$n > \frac{f(\overline{p})(1-\overline{p})}{\bar{a}},\ \bar{a} < 2q - 1, \text{ and } 0 \leq \gamma < \min\left\{\frac{\bar{a}n - f(\overline{p})\overline{p}}{(\bar{a} - 1 + 2q)\,n}, \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}\right\}, \text{ or}$$

(iii) Few voters, low availability:

$$\begin{aligned} \frac{f(\overline{p})\,(\bar{a}(1-\overline{p}) + 2q - 1)}{\bar{a}(2q-1)} < n < \frac{f(\overline{p})(1-\overline{p})}{\bar{a}},\ \bar{a} < 2q - 1,\ \overline{p} < \frac{1}{2}, \quad \text{and} \\ \frac{\bar{a}n - f(\overline{p})(1-\overline{p})}{(\bar{a} + 1 - 2q)\,n} < \gamma < \frac{\bar{a}n - f(\overline{p})\overline{p}}{(\bar{a} - 1 + 2q)\,n}. \end{aligned}$$

(b) Large regular block size (Many voters, any availability):

$$\begin{aligned} \frac{v}{c} &> \frac{n\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}, \\ n &> \frac{f(\overline{p})\,(\bar{a}(1-\overline{p}) + 2q - 1)}{\bar{a}(2q-1)}, \text{ and} \\ \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)} < \gamma &< \frac{\bar{a}(1-\overline{p})}{\bar{a}(1-\overline{p}) + 2q - 1}. \end{aligned}$$

These conditions are necessary and sufficient for the existence of an incomplete participation equilibrium by both types, that is, $t_L, t_R \in (0,1)$, which are given by equations (9) and (10). Furthermore, the average participation among discretionary voters $\bar{t}$ and the average total participation $\bar{t}_{total}$ are given by (11) and (12). Finally, in such equilibrium, the probability of being pivotal for either R of L is equal to the common cost to benefit ratio c/v (1), and the expected votes for R and L are equal (5).

Note that Proposition 1 in the main text is the restriction to case (b), which is the empirically most plausible one, as discussed in Proposition 2 above.

B. Proposal Classification

The ISS's functional classification of proposals into 257 types is fine enough to risk obscuring the economic meaning of each proposal-type. For example, ISS assigns a different proposal-type for "Amend Omnibus Stock Plan (M0524)" and "Approve Omnibus Stock Plan (M0522)", even though these two proposals address the same economic issue, executive compensation. For this reason, it is useful to work with a coarser, more economically meaningful, classification. Our classification groups proposals into 12 economically relevant types. We list these types along with their frequency in our sample in Table 3. The set of types is chosen to reflect leading issues arising in the literature on voting and corporate governance (see for example Knoeber (1986), LaPorta, de Silanes, Shleifer, and Vishny (1998), Grullon and Michaely (2002), Gompers, Ishii, and Metrick (2003), Bebchuk, Cohen, and Ferrell (2009), Becht, Franks, Mayer, and Rossi (2009), Bebchuk and Fried (2009), Ferri and Maber (2012)). Once the set of types is chosen, the proposals are classified based on their description in a straightforward way, as illustrated in the example above on M0524 and M0522. In Table 11, we list the top 3 proposals per category. Needless to say, this classification is not unique.

C. Tables and Figures

Table 2: **Univariate Statistics**

This table shows the univariate statistics for a sample of proposals voted upon in US firms from 2003-2011. Panel A shows the number of proposals and meetings. Panel B shows the firm characteristics (at the firm-year level). Panel C shows the summary statistics for ownership (at the meeting level).

Panel A: Number of observations per year

Date	Proposals	Meetings
2003	358	212
2004	2,566	1,389
2005	1,256	788
2006	1,776	1,021
2007	1,562	824
2008	1,491	862
2009	2,069	1,149
2010	4,149	1,756
2011	3,293	1,296

Panel B: Firm level statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
Assets	8568	18,780.92	109,456.10	0.76	2,265,792.00
Leverage	8568	0.23	0.23	-	3.64
M/B	8568	1.88	1.36	0.38	26.82
Return (monthly)	6256	0.08	0.47	(3.61)	3.62
Return (annual)	6191	0.009	0.032	-0.229	0.256

Panel C: Summary statistics for ownership

Variable	Obs	Mean	Std. Dev.	Min	Max
% Institutional ownership	9297	68.32	21.58	0.60	99.96
of which: N-PX	9297	20.13	10.77	0.01	58.96
% > 5% Ownership	8705	24.51	18.43	0	97.90
of which: institutional	8705	23.13	17.88	0	97.90
private	9297	2.24	5.43	0	65.95
% Management ownership	9297	5.18	5.56	0	79.43
Shares outstanding	9163	272,000,000	861,000,000	555,992	29,100,000,000

Table 3: **Proposals**

This table shows the univariate statistics for a sample of proposals voted upon in US firms from 2003-2011. Panel B (C) shows the frequency of proposals voted upon in US firms from 2003-2011 per proposal (sponsor) type.

Panel A: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
Total participation	77.35	11.78	0.00	100.00
Discretionary participation	73.33	14.39	0.00	100.00

Panel B: Proposals per proposal-type

Proposal Type	Frequency	Support (%)	Participation (%)	
			Total	Discretionary
Compensation	8,171	61.44	75.31	70.57
Say-on-pay	2,118	69.91	76.69	72.58
Say-on-pay frequency	1,818	N/A	75.96	71.49
Restructuring	1,301	69.42	81.35	78.51
Board	1,274	50.70	78.54	72.89
CSR	1,086	10.02	73.93	75.08
Defense	990	67.54	80.94	75.61
Governance	806	56.41	78.67	74.19
Merger	356	72.52	75.15	70.46
Business	237	45.26	76.59	73.23
Payout	21	69.53	80.66	76.66
Other proposals	336	46.97	76.49	73.09

Panel C: Proposals per sponsor type

Sponsor Type	Frequency	Support (%)	Participation (%)	
			Total	Discretionary
Management	14,912	60.17	78.25	74.09
Individual activist	1016	26.26	72.39	66.58
Institutional (pension fund)	665	25.69	74.29	70.45
Social group	451	10.76	74.24	74.92
Institutional (non-pension fund)	309	20.68	73.33	71.45
Union	295	25.78	75.14	71.80
Coalition	43	30.19	74.69	70.16
Employee	12	17.97	62.42	47.38
Corporate	5	9.16	74.27	74.24
Proxy advisor	2	37.11	69.70	65.98
Other sponsors	810	24.38	74.14	71.08

Table 4: **Algorithm input parameters**

This table shows univariate statistics for the input parameters for the algorithm.

Variable	N	Mean	Std. Dev.	Min	Max
$dSuL$	16,434	0.15	0.08	0.00	0.94
$dSuR$	16,450	0.56	0.12	0.01	0.97
$dSuL^2$	16,434	0.05	0.05	0.00	0.89
$dSuR^2$	16,450	0.36	0.13	0.00	0.94
γ	16,479	0.21	0.10	0.00	0.59
market cap (proxy of n)	16,479	17,200,000,000	46,400,000,000	3,196,664	555,000,000,000
q	16,479	0.89	0.07	0.51	1.00
N non-NPX institutions	17,764	532.24	650.71	0	5,933

Table 5: **Estimation Results**

This table shows the univariate statistics for the estimation results of the algorithm run for the bins of proposal type×decile of institutional ownership×decile of market cap. Panel A shows the percentage of proposals for which the algorithm detects an equilibrium for each equilibrium type. Panel B shows the summary statistics of the distance between the actual voting support and the underlying estimated preferences for the entire shareholder population. Panel C shows means for the distances for each equilibrium.

Panel A: Parameter estimates

Estimates	Baseline	95% confidence interval		GMM	95% confidence interval	
v/cn benefit to cost ratio per voter	2.07	2.02	2.10	1.96	1.88	2.04
[s.e.] $-$		[0.01689]			[0.03677]	
[s.e.] $+$			[0.02137]			[0.04271]
p popularity of R among discretionary voters						
$\overline{p}$	0.83	0.81	0.84	0.82	0.76	0.83
[s.e.] $-$		[0.00148]			[0.00708]	
[s.e.] $+$			[0.01058]			[0.03185]
std(p)	0.24	0.23	0.24	0.23	0.22	0.24
[s.e.] $-$		[0.00181]			[0.00255]	
[s.e.] $+$			[0.00463]			[0.00721]
Mean Absolute Error						
$\overline{dSuL}$	0.0229			0.0364		
$\overline{dSuR}$	0.0163			0.0341		
$\overline{dSuL^2}$	0.0445			0.0454		
$\overline{dSuR^2}$	0.0126			0.0322		
N	15,666			14,127		

Panel B: Distance between the underlying preferences and voting outcomes

	Obs	Mean	Std. Dev.	Min	Max
$\overline{q}-\overline{p}$	15,666	6.2%	7.8%	-36.1%	95.2%
$\lvert\overline{q}-\overline{p}\rvert$	15,666	7.0%	7.1%	0.0%	95.2%
$O_{\text{disc}}(\overline{p})-O_{\text{only-reg}}$	15,666	31.1%	14.8%	-85.8%	89.9%
$O_{\text{full}}(\overline{p})-O_{\text{only-reg}}$	15,666	52.6%	16.8%	-91.6%	98.0%
$O_{\text{disc}}(\overline{p})-O_{\text{full}}(\overline{p})$	15,666	-21.4%	8.0%	-58.6%	43.1%
$\lvert O_{\text{disc}}(\overline{p})-O_{\text{full}}(\overline{p})\rvert$	15,666	21.8%	7.0%	2.4%	58.6%
Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	15,666	3.7%	6.2%	0.0%	50.0%

Panel C: Selection effects per equilibrium

	Equilibrium	mm	$1m$	10	11	$m1$	$m0$
N		41	15,545	0	0	80	0
%		0.3%	99.2%	0.0%	0.0%	0.5%	0.0%
Participation	t_L	59.3%	100.0%			64.8%	
	t_R	90.7%	66.5%			100.0%	
Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$		18.7%	3.7%			24.7%	
$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	signed	22.4%	-22.0%			21.4%	
$\lvert O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})\rvert$	unsigned	25.2%	22.0%			21.4%	
$\overline{q} - \overline{p}$	signed	53.4%	5.8%			57.7%	
$\lvert\overline{q} - \overline{p}\rvert$	unsigned	53.4%	6.4%			57.7%	
$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$		-6.7%	31.4%			-21.8%	
Estimation error		0.0%	0.8%			0.0%	

Table 6: **Heterogeneity**

This table shows average parameter estimates by proposal type (Panel A), sponsor type (Panel B), and year (Panel C).

Panel A: Shareholder preferences by proposal type

Proposal type	$v/(cn)$	t_L	t_R	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	mm	$1m$	$m1$	Total
Board	0.84	100%	55%	9%	-25%	9%	0	706	2	708
Business	1.71	99%	56%	3%	-26%	19%	0	120	2	122
CSR	1.94	100%	58%	1%	-28%	27%	0	1,066	0	1,066
Compensation	1.01	100%	54%	7%	-25%	11%	0	724	5	729
Defense	1.29	95%	60%	8%	-19%	11%	26	363	13	402
Governance	0.84	96%	58%	13%	-21%	7%	9	305	23	337
Other	1.53	97%	58%	2%	-25%	21%	2	98	6	106
Payout	.	100%	62%	0%	-24%	30%	0	3	0	3
Restructuring	1.53	100%	57%	1%	-26%	21%	0	72	0	72
Total	1.32	99%	57%	6%	-25%	16%	37	3,457	51	3,545
Management proposals										
Board	2.94	100%	76%	0%	-16%	45%	0	466	1	467
Business	2.63	100%	76%	0%	-19%	51%	0	94	0	94
CSR	.	100%	87%	0%	-9%	54%	0	6	0	6
Compensation	2.08	100%	67%	4%	-22%	31%	0	7,162	0	7,162
Defense	3.08	100%	80%	0%	-13%	44%	0	544	0	544
Governance	2.78	99%	79%	1%	-13%	39%	0	319	9	328
Merger	2.72	100%	72%	0%	-22%	47%	0	326	1	327
Other	1.91	97%	73%	4%	-15%	24%	4	142	18	164
Payout	.	100%	74%	0%	-21%	52%	0	15	0	15
Restructuring	2.47	100%	77%	2%	-16%	41%	0	932	0	932
SOP	2.41	100%	70%	1%	-22%	42%	0	2,082	0	2,082
Total	2.27	100%	70%	3%	-20%	36%	4	12,088	29	12,121

Panel B: Shareholder preferences by sponsor type

sponsor_type	$v/(cn)$	t_L	t_R	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	mm	$1m$	$m1$	Total
Coalition	0.96	96%	60%	6%	-19%	5%	0	27	3	30
Corporate	2.21	100%	63%	0%	-26%	34%	0	4	0	4
Employee	0.84	100%	29%	0%	-38%	3%	0	12	0	12
Fund	1.35	99%	56%	4%	-26%	16%	1	293	4	298
Individual activ	1.17	98%	54%	5%	-26%	15%	35	968	3	1,006
Management	2.31	100%	70%	3%	-20%	36%	0	12,416	0	12,416
Pension Fund	1.04	100%	55%	7%	-25%	10%	0	655	2	657
Proxy advisor	0.53	100%	52%	6%	-30%	10%	0	2	0	2
Social group	1.90	100%	58%	1%	-27%	27%	0	457	0	457
Union	1.03	99%	56%	7%	-23%	10%	5	281	4	290

Panel C: Shareholder preferences by year

Year	$v/(cn)$	t_L	t_R	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	mm	$1m$	$m1$	Total
2003	1.99	100%	64%	3%	-24%	33%	0	331	1	332
2004	1.93	100%	66%	3%	-22%	30%	0	1,844	5	1,849
2005	1.90	100%	67%	6%	-23%	31%	8	1,044	4	1,056
2006	2.01	100%	67%	4%	-22%	31%	0	1,533	3	1,536
2007	2.13	100%	66%	2%	-21%	30%	0	1,391	1	1,392
2008	2.22	100%	67%	2%	-21%	30%	0	1,299	0	1,299
2009	2.08	100%	66%	3%	-22%	29%	0	1,990	0	1,990
2010	2.08	100%	68%	2%	-22%	34%	0	3,114	0	3,114
2011	2.34	100%	71%	2%	-21%	41%	0	1,908	0	1,908

Table 7: Counterfactuals

This table shows $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$, the average probability that the minority wins (Panel A) and the number of proposals per equilibrium (Panel B) for varying values of c, holding all other parameters constant. The column caption states the value of c as a multiple of the estimates presented in Table 5.

Panel A: Misalignment Probability																
Multiple	0.25×	0.5×	0.75×	1×	1.25×	1.5×	1.75×	2×	3×	4×	5×	10×	15×	20×	25×	30×
All	0.0%	$2{\cdot}10^{-4}$%	0.8%	3.8%	10.3%	16.2%	21.2%	25.5%	34.9%	34.5%	33.2%	26.3%	19.8%	14.5%	10.7%	8.1%
By Equilibrium:																
mm				18.7%	37.5%	41.9%	40.9%	41%	44%	41%	39%	37%	36%	36%	47%	47%
$1m$		1.8%	4.0%	3.7%	10.5%	16.3%	21.0%	24%	30%							
10					$6{\cdot}10^{-3}$%	$9.7{\cdot}10^{-2}$%	0.1%	16%	34%	32%	30%	26%	34%	35%	37%	35%
11	0%	0%	0%													
$m1$			8.4%	24.7%	27.0%	26.1%	28.1%	32%	44%							
$m0$							0.4 %	44%	46%	43%	40%	28%	21%	18%	11%	9%
By type:																
Board shareholder proposals (low distance)	0.0%	0.0%	2.2%	9%	21%	32%	38%	42%	44%	40%	34%	12%	5%	3%	3%	2%
Defense shareholder proposals (high distance)	0.0%	$9{\cdot}10^{-3}$%	1.9%	8%	17%	24%	29%	33%	38%	35%	31%	15%	9%	5%	3%	4%
Panel B: Equilibrium Incidence																
Multiple	0.25×	0.5×	0.75×	1×	1.25×	1.5×	1.75×	2×	3×	4×	5×	10×	15×	20×	25×	30×
mm	0	0		41	182	463	1091	2,295	4286	2821	1944	813	524	88	36	31
$1m$	0	2	3045	15545	15438	15174	14533	13,310	2896	0	0	0	0	0	0	0
10	0	0		0	2	7	27	42	7279	8845	7772	3965	193	106	49	3
11	60	60	11	0	0	0	0	0	0	0	0	0	0	0	0	0
$m1$	0	0	67	80	44	22	14	12	1	0	0	0	0	0	0	0
$m0$	0	0	0	0	0	0	1	7	1204	4000	5950	10886	14933	15390	15353	15075

Table 8: Comparison of Estimation Methods

This table compares the prediction accuracy of the algorithm compared to reduced form models reported in Panel A. The dependent variable and the relevant sample is reported in the table caption. Independent variable are the input variables to the algorithm γ, q, and market capitalization, $\overline{p}(1-\overline{p})$, the number of proposals in the meeting, and the dependent variables from Table 3 of Malenko and Shen (2016), most importantly: NegRec, which equals one if ISS gives a negative recommendation, and zero otherwise; BelowCutoff, which equals one if the firm is below the cutoff ($MaxTSR < 0$), and zero otherwise, and the interaction of these variables. Standard errors are reported in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively. Panel B reports the mean squared error (MSE) of our baseline estimations and the ones reported in Panel A columns 4-7, their difference, the test statistic and the p-value of the Diebold-Mariano test for equality of predictive accuracy.

Panel A: Reduced Form regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent variable	Total participation	$\overline{dSuL}$	$\overline{dSuR}$	$O_{\text{disc}}(\overline{p})$	Total participation	$\overline{dSuL}$	$\overline{dSuR}$	$O_{\text{disc}}(\overline{p})$	Total participation
Sample	All	All	All	All	Excl. 2011	Excl. 2011	Excl. 2011	Excl. 2011	All
Regression	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS	Tobit
Gamma	0.202***	-0.132***	0.146***	0.283***	0.206***	0.294***	-0.145***	0.147***	0.202***
	(12.25)	(-5.45871)	(4.97973)	(10.7302)	(11.6877)	(10.4963)	(-5.66793)	(4.81377)	(13.4097)
$\overline{q}$	0.018***	-0.451***	0.417***	0.723***	0.020***	0.729***	-0.451***	0.419***	0.018***
	(3.06882)	(-35.6601)	(30.5898)	(51.3033)	(3.11961)	(50.1657)	(-34.7959)	(29.6577)	(3.35901)
$\overline{p}\cdot(1-\overline{p})$	-0.165***	0.724***	-0.871***	-0.873***	-0.215***	-0.893***	0.728***	-0.903***	-0.165***
	(-5.70509)	(15.275)	(-16.255)	(-15.2818)	(-6.65427)	(-14.4223)	(14.3934)	(-15.6557)	(-6.24452)
Market cap	0.201048	1.29863	0.0417589	0.134928	0.294562	0.682407	1.10374	0.255816	0.220057
	(-0.002**)	(0)	(-0.001)	(0)	(-0.002**)	(0)	(-0.001)	(-0.001)	(-0.002**)
# proposals in meeting	-1.96551	-0.494096	-1.06753	-0.306639	-2.42255	-0.255197	-0.862115	-1.19468	-2.1517
	(-0.001)	(0.069***)	(-0.071***)	(-0.083***)	(-0.002)	(-0.067***)	(0.053***)	(-0.057***)	(-0.001)
NegRec	-0.001	0.069***	-0.071***	-0.083***	-0.002	-0.067***	0.053***	-0.057***	-0.001
	(-0.373947)	(11.8787)	(-11.6256)	(-12.4002)	(-0.807994)	(-9.58043)	(8.89368)	(-8.99107)	(-0.407723)
MaxTSR	0	0	0	0	0	0	0	0	0
	(-0.874149)	(0.392299)	(-0.846655)	(-0.920532)	(-0.539165)	(-1.35456)	(1.11853)	(-0.561241)	(-0.956944)
BelowCutoffMaxTSR	0.001***	0	0.001***	0	0.001***	0	0	0.001***	0.001***
	(3.37761)	(1.07129)	(3.53101)	(0.55448)	(2.91743)	(0.445572)	(1.35149)	(3.61571)	(3.69752)
Total Compensation	1.67076	-0.459484	1.77932	0.463802	1.54221	0.345444	-0.473349	1.61794	1.82905
	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)	(0)
TDC1 Change	0.808093	0.0971939	1.02828	1.50597	1.03692	1.53424	0.164914	1.21642	0.884254
	(0.003)	(0.008)	(-0.01)	(-0.015*)	(0.003)	(-0.013)	(0.007)	(-0.011)	(0.003)
% Stock Compensation	0.003	0.008	-0.01	-0.015*	0.003	-0.013	0.007	-0.011	0.003
	(0.456466)	(1.03957)	(-1.06516)	(-1.68834)	(0.458282)	(-1.44587)	(0.859745)	(-1.07654)	(0.499935)
Director holdings %	0	0	0	0	-0.001	-0.001	0.001	0	0
	(-0.878148)	(0.574338)	(-0.0687775)	(-0.318927)	(-1.0724)	(-0.604603)	(0.793207)	(-0.643289)	(-0.961415)
Log Equity	0.003	-0.002	-0.001	-0.006	0	-0.006	-0.002	-0.007	0.003
	(0.958112)	(-0.537401)	(-0.200036)	(-1.23872)	(-0.0640705)	(-1.12589)	(-0.385976)	(-1.23676)	(1.04903)
ROA	2.92192	-0.923531	2.83253	1.38013	3.44727	0.0676263	0.444505	2.24262	3.19879
	(0.002)	(0.003)	(-0.002)	(-0.003)	(0.002)	(-0.007**)	(0.005)	(-0.005)	(0.002)
Market/book	1.30587	0.953262	-0.607647	-1.14304	1.0401	-2.01845	1.59905	-1.50814	1.42981
	(0.020**)	(0.033)	(0.008)	(-0.027*)	(0.009)	(-0.025)	(0.027)	(-0.013)	(-0.016***)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Proposal type FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Sponsor type FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R-squared	0.655	0.468	0.551	0.529	0.647	0.534	0.478	0.564	0
N	11,602	10,851	10,907	11,602	10,131	10,131	9,466	9,521	–

Panel B: Diebold-Mariano comparison between reduced form and baseline forecasts

Forecasts	Participation	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$
MSE algorithm	0.011	0.032	0.032	0.070
MSE Malenko-Shen*	0.442	0.244	0.134	0.277
Difference	-0.431	-0.213	-0.102	-0.207
S (1)	-334.9	-168.7	-104.7	-104.1
p	<0.0000	<0.0000	<0.0000	<0.0000
Out-sample				
MSE baseline estimation	0.024	0.034	0.057	0.122
MSE reduced form	0.532	0.268	0.143	0.296
Difference	-0.507	-0.234	-0.086	-0.174
S (1)	-48.6	-21.8	-10.6	-8.1
p	<0.0000	<0.0000	<0.0000	<0.0000

Table 9: **Robustness**

Panel A reports, for the alternative estimations, the mean squared errors (MSE). *** denote a <1% significant Diebold-Mariano test, showing that the MSE of the variable denoted in the header for the respective estimation is lower than those of the Malenko-Shen estimation reported in row 2 (row 4 for out-of-sample tests). Panel B reports the incidence of each equilibrium for these estimations. Column 1 describes whether the algorithm matches the moments of deciles, the moments of quintiles, the median instead of the mean or instead of a proposal type×decile γ×decile size bin, an ISS recommendation×proposal type×decile γ×decile size bin. Column 2 reports whether the entire sample or a subsample was used, where High/Low activist (private, institutional) ownership or ownership concentration (High/Low forecast std) refer to above/below the median hedge fund ownership (private ownership, institutional ownership) or ownership concentration (analyst forecast standard deviation), Early/Late meetings refer to meetings held before/after the median proxy meeting date, Early/Late meetings/ISS refer to meetings held before/after ISS has issued recommendations for the resp. proposal type in both directions, and equity lending supply/demand refers to the quarterly lowest/highest quintile in equity lending supply/demand.

Panel A: Mean Squared Errors

		Mean Squared Errors			
Algorithm	Sample	t	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$
Base	All	0.011***	0.032***	0.032***	0.07***
Malenko-Shen	All	0.442***	0.244***	0.134***	0.277***
Base	2011 (out-sample)	0.024***	0.034***	0.057***	0.122***
Malenko-Shen	2011 (out-sample)	0.532***	0.268***	0.143***	0.296***
GMM	All	0.013***	0.022***	0.03***	0.072***
Quintiles	All	0.012***	0.022***	0.03***	0.072***
n-decile with count	All	0.012***	0.021***	0.029***	0.068***
Median	All	0.012***	0.023***	0.029***	0.07***
No $1m$	All	0.157***	0.066***	0.168***	0.211***
ISS	All	0.012***	0.015***	0.022***	0.044***
Base	Single proposal	0.013***	0.023***	0.031***	0.075***
Base	Bundle	0.011***	0.015***	0.02***	0.049***
Base	Low activist ownership	0.012***	0.02***	0.026***	0.064***
Base	High activist ownership	0.012***	0.02***	0.027***	0.064***
Base	Low private ownership	0.012***	0.02***	0.027***	0.065***
Base	High private ownership	0.012***	0.021***	0.025***	0.063***
Base	Low institutional ownership	0.014***	0.02***	0.029***	0.07***
Base	High institutional ownership	0.01***	0.019***	0.024***	0.056***
Base	Low ownership concentration	0.01***	0.015***	0.02***	0.048***
Base	High ownership concentration	0.015***	0.027***	0.036***	0.084***
Base	Early meeting	0.012***	0.02***	0.027***	0.064***
Base	Late meeting	0.014***	0.022***	0.028***	0.067***
Base	Early meeting/ISS	0.012***	0.017***	0.023***	0.055***
Base	Late meeting/ISS	0.012***	0.022***	0.029***	0.07***
Base	Low forecast std	0.012***	0.021***	0.029***	0.07***
Base	High forecast std	0.011***	0.017***	0.022***	0.053***
Base	Low equity lending demand	0.01***	0.017***	0.023***	0.056***
Base	High equity lending demand	0.013***	0.022***	0.029***	0.069***
Base	Low equity lending supply	0.016***	0.025***	0.034***	0.085***
Base	High equity lending supply	0.011***	0.018***	0.024***	0.056***

Panel B: Equilibrium incidence

Algorithm	Sample	mm	$1m$	10	11	$m1$	$m0$	Total
		Equilibrium						
Base	All	41	15,545	0	0	80	0	15,666
GMM	All	205	13,908	3	0	11	0	14,127
Quintiles	All	42	16,007	0	0	1	0	16,050
Median	All	45	15,525	0	0	87	0	15,657
n-decile with count	All	30	15,546	0	0	84	0	15,660
No $1m$	All	7,313	0	3,045	5,228	80	0	16,059
ISS	All	237	15,140	0	0	682	0	16,059
Base	Bundle	51	5,461	0	0	42	7	5,561
Base	Single proposal	63	9,004	0	0	24	0	9,091
Base	High activist ownership	48	7,118	0	0	70	7	7,243
Base	Low activist ownership	57	7,367	0	0	31	5	7,460
Base	High private ownership	4	2,086	1	0	93	0	8,663
Base	Low private ownership	32	13,355	0	0	62	0	5,705
Base	High institutional ownership	13	8,008	0	0	61	0	7,526
Base	Low institutional ownership	11	7,275	0	0	132	0	7,418
Base	High ownership concentration	18	6,682	0	0	196	0	6,896
Base	Low ownership concentration	10	8,050	0	0	68	0	8,128
Base	Early meeting	104	11,241	0	0	36	5	11,386
Base	Late meeting	12	3,288	0	0	34	4	3,338
Base	Early meeting/ISS	11	5,563	0	0	131	0	5,705
Base	Late meeting/ISS	2	5,113	0	0	57	0	5,172
Base	High analyst forecast std	8	9,106	0	0	71	0	9,185
Base	Low analyst forecast std	16	8,678	0	0	206	0	8,900
Base	Low equity lending demand	9	5,293	0	0	73	0	4,862
Base	High equity lending demand	40	6,936	0	0	43	0	7,019
Base	Low equity lending supply	42	2,194	0	0	55	0	2,291
Base	High equity lending supply	4	9,960	0	0	58	0	9,575

Table 10: **Proposal Splits**

This table shows the estimates per proposal type when the sample is split into bundled vs. non-bundled proposals (Panel A), high vs low activist ownership companies (Panel B), early vs late shareholder meetings in a particular year (Panel C), high vs low equity lending demand (Panel D) and high vs low equity lending supply (Panel E). In Panel A, we split the sample based on the number of proposals being voted on in a meeting: if a proposal is being voted on in a meeting that contains more than the median number of proposals in any given year, we call those proposals bundled. In Panel B, we split the sample into high and low activist ownership, based on the percentage of shares owned by hedge fund activists. We classify a company as a high activist ownership company if in any given year that the stake of the hedge fund activist ownership is higher than the median activist ownership across all companies in our sample. In Panel C, we split the sample into meetings that are held before the median month in any given year. In Panels D and E, we split the sample into stocks with high (low) equity lending demand (Panel D) and high (low) equity lending supply (Panel E). As described in the main text, we define equity lending supply as the difference between the value of a company's lendable shares and the number of lendable shares currently on loan divided by the firm's market capitalization. We define equity lending demand as the value of shares actually borrowed divided by the firm's market capitalization. We then compare the firms above and below median in terms of the demand and supply of equity lending.

Panel A: Bundled vs. non-bundled proposals

	Single proposal				**Bundle of proposals**			
Proposal type	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$
Shareholder proposals								
Board	0.91	11.1%	-20.0%	8.7%	0.85	7.0%	-25.2%	9.2%
Business	1.76	2.8%	-23.6%	21.5%	1.49	3.5%	-26.1%	17.9%
CSR	2.00	2.7%	-23.3%	25.9%	1.94	1.2%	-28.8%	26.5%
Compensation	1.08	8.1%	-22.1%	10.8%	1.00	6.6%	-25.8%	11.2%
Defense	1.29	7.7%	-20.6%	9.9%	1.20	5.0%	-21.0%	11.8%
Governance	0.97	9.4%	-14.9%	4.3%	0.85	13.4%	-19.7%	6.8%
Restructuring	1.40	1.0%	-29.4%	21.7%	1.40	0.1%	-23.6%	29.7%
Payout					2.72	1.0%	-25.5%	19.1%
Total	1.34	7.0%	-21.3%	14.2%	1.32	5.1%	-25.7%	16.1%
Management proposals								
Board	2.98	0.1%	-15.2%	47.6%	2.90	0.1%	-17.7%	43.7%
Business	2.95	0.1%	-18.0%	54.2%	2.72	0.1%	-20.2%	49.6%
CSR	3.19	0.1%	-12.8%	60.5%	3.33	0.1%	-8.3%	53.2%
Compensation	2.05	5.0%	-22.0%	30.0%	2.23	2.5%	-21.4%	34.5%
Defense	2.94	0.7%	-14.6%	42.6%	3.06	0.3%	-14.0%	42.0%
Governance	2.61	2.3%	-14.0%	37.1%	2.68	1.0%	-10.7%	34.5%
Merger	2.72	0.1%	-22.3%	47.6%	2.54	0.3%	-23.0%	40.9%
Payout	1.70	0.0%	-41.5%	47.3%	2.87	0.1%	-19.2%	51.8%
Restructuring	2.46	2.6%	-15.1%	42.1%	2.50	2.2%	-18.3%	42.5%
SOP	2.43	1.2%	-22.3%	42.5%	2.38	0.5%	-21.0%	38.7%
Total	2.23	3.7%	-21.0%	34.7%	2.44	1.7%	-19.3%	37.4%

Panel B: Low vs. high activist ownership ownership

	Shareholder proposals							
	Low activist ownership				**High activist ownership**			
Proposal type	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$
Board	0.86	7.4%	-25.7%	0.84	10.4%	-18.4%	6.6%	
Business	1.65	2.4%	-26.9%	21.9%	1.48	5.1%	-24.0%	14.6%
CSR	2.00	1.3%	-28.9%	27.8%	1.85	2.2%	-24.5%	23.2%
Compensation	1.04	6.1%	-26.1%	12.3%	1.01	10.2%	-21.1%	9.1%
Defense	1.13	8.6%	-16.8%	7.0%	1.38	6.6%	-22.9%	14.6%
Governance	0.99	12.5%	-19.2%	7.3%	0.75	11.4%	-17.8%	5.2%
Payout	2.50	0.1%	-27.5%	39.1%	3.17	0.0%	-15.9%	11.0%
Restructuring	1.36	0.6%	-28.3%	22.6%	1.38	1.1%	-25.4%	14.4%
Total	1.36	5.3%	-25.5%	16.6%	1.28	7.1%	-21.6%	13.4%
	Management proposals							
Board	2.94	0.1%	-16.6%	44.4%	2.96	0.1%	-15.9%	47.1%
Business	2.71	0.1%	-20.3%	54.2%	2.89	0.1%	-17.5%	47.0%
CSR	3.31	0.0%	-8.5%	58.3%	3.29	0.1%	-10.1%	46.7%
Compensation	2.13	4.5%	-22.1%	31.3%	2.06	4.2%	-21.9%	30.8%
Defense	3.02	0.4%	-13.9%	44.9%	3.05	0.3%	-12.9%	40.8%
Governance	2.64	2.2%	-12.1%	34.8%	2.71	1.0%	-13.7%	40.8%
Merger	2.66	0.1%	-24.2%	47.4%	2.73	0.2%	-21.3%	46.3%
Payout	2.94	0.0%	-18.9%	52.2%	2.38	0.2%	-25.5%	49.4%
Restructuring	2.49	2.5%	-15.7%	41.8%	2.51	1.8%	-16.0%	42.2%
SOP	2.43	1.2%	-23.0%	43.7%	2.41	0.8%	-21.2%	40.0%
Total	2.32	3.1%	-20.5%	36.0%	2.27	2.9%	-20.4%	35.0%

Panel C: Early vs. late shareholder meetings

Shareholder proposals								
	Early meetings				**Late meetings**			
Proposal type	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$
Board	0.84	10%	-24%	9%	1.06	7.2%	-18.8%	10.4%
Business	1.56	3%	-27%	20%	1.64	0.6%	-20.3%	13.9%
CSR	1.91	1%	-28%	26%	2.20	2.0%	-22.8%	30.4%
Compensation	1.01	7%	-25%	12%	0.99	7.6%	-19.7%	8.7%
Defense	1.23	9%	-19%	11%	1.35	5.0%	-21.0%	11.3%
Governance	0.85	14%	-21%	7%	0.99	6.3%	-17.4%	3.9%
Payout	2.75	0%	-24%	24%	2.67	0.0%	-22.9%	41.0%
Restructuring	1.41	1%	-27%	20%	1.40	0.9%	-23.3%	23.0%
Total	1.31	6%	-25%	16%	1.44	5.0%	-20.3%	15.7%
Management proposals								
Board	2.92	0%	-17%	45%	2.84	0.3%	-11.8%	36.2%
Business	2.83	0%	-19%	52%	2.72	0.1%	-21.4%	51.1%
CSR	3.27	0%	-10%	48%	3.49	0.0%	-3.5%	88.3%
Compensation	2.14	4%	-21%	32%	1.97	5.2%	-23.2%	28.5%
Defense	3.03	0%	-14%	45%	2.81	1.0%	-15.4%	37.9%
Governance	2.70	2%	-16%	38%	2.48	2.3%	-11.9%	26.9%
Merger	2.77	0%	-22%	48%	2.65	0.3%	-23.0%	46.0%
Payout	2.72	0%	-22%	49%	3.27	0.1%	-11.1%	65.7%
Restructuring	2.50	2%	-15%	41%	2.51	1.3%	-16.6%	43.4%
SOP	2.45	1%	-22%	42%	2.28	1.9%	-24.6%	41.2%
Total	2.33	3%	-20%	36%	2.17	3.7%	-21.6%	33.1%

Panel D: Low vs. high equity lending demand

Shareholder proposals

	Low demand				**High demand**			
Proposal type	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$
Board	0.85	6%	-25%	11%	0.87	13%	-19%	6%
Business	1.41	2%	-25%	17%	1.95	0%	-27%	28%
CSR	1.94	1%	-28%	-26%	1.98	2%	-25%	-26%
Compensation	1.07	7%	-24%	11%	0.98	10%	-24%	10%
Defense	1.25	6%	-20%	11%	1.21	8%	-20%	9%
Governance	0.81	15%	-19%	6%	0.97	9%	-21%	6%
Payout	2.72	0%	-24%	-30%	.	.	.	.
Restructuring	1.21	1%	-26%	20%	1.65	1%	-27%	17%
Total	1.34	5%	-25%	16%	1.32	7%	-22%	14%
Management proposals								
Board	2.89	0%	-14%	43%	3.04	0%	-15%	50%
Business	2.96	0%	-14%	56%	2.94	0%	-17%	47%
CSR	3.33	0%	-8%	53%	3.19	0%	-13%	61%
Compensation	2.30	3%	-20%	34%	1.98	4%	-23%	29%
Defense	2.02	0%	-14%	40%	2.94	0%	-15%	43%
Governance	2.72	0%	-10%	36%	2.60	1%	-14%	37%
Merger	2.73	0%	-14%	46%	2.67	1%	-23%	44%
Payout	2.66	0%	-23%	60%	1.24	0%	-24%	38%
Restructuring	2.67	1%	-14%	45%	2.48	2%	-17%	43%
SOP	2.43	2%	-21%	42%	2.42	1%	-22%	42%
Total	2.44	2%	-19%	37%	2.19	3%	-21%	34%

Panel E: Low vs. high equity lending supply

Shareholder proposals

	Low supply				High supply			
Proposal type	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	$v/(cn)$	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$
Board	1.02	6%	-28%	15%	0.82	9%	-23%	8%
Business	1.29	1%	-30%	22%	1.62	2%	-25%	20%
CSR	1.70	2%	-32%	29%	1.98	1%	-27%	26%
Compensation	1.07	6%	-26%	13%	1.00	7%	-25%	10%
Defense	0.62	12%	1%	8%	1.31	5%	-24%	14%
Governance	1.04	6%	-22%	7%	0.85	13%	-20%	6%
Payout	2.33	0%	-32%	37%	2.92	0%	-19%	26%
Restructuring	1.03	1%	-37%	21%	1.45	1%	-25%	19%
Total	1.22	5%	-25%	16%	1.34	6%	-25%	15%

Management proposals

Board	2.84	0%	-12%	42%	2.96	0%	-15%	42%
Business	2.92	0%	-16%	61%	3.02	0%	-14%	45%
CSR	3.35	0%	-7%	58%	3.26	0%	-11%	51%
Compensation	1.93	9%	-25%	31%	2.12	3%	-21%	31%
Defense	2.91	1%	-13%	50%	3.04	0%	-14%	40%
Governance	2.23	2%	-6%	19%	2.75	1%	-15%	40%
Merger	2.51	0%	-26%	49%	2.81	0%	-20%	43%
Payout	.	.	.		1.72	0%	-36%	45%
Restructuring	2.46	3%	-20%	50%	2.60	1%	-14%	42%
SOP	2.31	2%	-26%	50%	2.49	0%	-20%	39%
Total	2.16	6%	-22%	38%	2.31	2%	-20%	34%

Table 11: **Proposal type classification**

Proposal Type	Proposal Descriptions
Compensation	Amend Omnibus Stock Plan, Advisory Vote to Ratify Named Exec. Officers' Comp., Approve Omnibus Stock Plan
Say-On-Pay	Advisory Vote on Say-On-Pay Frequency, Bundled Say-On-Pay/Golden Parachute Advisory
Restructuring	Increase Authorized Common Stock, Company Specific-Equity-Related, Approve Reverse Stock Split
Board	Require a Majority Vote for the Election of Board, Require Independent Board Chairman, Restore or Provide for Cumulative Voting
CSR	Political Contributions and Lobbying, Social Proposal, Improve Human Rights Standards or Policy
Defense	Declassify the Board of Directors, Reduce Supermajority Vote Requirement, Submit Shareholder Rights Plan (Poison Pill)
Governance	Amend Articles/Bylaws/Charter-NonRoutine, Amend Articles/Bylaws/Charter-Call Special Meeting, Company Specific-Gov. Related
Merger	Approve Merger Agreement, Approve Acquisition OR Issue Shares in Connection with Acquisition, Approve Sale of Company Assets
Business	Change Company Name, Claw-Back of Payments under Restatement, Company-Specific–Organization-Related
Payout	Approve Allocation of Income and Divide, Approve Dividends, Initiate Payment of Cash Dividend
Other	Company-Specific–Shareholder Miscellaneous, Other Business, Company-Specific-Miscellaneous

Figure 1: Equilibria regions under agreement and disagreement across the group of voters ($\overline{a} = 1$, $h - l = 0.2$, $q = 0.9$).

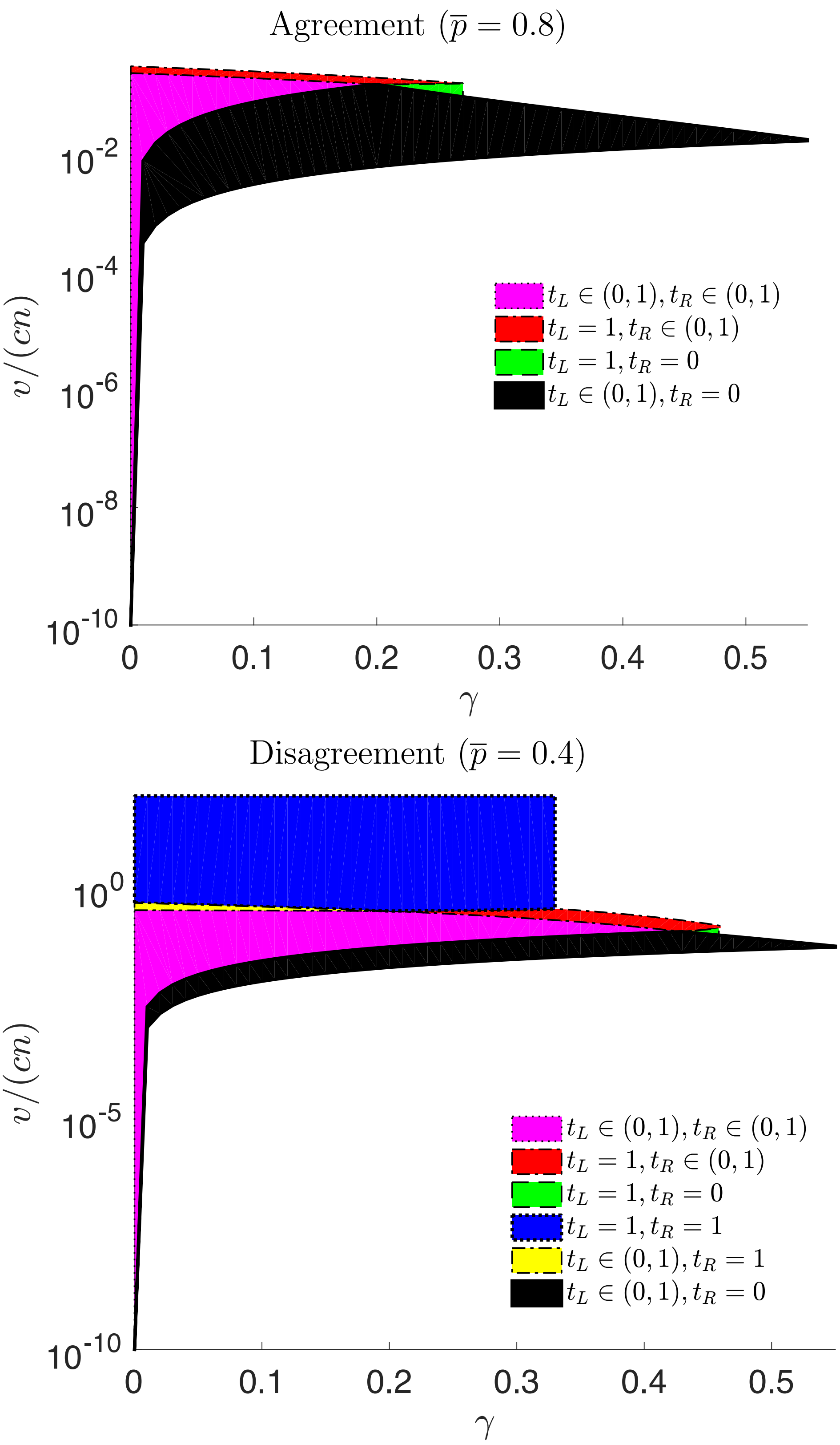
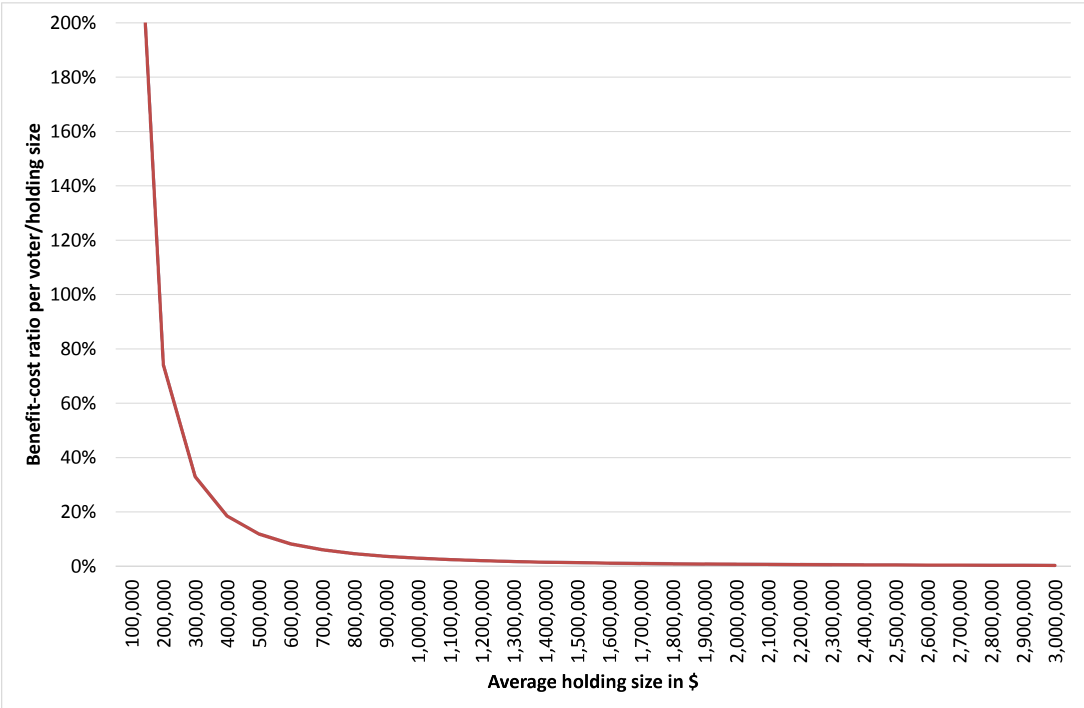


Figure 2: **Benefit-to-cost ratio vs. average holding size.**



Internet Appendix

"Freeriders and Underdogs: Participation in Corporate Voting"

Dragana Cvijanovic, Moqi Groen-Xu, and Konstantinos E. Zachariadis

In this Online Appendix, we present the derivations of the equilibria where at least one type of discretionary voter has a 'corner' participation rate.

First, we list the propositions for all the equilibria, and then, we present the corresponding proofs. Throughout, assume that $q \in (1/2, 1)$, $\overline{a} \in (0, 1]$, and $g(a) = \delta(a - \overline{a})$.

Proposition 3 (Equilibrium $1m$). *Assume that $p \sim \mathcal{U}[l, h]$, $(l, h) \subseteq (0, 1)$ and let $d \equiv h - l$. If and only if*

$$n \in N_{1m} \equiv \left(\max\left\{ \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d}, \frac{1}{\overline{p}d} \frac{l^2}{(1-\gamma)\overline{a} - (2q-1)\gamma} \right\}, \infty \right), \quad \text{(IA.1)}$$

$$\gamma \in \Gamma_{1m} \equiv \left(\frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p}) + 2q - 1} \mathbb{I}\left(\overline{p} < \frac{1}{2}\right), \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \right), \quad \text{(IA.2)}$$

$$\begin{aligned} \frac{v}{cn} \in V_{1m} \equiv & \left(\max\left\{ \frac{(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma}, \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\overline{p}} d, \frac{1}{n} \right\}, \right. \\ & \left. \min\left\{ \frac{4(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma}, \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{l^2} \overline{p}d \right\} \right), \end{aligned} \quad \text{(IA.3)}$$

where $\mathbb{I}$ the indicator function and there exists an equilibrium with $t_L = 1$ and $t_R \in (0, 1)$ given by

$$t_R = \sqrt{\frac{\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\frac{dc}{v} n\overline{p}}}{} \frac{1}{(1-\gamma)\overline{a}}} - 1. \quad \text{(IA.4)}$$

Proposition 4 (Equilibrium 10). *Assume that $p \in (l, h) \subseteq (0, 1)$ (not necessarily uniform). If and only if:*

$$n \in N_{10} \equiv \left(\frac{f(p^*)\, p^*}{(1-\gamma)\overline{a}\overline{p}}, \infty \right), \quad \text{(IA.5)}$$

$$\gamma \in \Gamma_{10} \equiv \left(\frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}}, \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \right), \quad \text{(IA.6)}$$

$$\frac{v}{cn} \in V_{10} \equiv \left(\max\left\{ \frac{1}{n}, \frac{(1-\gamma)\overline{a}(1-\overline{p})}{f(p^*)(1-p^*)} \right\}, \frac{(1-\gamma)\overline{a}\overline{p}}{f(p^*)\, p^*} \right), \quad \text{(IA.7)}$$

, there exists an equilibrium with $t_L = 1$ and $t_R = 0$, where from (4); here, $p^* = 1 - (2q -$

$1)\gamma/\left((1-\gamma)\overline{a}\right)$.

Proposition 5 (Equilibrium 11)**.** *Assume that $p\in(l,h)\subseteq(0,1)$ (not necessarily uniform). If and only if:*

$$l<\frac{1}{2},\ h<\frac{\overline{a}+2q-1}{2\overline{a}},$$

and

$$\begin{aligned}
n\in N_{11} &\equiv (0,\infty), && \text{(IA.8)}\\
\gamma\in\Gamma_{11} &\equiv \left(\max\left\{0,\frac{\overline{a}(1-2h)}{\overline{a}(1-2h)+2q-1}\right\},\frac{\overline{a}(1-2l)}{\overline{a}(1-2l)+2q-1}\right), && \text{(IA.9)}\\
\frac{v}{cn}\in V_{11} &\equiv \left(\max\left\{\frac{(1-\gamma)\overline{ap}}{f\left(p^*\right)\frac{p^*}{2}},\frac{(1-\gamma)\overline{a}(1-\overline{p})}{f\left(p^*\right)\frac{1-p^*}{2}},\frac{1}{n}\right\},\infty\right), && \text{(IA.10)}
\end{aligned}$$

, there exists an equilibrium with $t_L=1$ and $t_R=1$, where from (4); here, $p^*=1/2-(2q-1)\gamma/\left(2(1-\gamma)\overline{a}\right)$.

Proposition 6 (Equilibrium $m1$)**.** *Assume that $p\sim\mathcal{U}[l,h]$, $(l,h)\subseteq(0,1)$ and let $d\equiv h-l$. Iff and only if*

$$\overline{p}<\frac{1}{2},$$

and

$$\begin{aligned}
n\in N_{m1} &\equiv \left(\max\left\{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})},\frac{(1-h)^2}{d(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}\right\},\infty\right), && \text{(IA.11)}\\
\gamma\in\Gamma_{m1} &\equiv \left(0,\frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}\right), && \text{(IA.12)}\\
\frac{v}{cn}\in V_{m1} &\equiv \left(\max\left\{\frac{1}{n},d\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{(1-\overline{p})}\right\},\right.\\
&\quad \left.\min\left\{d\frac{4(1-\gamma)^2\overline{a}^2(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma},d\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}\right\}\right), && \text{(IA.13)}
\end{aligned}$$

, there exists an equilibrium with $t_L\in(0,1)$ given by

$$t_L = \sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}}-1, \qquad \text{(IA.14)}$$

and $t_R=1$.

Proposition 7 (Equilibrium m0)**.** *Assume that* $p \sim \mathcal{U}[l,h]$*,* $(l,h) \subseteq (0,1)$ *and let* $d \equiv h - l$*, If and only if*

$$
\begin{aligned}
n \in N_{m0} &\equiv \left(\max\left\{\frac{(2q-1)\gamma}{d(1-\gamma)^2\overline{a}^2(1-\overline{p})}, \frac{1-\overline{p}}{d(2q-1)\gamma}\right\}, \infty\right), && \text{(IA.15)}\\
\gamma \in \Gamma_{m0} &\equiv \left(0, \frac{\overline{a}}{2q-1+\overline{a}}\right), && \text{(IA.16)}\\
\frac{v}{cn} \in V_{m0} &\equiv \left(\max\left\{\frac{1}{n}, d(1-\overline{p})(2q-1)\gamma\right\}, \min\left\{d\frac{(1-\gamma)^2\overline{a}^2(1-p)}{(2q-1)\gamma}, \frac{d(2q-1)\gamma}{1-\overline{p}}\right\}\right), && \text{(IA.17)}
\end{aligned}
$$

, there exists an equilibrium with $t_L \in (0,1)$ *given by*

$$
t_L = \sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}}\frac{1}{(1-\gamma)\overline{a}}} \qquad \text{(IA.18)}
$$

and $t_R = 0$*.*

Proposition 8 (Equilibrium 00)**.** *If and only if*

$$
\begin{aligned}
\gamma \in \Gamma_{00} &\equiv \left(\frac{1}{2q}, \infty\right), \ \textit{or} && \text{(IA.19)}\\
\frac{v}{cn} \in V_{00} &\equiv \left(0, \frac{1}{n}\right), && \text{(IA.20)}
\end{aligned}
$$

, there exists an equilibrium with $t_L = 0$ *and* $t_R = 0$*.*

The last proposition, that is, the no participation equilibrium, follows directly from a violation of either Assumption 1 or Assumption 2.

IA.0.1 Proofs of Propositions 3–7

Note that: $t_L = t_R = 0$ is ruled out from Assumptions 1 and 2; also recall that $t_L = 0$ cannot happen in equilibrium, as this would imply that p^* in (4) is negative, but $p^* \in (0,1)$. Hence, the equilibria we need to inquire about are:

$$t_L = 1, t_R \in (0,1), \tag{1m}$$

$$t_L = 1, t_R = 0, \tag{10}$$

$$t_L = 1, t_R = 1, \tag{11}$$

$$t_L \in (0,1), t_R = 1, \tag{m1}$$

$$t_L \in (0,1), t_R = 0. \tag{m0}$$

For equilibria 1m, 10, and 11 where $t_L = 1$, we have:

$$K = 1 - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}.$$

We need $K > 0$, otherwise $p^* < 0$, i.e.,

$$\begin{aligned} 1 > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} &\iff \overline{a} - \overline{a}\gamma > (2q-1)\gamma \\ \iff \overline{a} > (2q-1+\overline{a})\gamma &\iff \gamma < \frac{\overline{a}}{2q-1+\overline{a}}. \end{aligned} \tag{IA.21}$$

Now,

$$p^* = K\frac{1}{1+t_R}.$$

We also need $p^* < 1$, i.e.,

$$\begin{aligned} K\frac{1}{1+t_R} < 1 &\iff t_R > K - 1 \\ &\iff t_R > -\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}. \end{aligned}$$

Also note that $K < 1$.

For $t_L = 1$, we need:

$$\begin{aligned}
\mathbb{P}\left[\text{pivotal}|L\right] &> \frac{c}{v} \\
\iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})}\frac{f(p^*)(1-p^*)}{1+t_R} &> \frac{c}{v} \\
\iff \frac{f(p^*)(1-p^*)}{1+t_R} &> \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
\iff f\left(K\frac{1}{1+t_R}\right)\frac{1+t_R-K}{(1+t_R)^2} &> \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}). \qquad \text{(IA.22)}
\end{aligned}$$

Equilibrium 1m: For $t_R \in (0,1)$, we need:

$$\begin{aligned}
\mathbb{P}\left[\text{pivotal}|R\right] &= \frac{c}{v} \\
\iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}\frac{f(p^*)p^*}{1+t_R} &= \frac{c}{v} \\
\iff f\left(K\frac{1}{1+t_R}\right)\frac{K}{(1+t_R)^2} &= \frac{c}{v}(1-\gamma)n\overline{ap}. \qquad \text{(IA.23)}
\end{aligned}$$

Let us assume that $p \sim \mathcal{U}[l,h], 0 \leq l < h \leq 1$; then, $\overline{p} = \frac{h+l}{2}, f(p) = \frac{1}{h-l} = \frac{1}{d}$ and (IA.23) becomes:

$$\begin{aligned}
\frac{1}{d}\frac{K}{(1+t_R)^2} &= \frac{c}{v}(1-\gamma)n\overline{ap} \\
\iff (1+t_R)^2 &= \frac{K}{\frac{dc}{v}(1-\gamma)n\overline{ap}} = \frac{\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{(1-\gamma)\overline{a}}}{\frac{dc}{v}(1-\gamma)n\overline{ap}} \\
\iff (1+t_R)^2 &= \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\frac{dc}{v}n\overline{p}(1-\gamma)^2\overline{a}^2} \\
\iff t_R &= \sqrt{\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\frac{dc}{v}n\overline{p}}}\frac{1}{(1-\gamma)\overline{a}} - 1. \qquad \text{(IA.24)}
\end{aligned}$$

Need: $t_R < 1$

$$\begin{aligned}
&\iff \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\frac{dc}{v}n\overline{p}}\frac{1}{(1-\gamma)^2\overline{a}^2} < 4 \\
&\iff \frac{v}{c} < d\frac{4(1-\gamma)^2\overline{a}^2n\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \qquad \text{(IA.25)}
\end{aligned}$$

and $t_R > 0$

$$
\begin{aligned}
&\iff \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\frac{dc}{v} n\overline{p}} > (1-\gamma)^2\overline{a}^2 \\
&\iff \frac{v}{c} > d\frac{(1-\gamma)^2\overline{a}^2 n\overline{p}}{(1-\gamma)\overline{a} - (2q-1)\gamma}. \qquad \text{(IA.26)}
\end{aligned}
$$

In addition, from (IA.22), for $p \sim \mathcal{U}[l,h]$ we have that:

$$
\begin{aligned}
&\frac{1}{d}\frac{1+t_R-K}{(1+t_R)^2} > \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
\iff &\frac{1}{1+t_R} - \frac{K}{(1+t_R)^2} > \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
\iff &\sqrt{\frac{\frac{dc}{v}n\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{dc}{v}(1-\gamma)n\overline{a}\overline{p} > \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
\iff &\sqrt{\frac{v}{dcn}\frac{\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}} > \overline{p} + 1 - \overline{p} \\
\iff &\frac{v}{c} > nd\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}. \qquad \text{(IA.27)}
\end{aligned}
$$

We need to check whether (IA.25)-(IA.27) are consistent with each other:

$$
\begin{aligned}
&4(1-\gamma)^2\overline{a}^2\overline{p}^2 > [(1-\gamma)\overline{a}-(2q-1)\gamma]^2 \\
\iff &2(1-\gamma)\overline{ap} > (1-\gamma)\overline{a} - (2q-1)\gamma \\
\iff &(1-\gamma)\overline{a}(1-2\overline{p}) - (2q-1)\gamma < 0 \\
\iff &\overline{a}(1-2\overline{p}) < \gamma\left[\overline{a}(1-2\overline{p}) + 2q - 1\right].
\end{aligned}
$$

Case A: $1 - 2\overline{p} > 0 \implies \overline{p} < \frac{1}{2}$ then:

$$
\text{need} \quad \gamma > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p}) + 2q - 1}. \qquad \text{(IA.28)}
$$

For this to be consistent with (IA.21), we need:

$$\begin{aligned}
&\frac{\overline{a}}{2q-1+\overline{a}} > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1} \\
&\iff \overline{a}(1-2\overline{p})+2q-1 > (1-2\overline{p})(2q-1)+\overline{a}(1-2\overline{p}) \\
&\iff 1 > 1-2\overline{p} \iff 2\overline{p} > 0 \iff \overline{p} > 0. \qquad \text{OK.}
\end{aligned}$$

Case B: $1-2\overline{p} < 0 \iff \overline{p} > \frac{1}{2}$,

$$\begin{aligned}
&\text{but} \quad \overline{a}(1-2\overline{p})+2q-1 > 0 \\
&\iff 2\overline{p} < \frac{2q-1+\overline{a}}{\overline{a}} \\
&\iff \overline{p} < \frac{2q-1}{2\overline{a}} + \frac{1}{2}.
\end{aligned}$$

Hence, for $\frac{1}{2} < \overline{p} < \frac{1}{2} + \frac{2q-1}{2\overline{a}}$ then we are OK without (IA.28).

Case C: $1-2\overline{p} < 0 \iff \overline{p} > \frac{1}{2}$,

$$\begin{aligned}
&\text{but} \quad \overline{a}(1-2\overline{p})+2q-1 < 0 \\
&\iff \overline{p} > \frac{2q-1}{2\overline{a}} + \frac{1}{2} \quad \text{then need} \\
&\iff \overline{a}(2\overline{p}-1) > \gamma\left[\overline{a}(2\overline{p}-1)-(2q-1)\right] \\
&\iff \gamma < \frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)}.
\end{aligned} \tag{IA.29}$$

(IA.29) vs. (IA.21):

$$\begin{aligned}
&\frac{2\overline{p}-1}{\overline{a}(2\overline{p}-1)-(2q-1)} < \frac{1}{2q-1+\overline{a}} \\
&\iff (2\overline{p}-1)(2q-1)+(2\overline{p}-1)\overline{a} < (2\overline{p}-1)\overline{a}-(2q-1) \\
&\iff 2\overline{p}-1 < -1 \iff 2\overline{p} < 0 \iff \overline{p} < 0. \qquad \text{NO.}
\end{aligned}$$

Hence, (IA.29) is weaker than (IA.21) and does not take precedence.

Now, we consider (IA.26) vs. (IA.27):

$$\begin{aligned}&(1-\gamma)^2\overline{a}\overline{p}^2 > [(1-\gamma)\overline{a}-(2q-1)\gamma]^2\\ \iff &\overline{a}(1-\overline{p}) < \gamma\left[\overline{a}(1-\overline{p})+2q-1\right]\\ \iff &\gamma > \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1}.\end{aligned}$$

Is this consistent with (IA.21)?

$$\begin{aligned}&\frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1} < \frac{\overline{a}}{2q-1+\overline{a}}\\ \iff &(1-\overline{p})(2q-1)+(1-\overline{p})\overline{a} < \overline{a}(1-\overline{p})+2q-1\\ \iff &1-\overline{p} < 1 \iff \overline{p} > 0. \qquad \text{OK.}\end{aligned}$$

Hence, both (IA.26) and (IA.27) can be true.

Summary up to this point:

Then, $t_L = 1, t_R \in (0,1)$ exists iff:

$$max\left\{\frac{(1-\gamma)^2\overline{a}^2 n\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}nd\right\} < \\ < \frac{v}{c} < d\frac{4(1-\gamma)^2\overline{a}^2 n\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \quad \text{(IA.30)}$$

$$\begin{aligned}\text{and if} \quad &\overline{p} < \frac{1}{2} \qquad \gamma > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p}+2q-1)}, \gamma < \frac{\overline{a}}{2q-1+\overline{a}},\\ \text{if} \quad &\frac{1}{2} < \overline{p} < \frac{1}{2}+\frac{2q-1}{\overline{a}} \qquad \gamma < \frac{\overline{a}}{2q-1+\overline{a}},\\ \text{if} \quad &\overline{p} > \frac{1}{2}+\frac{2q-1}{2\overline{a}} \qquad \gamma < \frac{\overline{a}}{2q-1+\overline{a}},\end{aligned}$$

, then t_R is given by (IA.4).

Next, for the requirement that $\gamma < \frac{1}{2q}$, check with (IA.21):

$$
\begin{aligned}
\frac{\overline{a}}{2q - 1 + \overline{a}} &< \frac{1}{2q} \\
\iff 2q\overline{a} &< 2q - 1 + \overline{a} \\
\iff (2q-1)\overline{a} &< 2q - 1 \\
\iff \overline{a} &< 1. \qquad \text{OK.}
\end{aligned}
$$

Next, for the requirement that $\frac{v}{c} \geq 1$, check with (IA.30), for the existence we need:

$$
\begin{aligned}
d4(1-\gamma)^2\overline{a}n\overline{p} &> (1-\gamma)\overline{a} - (2q-1)\gamma \\
\iff n &> \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d}.
\end{aligned}
\tag{IA.31}
$$

Next, for the lower bound, check with (IA.26):

$$
\begin{aligned}
\text{relative to 1 (one)} \rightarrow \quad & n > \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{(1-\gamma)^2\overline{a}^2\overline{p}d}, \\
\text{and relative to (IA.27)} \rightarrow \quad & \gamma > \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1}.
\end{aligned}
$$

In this case, the first lower bound given by (IA.26) is active.

$$
\begin{aligned}
\text{relative to 1 (one)} \rightarrow \quad & n > \frac{\overline{p}}{d\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]}, \\
\text{and relative to (IA.26)} \rightarrow \quad & \gamma < \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1}.
\end{aligned}
$$

In this case, the second lower bound given by (IA.27) is active.

$$
\begin{aligned}
\text{relative to (IA.26)} \rightarrow \quad & n < \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{(1-\gamma)^2\overline{a}^2\overline{p}d}, \\
\text{and relative to (IA.27)} \rightarrow \quad & n < \frac{\overline{p}}{\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]d}.
\end{aligned}
$$

In this case, 1 (one) is the relative low bound.

Recall that:

$$p^* = \frac{K}{1 + t_R},$$

where:

$$K = 1 - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}.$$

Since $p \in [l, h]$ for equilibrium 1m $[t_L = 1, t_R \in (0,1)]$, we need to also check that:

$$0 \leq l < p^* < h \leq 1. \tag{IA.32}$$

We have:

$$\begin{aligned}
t_R &= \sqrt{\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\frac{dc}{v} n\overline{p}}} \frac{1}{(1-\gamma)\overline{a}} - 1 \\
\iff \frac{1}{1+t_R} &= \sqrt{\frac{\frac{dc}{v} n\overline{p}}{(1-\gamma)\overline{a} - (2q-1)\gamma}} (1-\gamma)\overline{a} \\
\iff \frac{K}{1+t_R} &= \sqrt{\frac{\frac{dc}{v} n\overline{p}}{(1-\gamma)\overline{a} - (2q-1)\gamma}} \left[(1-\gamma)\overline{a} - (2q-1)\gamma\right] \\
\iff \frac{K}{1+t_R} &= \sqrt{\frac{dc}{v} n\overline{p} \left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]} \\
\iff \frac{1+t_R}{K} &= \sqrt{\frac{v}{nc} \frac{1}{d\overline{p}} \frac{1}{\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]}} \\
\iff \left(\frac{1+t_R}{K}\right)^2 &= \frac{v}{nc} \frac{1}{d\overline{p}} \frac{1}{\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]}.
\end{aligned}$$

Hence, (IA.32) becomes:

$$\begin{aligned}
& l < p^* < h \qquad \text{(IA.33)} \\
\iff & \frac{1}{h^2} < \frac{1}{p^{*2}} < \frac{1}{l^2} \\
\iff & \frac{1}{h^2} < \frac{v}{nc} \frac{1}{d\overline{p}} \frac{1}{\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]} < \frac{1}{l^2} \\
\iff & \frac{d\overline{p}\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]}{h^2} \qquad \text{(IA.34)} \\
& < \frac{v}{nc} < \\
& \frac{d\overline{p}\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]}{l^2}. \qquad \text{(IA.35)}
\end{aligned}$$

How does this reconcile with (IA.30)?

(IA.34) vs. second term in max for the lower bound in (IA.30):

$$
\begin{aligned}
&\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}d \overset{?}{>} d\overline{p}\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{h^2} \\
\iff& \frac{1}{\overline{p}} > \frac{\overline{p}}{h^2} \\
\iff& \overline{p}^2 < h^2 \\
\iff& \left(\frac{h+l}{2}\right)^2 < h^2,
\end{aligned}
$$

which is always true; hence, (IA.34) is never relevant.

(IA.35) vs. upper bound in (IA.30):

$$
\begin{aligned}
&\frac{d\overline{p}\left[(1-\gamma)\overline{a}-(2q-1)\gamma\right]}{l^2} \overset{?}{>} \frac{4(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma} \\
\iff& \left[(1-\gamma)\overline{a}-(2q-1)\gamma\right]^2 > 4(1-\gamma)^2\overline{a}^2 l^2 \\
\iff& (1-\gamma)\overline{a}-(2q-1)\gamma > 2(1-\gamma)\overline{a}l \\
\iff& (1-\gamma)\overline{a}\left[1-2l\right] > (2q-1)\gamma \\
\iff& (1-2l)\overline{a} > (2q-1+\overline{a}-2l\overline{a})\gamma \\
\iff& (1-2l)\overline{a} > (2q-1+\overline{a}(1-2l))\gamma.
\end{aligned}
$$

Case i) $1-2l>0 \iff l<\frac{1}{2}$ then we need

$$
\gamma < \frac{(1-2l)\overline{a}}{2q-1+\overline{a}(1-2l)}
$$

for the current upper bound to be relevant.

Case ii) $1 - 2l < 0 \iff l > \frac{1}{2}$, then:

ii-1) $\quad 2q - 1 + \overline{a}(1 - 2l) > 0 \implies \overline{a} < \dfrac{2q-1}{2l-1}$ then (IA.35) is relevant upper bound.

ii-2) $\quad 2q - 1 + \overline{a}(1 - 2l) < 0 \implies \overline{a} > \dfrac{2q-1}{2l-1}$,

then we need $\gamma > \dfrac{(2l-1)\overline{a}}{1 - 2q + \overline{a}(2l-1)}$ for the current upper bound to be relevant.

We need to ensure that the new upper bound is larger than the existing lower bound of v/c:

vs 1 (one):

$$\begin{aligned} &\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{l^2} n\overline{p}d > 1 \\ &\implies n > \frac{1}{\overline{p}d} \frac{l^2}{(1-\gamma)\overline{a} - (2q-1)\gamma}. \end{aligned} \tag{IA.36}$$

vs. the first term in max for the lower bound in (IA.30):

$$\begin{aligned} &\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{l^2} n\overline{p}d > \frac{(1-\gamma)^2 \overline{a}^2 n\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma} \\ &\implies (1-\gamma)\overline{a} - (2q-1)\gamma > (1-\gamma)\overline{a}l \\ &\iff (1-\gamma)\overline{a}(1-l) > (2q-1)\gamma \\ &\iff \overline{a}(1-l) > [2q - 1 + \overline{a}(1-l)]\,\gamma \\ &\iff \gamma < \frac{\overline{a}(1-l)}{2q - 1 + \overline{a}(1-l)}. \end{aligned}$$

vs. the second term in max for the lower bound in (IA.30):

$$\begin{aligned} &\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{l^2} n\overline{p}d > \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\overline{p}} nd \\ &\implies \overline{p}^2 > l^2 \\ &\implies \overline{p} > l. \checkmark \end{aligned}$$

How does (IA.36) compare with the existing bound on n?

$$\frac{1}{\overline{p}d}\frac{l^2}{(1-\gamma)\overline{a}-(2q-1)\gamma} > \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d}$$
$$\implies 2l(1-\gamma)\overline{a} > (1-\gamma)\overline{a}-(2q-1)\gamma$$
$$\iff (1-\gamma)\overline{a}(1-2l) < (2q-1)\gamma.$$

If $1-2l<0 \implies l>\frac{1}{2}$ always true.

If $1-2l>0 \implies l<\frac{1}{2}$, then we need:

$$\gamma > \frac{\overline{a}(1-2l)}{2q-1+\overline{a}(1-2l)},$$

which is acceptable if:

$$2l > l$$
$$\implies l > 0 \quad \text{OK.}$$

Therefore, both can be true for $l<\frac{1}{2}$.

Hence, the upper bound in (IA.30) needs to be amended to:

$$\min\left\{\frac{4(1-\gamma)^2\overline{a}^2 n\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{l^2}n\overline{p}d\right\}.$$

This concludes the proof of Proposition 3.

Note that at this equilibrium, we have:

$$t_{total} = \gamma\overline{t} + (1-\gamma), \quad \text{where:}$$
$$\overline{t} = \overline{a}(\overline{p}t_R + (1-\overline{p})t_L)$$
$$\iff \overline{t} = \overline{a}\left(\sqrt{\frac{v}{cnd}\overline{p}\left[(1-\gamma)\overline{a}-(2q-1)\gamma\right]}\frac{1}{(1-\gamma)\overline{a}} - \overline{p} + (1-\overline{p})\right)$$
$$\iff \overline{t} = \sqrt{\frac{v}{cnd}\overline{p}\left(\frac{\overline{a}}{(1-\gamma)} - \frac{(2q-1)\gamma}{(1-\gamma)^2}\right)} + \overline{a}(1-2\overline{p}).$$

And:

$$\text{sign}\left(\frac{\partial \bar{t}}{\partial \gamma}\right) = \text{sign}\left(\frac{\partial\left(\sqrt{\frac{v}{cnd}\overline{p}\left(\frac{\overline{a}}{(1-\gamma)} - \frac{(2q-1)\gamma}{(1-\gamma)^2}\right)} + \overline{a}(1-2\overline{p})\right)}{\partial \gamma}\right) \quad \text{and}$$

$$\begin{aligned}
\frac{\partial\left(\frac{\overline{a}}{(1-\gamma)} - \frac{(2q-1)\gamma}{(1-\gamma)^2}\right)}{\partial \gamma} &= \frac{\overline{a}}{(1-\gamma)^2} - \frac{(2q-1)(1-\gamma)^2 + (2q-1)\gamma 2(1-\gamma)}{(1-\gamma)^4} \\
&= \frac{\overline{a}}{(1-\gamma)^2} - \frac{(2q-1)\left[1-\gamma+2\gamma\right]}{(1-\gamma)^3} \\
&= \frac{\overline{a}(1-\gamma) - (2q-1)(1+\gamma)}{(1-\gamma)^3}
\end{aligned}$$

$$\begin{aligned}
\Longleftrightarrow \frac{\partial \bar{t}}{\partial \gamma} < 0 \Longleftrightarrow\ & \overline{a}(1-\gamma) < (2q-1)(1+\gamma) \\
\Longleftrightarrow\ & \overline{a} - \overline{a}\gamma < (2q-1) + (2q-1)\gamma \\
\Longleftrightarrow\ & \overline{a} - (2q-1) < (2q-1+\overline{a})\gamma \\
\Longleftrightarrow\ & \gamma > \frac{\overline{a} - (2q-1)}{2q-1+\overline{a}}.
\end{aligned}$$

If $\overline{a} < 2q-1$ always true!

If $\overline{a} > 2q-1$, then for a very small γ, $\bar{t}(t_R)$ might increase with γ. (as in equilibrium mm; see Proposition 1). ■

Equilibrium 10. For $t_R = 0$, we need from (IA.23):

$$\begin{aligned}
\mathbb{P}\left[\text{pivotal}|R\right] &< \frac{c}{v} \\
\Longleftrightarrow \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}\frac{f(p^*)p^*}{1+0} &< \frac{c}{v},
\end{aligned}$$

where from $p^* = \frac{K}{1+t_R}$ for $t_R = 0$, we have $p^* = K$

$$\begin{aligned}
&\Longleftrightarrow \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}f(K)K < \frac{c}{v} \\
&\Longleftrightarrow \frac{c}{v} > \frac{f(K)K}{(1-\gamma)n\overline{ap}}, \text{where } K = 1 - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}.
\end{aligned}$$

While for $t_L = 1$ we need from (IA.22):

$$\frac{c}{v} < \frac{f(K)(1-K)}{(1-\gamma)n\overline{a}(1-\overline{p})},$$

For existence, we need:

$$\begin{aligned}
&\frac{f(K)K}{(1-\gamma)n\overline{ap}} < \frac{f(K)(1-K)}{(1-\gamma)n\overline{a}(1-\overline{p})} \\
\iff &\left(1 - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\right)(1-\overline{p}) < \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\overline{p} \\
\iff &1 - \overline{p} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} < 0 \\
\iff &(1-\overline{p})(1-\gamma)\overline{a} - (2q-1)\gamma < 0 \\
\iff &\left[2q - 1 + \overline{a}(1-\overline{p})\right]\gamma > (1-\overline{p})\overline{a} \\
\iff &\gamma > \frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}}.
\end{aligned}$$

Is this consistent with (IA.21), i.e., for existence we need:

$$\begin{aligned}
&\frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}} < \frac{\overline{a}}{2q-1+\overline{a}} \\
\iff &(1-\overline{p})\overline{a}\left[2q-1+\overline{a}\right] < (2q-1+(1-\overline{p})\overline{a})\overline{a} \\
\iff &(1-\overline{p}-1)\overline{a}(2q-1) < 0 \\
\iff &-\overline{pa}(2q-1) < 0. \qquad \text{OK.}
\end{aligned}$$

Assumption (IA.21) on γ is satisfied, then what about A1 that $\frac{v}{c} \geq 1$ or $\frac{c}{v} \leq 1$. Hence, for existence, we need:

$$\begin{aligned}
&\frac{f(K)K}{(1-\gamma)n\overline{ap}} \leq 1 \\
\iff &n \geq \frac{f(K)K}{(1-\gamma)\overline{ap}}.
\end{aligned}$$

Given $p \in [l, h]$, we also need to ensure that:

$$l \leq K \leq h \text{ where:}$$

$$K = 1 - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}.$$

So we have:

$$\begin{aligned}
& 1 - h \leq 1 - K \leq 1 - l \\
\Longleftrightarrow\ & 1 - h \leq \frac{2q-1}{\overline{a}} \frac{\gamma}{1-\gamma} \leq 1 - l \\
\Longleftrightarrow\ & \frac{\overline{a}(1-h)}{2q-1} \leq \frac{1}{\frac{1}{\gamma} - 1} \leq \frac{(1-l)\overline{a}}{2q-1} \\
\Longleftrightarrow\ & \frac{2q-1}{\overline{a}(1-l)} \leq \frac{1}{\gamma} - 1 \leq \frac{2q-1}{\overline{a}(1-h)} \\
\Longleftrightarrow\ & \frac{2q-1+\overline{a}(1-l)}{\overline{a}(1-l)} \leq \frac{1}{\gamma} \leq \frac{2q-1+\overline{a}(1-h)}{\overline{a}(1-h)} \\
\Longleftrightarrow\ & \frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)} && \text{(IA.37)} \\
& \leq \gamma \leq \\
& \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)}. && \text{(IA.38)}
\end{aligned}$$

How does (IA.37) compare with the current lower bound?

$$\begin{aligned}
& \frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)} < \frac{\overline{a}(1-\overline{p})}{2q-1+\overline{a}(1-\overline{p})} \\
\Longleftrightarrow\ & \frac{2q-1}{\overline{a}(1-h)} > \frac{2q-1}{\overline{a}(1-\overline{p})} \\
\Longleftrightarrow\ & 1 - h < 1 - \overline{p} \\
\Longleftrightarrow\ & \overline{p} < h. \checkmark
\end{aligned}$$

Therefore, the current lower bound stands.

How does (IA.38) compare with the current upper bound?

$$\begin{aligned}
& \frac{\overline{a}}{2q-1+\overline{a}} < \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \\
\Longleftrightarrow\ & \frac{2q-1}{\overline{a}} > \frac{2q-1}{\overline{a}(1-l)} \\
\Longleftrightarrow\ & 1 < 1 - l \\
\Longleftrightarrow\ & l < 0. \quad \text{X}
\end{aligned}$$

Therefore, (IA.38) takes precedence.

This concludes the proof of Proposition 4. ■

Equilibrium 11. For $t_R = 1$, we need from (IA.23):

$$\begin{aligned} \mathbb{P}\left[\text{pivotal}|R\right] &> \frac{c}{v} \\ \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}}\frac{f(p^*)p^*}{2} &> \frac{c}{v}, \end{aligned}$$

and $p^* = \frac{K}{2}$ for $t_R = 1$; hence:

$$\begin{aligned} \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}}\frac{f\left(\frac{K}{2}\right)K}{4} &> \frac{c}{v} \\ \iff \frac{c}{v} &< \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}}f\left(\frac{K}{2}\right)\frac{K}{4}. \end{aligned}$$

For $t_L = 1$, we need from (IA.22):

$$\begin{aligned} f(\frac{K}{2})\frac{2-K}{4} &> \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\ \iff \frac{c}{v} &< \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})}f\left(\frac{K}{2}\right)\frac{2-K}{4}. \end{aligned}$$

In addition, we also need $\frac{c}{v} \leq 1$.

Given $p \in [l, h]$, we further need to ensure that:

$$\begin{aligned} & l \leq \frac{K}{2} \leq h \\ \iff & 1-2h \leq 1-K \leq 1-2l \\ \iff & 1-2h \leq \frac{2q-1}{\overline{a}}\frac{\gamma}{1-\gamma} \leq 1-2l. \end{aligned}$$

Therefore, for equilibrium to exist, we need $1-2l \geq 0 \iff l \leq \frac{1}{2}$ (and $1-2h \leq 1 \implies h \geq 0$ OK).

Case 1: $1-2h<0 \implies h \geq \frac{1}{2}$; then, we just need:

$$\begin{aligned}
&\frac{2q-1}{\overline{a}}\frac{\gamma}{1-\gamma} \leq 1-2l \\
\iff &\frac{1}{\frac{1}{\gamma}-1} \leq \frac{(1-2l)\overline{a}}{2q-1} \\
\iff &\frac{1}{\gamma}-1 \geq \frac{2q-1}{(1-2l)\overline{a}} \\
\iff &\frac{1}{\gamma} \geq \frac{2q-1+(1-2l)\overline{a}}{(1-2l)\overline{a}} \\
\iff &\gamma \leq \frac{(1-2l)\overline{a}}{(1-2l)\overline{a}+2q-1},
\end{aligned}$$

which also takes precedence over the current upper bound.

Case 2: $1-2h>0 \implies h \leq \frac{1}{2}$; then, we also need:

$$\gamma \geq \frac{(1-2h)\overline{a}}{(1-2h)\overline{a}+2q-1}.$$

We also need to check what happens when $1-2h<0 \implies h>\frac{1}{2}$ and $1-2h<\frac{1-2q}{\overline{a}} \implies h>\frac{\overline{a}+(2q-1)}{2\overline{a}}$.

We need:

$$\begin{aligned}
&\frac{(2h-1)\overline{a}}{\overline{a}(2h-1)+1-2q} < \frac{\overline{a}(1-2l)}{\overline{a}(1-2l)+2q-1} \\
\iff &(2h-1)(2q-1) < (1-2l)(1-2q) \\
\iff &2h-1 < 2l-1 \\
\iff &h < l. \quad \text{X}
\end{aligned}$$

Hence, in that case equilibrium does not exist.

Therefore, we need:

$$h < \frac{\overline{a}+2q-1}{2\overline{a}}.$$

This concludes the proof of Proposition 5. ■

Equilibrium m1. Here, we want to inquire whether $t_L \in (0,1), t_R = 1$ can be an equilibrium: First, we have that:

$$\begin{aligned} p^* &= \frac{t_L}{1+t_L} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \frac{1}{1+t_L} \\ \iff 1 - p^* &= \frac{1}{1+t_L} \underbrace{\left[1 + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\right]}_{L}. \end{aligned}$$

We have $L > 0$ but also need:

$$1 - p^* < 1 \iff 1 + t_L > 1 + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff t_L > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \tag{IA.39}$$

and for an equilibrium to exist: $\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} < 1$

$$\iff (2q-1+\overline{a})\gamma < \overline{a} \iff \gamma < \frac{\overline{a}}{2q-1+\overline{a}}. \tag{IA.40}$$

$$\mathbb{P}\left[\text{pivotal}|R\right] > \frac{c}{v} \iff \frac{1}{(1-\gamma)n} \frac{1}{\overline{a}\overline{p}(1+t_L)} f(p^*)p^* > \frac{c}{v}, \tag{IA.41}$$

$$\text{and } \mathbb{P}\left[\text{pivotal}|L\right] = \frac{c}{v} \iff \frac{1}{(1-\gamma)n} \frac{1}{\overline{a}(1-\overline{p})(1+t_L)} f(p^*)(1-p^*) = \frac{c}{v}. \tag{IA.42}$$

Assume that $p \sim \mathcal{U}[l,h]$ then (IA.42) becomes:

$$\begin{aligned} &\frac{1}{(1-\gamma)n} \frac{1}{\overline{a}(1-\overline{p})} \frac{1}{(1+t_L)} \frac{1}{d} \frac{1}{1+t_L} L = \frac{c}{v} \\ \iff &(1+t_L)^2 = \frac{L}{\frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p})} = \frac{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{(1-\gamma)\overline{a}}}{\frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p})} \\ \iff &(1+t_L)^2 = \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})(1-\gamma)^2\overline{a}^2} \\ \iff &t_L = \sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}} \frac{1}{(1-\gamma)\overline{a}} - 1. \end{aligned} \tag{IA.43}$$

Need: $t_L < 1$

$$\iff \sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}} < 2$$
$$\iff \frac{v}{c} < d\frac{4(1-\gamma)^2\overline{a}^2n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}, \tag{IA.44}$$

and $t_L > 0$

$$\iff \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})} > (1-\gamma)^2\overline{a}^2$$
$$\iff \frac{v}{c} > d\frac{(1-\gamma)^2\overline{a}^2n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}. \tag{IA.45}$$

In addition, from (IA.41), we have for $p \sim \mathcal{U}[l,h]$ that:

$$\begin{aligned}
&\frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}(1+t_L)^2}\frac{1}{d}\left(t_L - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\right) > \frac{c}{v} \\
\iff &\frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}\frac{1}{d}\left(\frac{1}{1+t_L} - \frac{L}{(1+t_L)^2}\right) > \frac{c}{v} \\
\iff &\left(\frac{1}{1+t_L} - \frac{L}{(1+t_L)^2}\right) > \frac{dc}{v}(1-\gamma)n\overline{ap} \\
\iff &\sqrt{\frac{\frac{dc}{v}n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) > \frac{dc}{v}(1-\gamma)n\overline{ap} \\
\iff &\sqrt{\frac{\frac{v}{cdn}(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}} > 1 \iff \frac{v}{c} > nd\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{1-\overline{p}}.
\end{aligned} \tag{IA.46}$$

Let us see how (IA.39) changes in terms of the computed t_L in (IA.14). We have:

$$\sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}} - 1 > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}$$
$$\iff \sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}} > (2q-1)\gamma + (1-\gamma)\overline{a}$$
$$\iff \frac{v}{c} > nd(1-\overline{p})\left[(2q-1)\gamma + (1-\gamma)\overline{a}\right]. \tag{IA.47}$$

Note that since $\frac{1}{1-\overline{p}} > 1-\overline{p} \iff (1-\overline{p})^2 < 1 \iff 1-\overline{p} < 1 \iff \overline{p} > 0$, (IA.46) always supersedes (IA.47) and we do not need to worry about (IA.47).

Now, we need to check whether (IA.44) & (IA.46) are consistent with each other:

$$
\begin{aligned}
& 4(1-\gamma)^2\overline{a}^2(1-\overline{p})^2 \overset{?}{>} [(1-\gamma)\overline{a}+(2q-1)\gamma]^2 \\
\iff & 2(1-\gamma)\overline{a}(1-\overline{p}) \overset{?}{>} (1-\gamma)\overline{a}+(2q-1)\gamma \\
\iff & (1-\gamma)\overline{a}(2-2\overline{p}-1) > (2q-1)\gamma \\
\iff & (1-\gamma)\overline{a}(1-2\overline{p}) > (2q-1)\gamma \\
\iff & \overline{a}(1-2\overline{p}) > \gamma\left[\overline{a}(1-2\overline{p})+2q-1\right].
\end{aligned}
$$

Case A: $1-2\overline{p}>0 \implies \overline{p}<\frac{1}{2}$; then, we need:

$$
\gamma < \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}. \tag{IA.48}
$$

How does this compare with (IA.40)?

$$
\frac{\overline{a}}{2q-1+\overline{a}} > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1} \iff \cdots \iff \overline{p}>0. \qquad \text{OK.}
$$

Therefore, (IA.48) supersedes (IA.40) in this case.

Case B: $1-2\overline{p}<0 \implies \overline{p}>\frac{1}{2}$,

$$
\text{but} \quad \overline{a}(1-2\overline{p})+2q-1>0
$$

$$
\implies \overline{p} < \frac{1}{2}+\frac{2q-1}{2\overline{a}},
$$

then, there is no equilibrium for $\frac{1}{2}<\overline{p}<\frac{1}{2}+\frac{2q-1}{2\overline{a}}$.

Case C: $1-2\overline{p}<0 \implies \overline{p}>\frac{1}{2}$ and

$$
\overline{a}(1-2\overline{p})+2q-1<0 \iff \overline{p} > \frac{1}{2}+\frac{2q-1}{2\overline{a}},
$$

; then, we need

$$\begin{aligned} & \overline{a}(2\overline{p}-1) < \gamma\left[\overline{a}(2\overline{p}-1)-(2q-1)\right] \\ \iff & \gamma > \frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)}. \end{aligned} \tag{IA.49}$$

Is this consistent with (IA.40)? For equilibrium to exist, we need:

$$\begin{aligned} & \frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)} < \frac{\overline{a}}{2q-1+\overline{a}} \\ \iff & (2\overline{p}-1)(2q-1)+\overline{a}(2\overline{p}-1) < (2\overline{p}-1)\overline{a}-(2q-1) \\ \iff & (2q-1)(2\overline{p}-1+1) < 0 \\ \iff & (2q-1)2\overline{p} < 0. \qquad \text{NO,} \end{aligned}$$

; hence, there is no equilibrium for $\overline{p} > \frac{1}{2} + \frac{2q-1}{2\overline{a}}$.

How about (IA.45) vs. (IA.46):

$$\begin{aligned} & (1-\gamma)^2\overline{a}^2(1-\overline{p})^2 \overset{?}{>} [(1-\gamma)\overline{a}+(2q-1)\gamma]^2 \\ \iff & (1-\gamma)\overline{a}(1-\overline{p}) > (1-\gamma)\overline{a}+(2q-1)\gamma \\ \iff & (1-\gamma)\overline{a}(1-\overline{p}-1) > (2q-1)\gamma \\ \iff & -(1-\gamma)\overline{a}\overline{p} > (2q-1)\gamma. \qquad \text{NO.} \end{aligned}$$

Therefore, (IA.46) supersedes (IA.45) and is the only lower bound on $\frac{v}{c}$.

How about A1 that $\gamma < \frac{1}{2q}$ vs. (IA.48)? We have:

$$\begin{aligned} & \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1} < \frac{1}{2q} \quad \text{for} \quad \overline{p} < \frac{1}{2} \implies 1-2\overline{p} > 0 \\ \iff & \overline{a}(2q-1)(1-2\overline{p}) < 2q-1 \\ \iff & \overline{a}(1-2\overline{p}) < 1. \qquad \text{OK.} \end{aligned}$$

Therefore, (IA.48) is the valid γ threshold.

How about A2 that $\frac{v}{c} \geq 1$? Check with (IA.44) for existence:

$$\begin{aligned} 1 &< d\frac{4(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma} \\ \Longleftrightarrow n &> \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})}. \end{aligned} \tag{IA.50}$$

How about with a single lower bound (IA.46)?

$$\text{relative to 1 (one)} \rightarrow n > \frac{(1-\overline{p})}{d\left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]},$$

; in this case, (IA.46) is relevant; otherwise, 1 is relevant.

For the case $t_L \in (0,1), t_R = 1$, we have:

$$p^* = \frac{t_L}{1+t_L} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{1+t_L},$$

where:

$$\begin{aligned} t_L &= \sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}} - 1 \\ \Longleftrightarrow 1+t_L &= \sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}} \\ \Longleftrightarrow \frac{1}{1+t_L} &= (1-\gamma)\overline{a}\sqrt{\frac{\frac{dc}{v}n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}}. \end{aligned}$$

In addition:

$$
\begin{aligned}
& t_L - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \\
=& \frac{1}{(1-\gamma)\overline{a}} \left[\sqrt{\frac{(1-\gamma)\overline{a} + (2q-1)\gamma}{\frac{dc}{v} n(1-\overline{p})}} - ((2q-1)\gamma + (1-\gamma)\overline{a}) \right] \\
=& \frac{\sqrt{(1-\gamma)\overline{a} + (2q-1)\gamma}}{(1-\gamma)\overline{a}} \left[\sqrt{\frac{v}{dcn} \frac{1}{1-\overline{p}}} - \sqrt{(1-\gamma)\overline{a} + (2q-1)\gamma} \right] \\
\Longrightarrow & \frac{t_L - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}}{1+t_L} \\
=& \sqrt{\frac{dcn}{v}(1-\overline{p})} \left[\sqrt{\frac{v}{dcn} \frac{1}{1-\overline{p}}} - \sqrt{(1-\gamma)\overline{a} + (2q-1)\gamma} \right] \\
=& 1 - \sqrt{\frac{dcn}{v}(1-\overline{p}) \left((1-\gamma)\overline{a} + (2q-1)\gamma\right)}.
\end{aligned}
$$

Therefore, given that $p \in [l, h]$, we must also have,

$$
\begin{aligned}
& l < p^* < h \\
\Longleftrightarrow\ & 1 - h < 1 - p^* < 1 - l \\
\Longleftrightarrow\ & (1-h)^2 < (1-p^*)^2 < (1-l)^2 \\
\Longleftrightarrow\ & (1-h)^2 < \frac{dcn}{v}(1-\overline{p}) \left((1-\gamma)\overline{a} + (2q-1)\gamma\right) < (1-l)^2 \\
\Longleftrightarrow\ & \frac{d(1-\overline{p}) \left((1-\gamma)\overline{a} + (2q-1)\gamma\right)}{(1-l)^2} \qquad \text{(IA.51)} \\
& < \frac{v}{nc} < \\
& \frac{d(1-\overline{p}) \left((1-\gamma)\overline{a} + (2q-1)\gamma\right)}{(1-h)^2}. \qquad \text{(IA.52)}
\end{aligned}
$$

(IA.51) vs (IA.46):

$$
\begin{aligned}
& \frac{(1-\gamma)\overline{a} + (2q-1)\gamma}{1-\overline{p}} > \frac{(1-\overline{p}) \left[(1-\gamma)\overline{a} + (2q-1)\gamma\right]}{(1-l)^2} \\
\Longleftrightarrow\ & (1-l)^2 > (1-\overline{p})^2 \\
\Longleftrightarrow\ & 1 - l > 1 - \overline{p} \\
\Longleftrightarrow\ & l < \overline{p} \\
\Longleftrightarrow\ & l < \frac{h+l}{2} \qquad \text{OK.}
\end{aligned}
$$

How about (IA.52) vs. the upper bound in (IA.44)?

$$\begin{aligned}
&\frac{4(1-\gamma)^2\overline{a}^2(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma} \overset{?}{<} \frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2} \\
\iff &4(1-\gamma)^2\overline{a}(1-h)^2 < [(1-\gamma)\overline{a}+(2q-1)\gamma]^2 \\
\iff &2(1-\gamma)\overline{a}(1-h) < (1-\gamma)\overline{a}+(2q-1)\gamma \\
\iff &(1-\gamma)\overline{a}\,[2(1-h)-1] < (2q-1)\gamma \\
\iff &[2(1-h)-1]\,\overline{a} < [2q-1+(2(1-h)-1)\,\overline{a}]\,\gamma.
\end{aligned}$$

The sign depends on whether:

$$2(1-h)-1>0 \implies 1-h>\frac{1}{2} \implies h<\frac{1}{2}$$

or not.

Hence, we need to amend the upper bound for v/c to:

$$\min\left\{\frac{4(1-\gamma)^2\overline{a}^2(1-\overline{p})nd}{(1-\gamma)\overline{a}+(2q-1)\gamma}, \frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)nd}{(1-h)^2}\right\}.$$

We need to ensure that the new upper bound is larger than the existing lower bound :

vs1 (one):

$$\begin{aligned}
&\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}nd > 1 \\
\implies n > &\frac{(1-h)^2}{d(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}. \qquad \text{(IA.53)}
\end{aligned}$$

vs (IA.46):

$$\begin{aligned}
&\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}nd > nd\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{(1-\overline{p})} \\
\implies &(1-\overline{p})^2 > (1-h)^2 \\
\implies &1-\overline{p} > 1-h \\
\implies &\overline{p} < h. \checkmark
\end{aligned}$$

How does (IA.53) compare with the existing bound on n?

$$\begin{aligned}
&\frac{(1-h)^2}{d(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)} > \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})} \\
\implies & 2(1-h)(1-\gamma)\overline{a} > (1-\gamma)\overline{a}+(2q-1)\gamma \\
\implies & (1-\gamma)\overline{a}(2-2h-1) > (2q-1)\gamma \\
\implies & (1-\gamma)\overline{a}(1-2h) > (2q-1)\gamma.
\end{aligned}$$

If $1-2h<0 \implies h>\frac{1}{2}$, the current holds.

If $1-2h>0 \implies h<\frac{1}{2}$, then the current holds if:

$$\begin{aligned}
&(1-\gamma)\overline{a}(1-2h) < (2q-1)\gamma \\
\implies & \overline{a}(1-2h) < (2q-1+\overline{a}(1-2h))\gamma \\
\implies & \gamma > \frac{\overline{a}(1-2h)}{2q-1+\overline{a}(1-2h)}.
\end{aligned}$$

Check:

$$\begin{aligned}
&\frac{\overline{a}(1-2h)}{2q-1+\overline{a}(1-2h)} < \frac{\overline{a}(1-2\overline{p})}{2q-1+\overline{a}(1-2\overline{p})} \\
\iff & \frac{1}{\frac{2q-1}{\overline{a}(1-2h)}+1} < \frac{1}{\frac{2q-1}{\overline{a}(1-2\overline{p})}+1} \\
\implies & \frac{2q-1}{\overline{a}(1-2h)} > \frac{2q-1}{\overline{a}(1-2\overline{p})} \\
\implies & 1-2h < 1-2\overline{p} \\
\implies & h > \overline{p}. \checkmark
\end{aligned}$$

Therefore, both can be true for $h<\frac{1}{2}$.

This concludes the proof of Proposition 6.

Note that at this equilibrium we have:

$$
\begin{aligned}
& t_{total} = \gamma \bar{t} + (1-\gamma) \quad \text{(as always) where:} \\
& \bar{t} = \bar{a}(\bar{p} t_R + (1-\bar{p}) t_L) \\
\iff & \bar{t} = \bar{a}\left(\bar{p} - 1 + \bar{p} + \sqrt{\frac{v}{cnd}(1-\bar{p})\left[(1-\gamma)\bar{a} + (2q-1)\gamma\right]} \frac{1}{(1-\gamma)\bar{a}} \right) \\
\iff & \bar{t} = \bar{a}(2\bar{p} - 1) + \sqrt{\frac{v}{cnd}(1-\bar{p})\left[(1-\gamma)\bar{a} + (2q-1)\gamma\right]} \frac{1}{(1-\gamma)} \\
\iff & \bar{t} = \sqrt{\frac{v}{cnd}(1-\bar{p})\left[\frac{\bar{a}}{1-\gamma} + \frac{(2q-1)\gamma}{(1-\gamma)^2} \right]} + \bar{a}(2\bar{p}-1).
\end{aligned}
$$

And:

$$
\text{sign}\left(\frac{\partial \bar{t}}{\partial \gamma}\right) = \text{sign}\left(\frac{\partial\left(\sqrt{\frac{v}{cnd}(1-\bar{p})\left[\frac{\bar{a}}{1-\gamma} + \frac{(2q-1)\gamma}{(1-\gamma)^2}\right]} + \bar{a}(2\bar{p}-1) \right)}{\partial \gamma} \right) \quad \text{and}
$$

$$
\begin{aligned}
\frac{\partial\left(\frac{\bar{a}}{1-\gamma} + \frac{(2q-1)\gamma}{(1-\gamma)^2} \right)}{\partial \gamma} &= \frac{\bar{a}}{(1-\gamma)^2} + \frac{(2q-1)(1-\gamma)^2 + (2q-1)\gamma 2(1-\gamma)}{(1-\gamma)^4} \\
&= \frac{\bar{a}}{(1-\gamma)^2} + \frac{(2q-1)(1-\gamma) + (2q-1)2\gamma}{(1-\gamma)^3} \\
&= \frac{\bar{a}}{(1-\gamma)^2} + \frac{(2q-1)(1-\gamma+2\gamma)}{(1-\gamma)^3} \\
&= \frac{\bar{a}}{(1-\gamma)^2} + \frac{(2q-1)(1+\gamma)}{(1-\gamma)^3} \\
&= \frac{\bar{a}(1-\gamma) + (2q-1)(1+\gamma)}{(1-\gamma)^3} > 0 \quad \text{always,}
\end{aligned}
$$

; hence, in this equilibrium, $\frac{\partial \bar{t}}{\partial \gamma} > 0$ for all the parameter values in the regions where equilibrium exists. ■

Equilibrium m0. Now, we inquire about the existence of equilibrium with $t_L \in (0,1)$ and $t_R = 0$ Then, p^* becomes:

$$
\begin{aligned}
& p^* = 1 - \frac{(2q-1)\gamma}{1-\gamma} \frac{1}{\bar{a} t_L} \\
\iff & 1 - p^* = \frac{(2q-1)\gamma}{1-\gamma} \frac{1}{\bar{a} t_L}.
\end{aligned}
$$

We have $1 - p^* > 0$ but also need:

$$\begin{aligned} 1 - p^* < 1 &\iff (2q-1)\gamma < (1-\gamma)\overline{a}t_L \\ &\iff t_L > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \end{aligned} \tag{IA.54}$$

and for an equilibrium to exist:

$$\gamma < \frac{\overline{a}}{2q-1+\overline{a}}. \tag{IA.55}$$

$$\mathbb{P}\left[\text{pivotal}|R\right] < \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}\overline{p}}\frac{1}{t_L}f(p^*)p^* < \frac{c}{v}, \tag{IA.56}$$

$$\mathbb{P}\left[\text{pivotal}|L\right] = \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}(1-\overline{p})}\frac{1}{t_L}f(p^*)(1-p^*) = \frac{c}{v}. \tag{IA.57}$$

Assume that $p \sim \mathcal{U}[l,h]$; then, (IA.57) becomes:

$$\frac{1}{(1-\gamma)^2 n\overline{a}^2(1-\overline{p})}\frac{1}{t_L^2}\frac{1}{d}(2q-1)\gamma = \frac{c}{v}$$

$$\begin{aligned} &\iff t_L^2 = \frac{(2q-1)\gamma}{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})d\frac{c}{v}} \\ &\iff t_L = \sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}}}\frac{1}{(1-\gamma)\overline{a}}. \end{aligned} \tag{IA.58}$$

Need $t_L < 1$:

$$\begin{aligned} &\iff \frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}} < (1-\gamma)^2\overline{a}^2 \\ &\iff \frac{v}{c} < d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma}, \end{aligned} \tag{IA.59}$$

and $t_L > 0$, which is always true!

In addition, from (IA.56) we have for $p \sim \mathcal{U}[l,h]$ that:

$$
\begin{aligned}
& \frac{1}{(1-\gamma)n\overline{a}\overline{p}} \frac{1}{d} \frac{1}{t_L} p^* < \frac{c}{v} \\
\iff & \frac{1}{(1-\gamma)n\overline{a}\overline{p}} \frac{1}{d} \frac{1}{t_L} \left(1 - \frac{(2q-1)\gamma}{1-\gamma} \frac{1}{\overline{a}t_L}\right) < \frac{c}{v} \\
\iff & \frac{1}{(1-\gamma)n\overline{a}\overline{p}} \frac{1}{d} \left(\frac{1}{t_L} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \frac{1}{t_L^2}\right) < \frac{c}{v} \\
\iff & \sqrt{\frac{n(1-\overline{p})d\frac{c}{v}}{(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})d\frac{c}{v}}{(2q-1)\gamma} < d\frac{c}{v}(1-\gamma)n\overline{a}\overline{p} \\
\iff & \sqrt{\frac{v}{dcn} \frac{1-\overline{p}}{(2q-1)\gamma}} < 1 - \overline{p} + \overline{p} \\
\iff & \frac{v}{c} < d\frac{n(2q-1)\gamma}{1-\overline{p}}.
\end{aligned}
\tag{IA.60}
$$

Let us see how (IA.54) changes in terms of the computed t_L in (IA.18). We have:

$$
\begin{aligned}
& \sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}}} \frac{1}{(1-\gamma)\overline{a}} > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \\
\iff & \sqrt{\frac{1}{n(1-\overline{p})d\frac{c}{v}}} > \sqrt{(2q-1)\gamma} \\
\iff & \frac{v}{c} > dn(1-\overline{p})(2q-1)\gamma.
\end{aligned}
\tag{IA.61}
$$

Between (IA.59) and (IA.60), which is the relevant bound for v/c?

$$
\begin{aligned}
& d\frac{n(2q-1)\gamma}{1-\overline{p}} < d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} \\
\iff & (2q-1)\gamma < (1-\gamma)\overline{a}(1-\overline{p}) \\
\iff & \gamma\left[2q-1+\overline{a}(1-\overline{p})\right] < \overline{a}(1-\overline{p}) \\
\iff & \gamma < \frac{\overline{a}(1-\overline{p})}{2q-1+\overline{a}(1-\overline{p})}
\end{aligned}
\tag{IA.62}
$$

Hence, if (IA.62) is true, then (IA.60) is relevant; otherwise, (IA.59) is relevant. Note, that (IA.62) is consistent with (IA.55).

For the existence of this equilibrium, we need both (IA.59) and (IA.60) to be larger than (IA.61).

Hence, we have that for (IA.59) to be larger than (IA.61) we need:

$$
\begin{aligned}
& d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} > dn(1-\overline{p})(2q-1)\gamma \\
\Longleftrightarrow\ & (1-\gamma)^2\overline{a}^2 > (2q-1)^2\gamma^2 \\
\Longleftrightarrow\ & (2q-1+\overline{a})\gamma < \overline{\alpha} \\
\Longleftrightarrow\ & \gamma < \frac{\overline{\alpha}}{2q-1+\overline{a}},
\end{aligned}
$$

, which is always true according to (IA.55). For (IA.60) to be larger than (IA.61), we need:

$$
\begin{aligned}
& d\frac{n(2q-1)\gamma}{1-\overline{p}} > dn(1-\overline{p})(2q-1)\gamma \\
\Longleftrightarrow\ & (1-\overline{p})^2 < 1 \\
\Longleftrightarrow\ & \overline{p} > 0. \qquad \text{OK}
\end{aligned}
$$

Now, we check the consistency of the upper bounds for v/c (IA.59) and (IA.60) with respect to Assumption 2 (i.e., $v/c > 1$):

1 (one) vs. (IA.59):

$$
n > \frac{(2q-1)\gamma}{d(1-\gamma)^2\overline{a}^2(1-\overline{p})}. \tag{IA.63}
$$

1 (one) vs. (IA.60):

$$
n > \frac{1-\overline{p}}{d(2q-1)\gamma}. \tag{IA.64}
$$

Note that the upper bound of γ (IA.55) takes precedence over Assumption 1 (i.e., $q\gamma < 1/2$).

Given that $p \in [l, h]$, we need to also ensure that $0 \leq l < p^* < h \leq 1$ when $t_L \in (0,1)$ and

$t_R = 0$. We have:

$$\begin{aligned}
&t_L = \sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})\frac{dc}{v}}}\frac{1}{(1-\gamma)\overline{a}} \\
\iff &\frac{1}{t_L} = (1-\gamma)\overline{a}\sqrt{\frac{\frac{dcn}{v}(1-\overline{p})}{(2q-1)\gamma}} \\
\iff &\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{t_L} = \sqrt{(2q-1)\gamma\frac{dcn}{v}(1-\overline{p})} \\
\iff &1-p^* = \sqrt{(2q-1)\gamma\frac{dcn}{v}(1-\overline{p})}.
\end{aligned}$$

Hence,

$$\begin{aligned}
&l < p^* < h \\
\iff &1-h < 1-p^* < 1-l \\
\iff &(1-h)^2 < (1-p^*)^2 < (1-l)^2 \\
\iff &(1-h)^2 < (2q-1)\gamma\frac{dcn}{v}(1-\overline{p}) < (1-l)^2 \\
\iff &\frac{nd(2q-1)\gamma(1-\overline{p})}{(1-l)^2} \qquad \text{(IA.65)} \\
&< \frac{v}{c} < \\
&\frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2}. \qquad \text{(IA.66)}
\end{aligned}$$

(IA.65) vs (IA.61):

$$\begin{aligned}
&\frac{nd(2q-1)\gamma(1-\overline{p})}{(1-l)^2} \overset{?}{<} nd(1-\overline{p})(2q-1) \\
\iff &(1-l)^2 > 0,
\end{aligned}$$

is always true, and hence, (IA.65) never relevant.

(IA.66) vs (IA.59):

$$\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} \overset{?}{<} \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2}$$

$$\iff (1-\gamma)\overline{a}(1-h) < (2q-1)\gamma$$

$$\iff \gamma > \frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)}, \tag{IA.67}$$

Therefore, we need to inquire further. How does (IA.67) compare with the existing upper bound on γ (IA.55)?

$$\frac{\overline{a}}{2q-1+\overline{a}} \overset{?}{<} \frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)}.$$

$$\iff 2q-1+\overline{a}(1-h) < (2q-1)(1-h)+\overline{a}(1-h)$$

$$\iff 1-h > 1$$

$$\iff h < 0. \quad \times$$

Hence, (IA.67)<(IA.55) always occurs and so both (IA.59) and (IA.66) can be relevant.

(IA.66) vs (IA.60):

$$\frac{d(2q-1)\gamma n}{1-\overline{p}} \overset{?}{<} \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2}$$

$$\iff 1-h < 1-\overline{p}$$

$$\iff \overline{p} < h, \quad \checkmark$$

, which is always true, so that (IA.60)<(IA.66) for all parameter values and so (IA.66) is never the relevant upper bound for v/c.

This concludes the proof of Proposition 7.

Note that at this equilibrium, we have:

$$\begin{aligned}
& t_{total} = \gamma \bar{t} + (1-\gamma) \quad \text{(as always) where:} \\
& \bar{t} = \overline{a}(\overline{p} t_R + (1-\overline{p}) t_L) \\
\iff & \bar{t} = \overline{a}(1-\overline{p}) t_L \\
\iff & \bar{t} = \overline{a}(1-\overline{p}) \sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p}) d \frac{c}{v}} \frac{1}{(1-\gamma)\overline{a}}} \\
\iff & \bar{t} = \sqrt{(2q-1)\frac{v}{dcn}(1-\overline{p})\frac{\gamma}{(1-\gamma)^2}}.
\end{aligned}$$

Hence:

$$\begin{aligned}
\text{sign} \frac{\partial \bar{t}}{\partial \gamma} &= \text{sign} \frac{\partial \left(\frac{\gamma}{(1-\gamma)^2} \right)}{\partial \gamma} \quad \text{and} \\
\frac{\partial \left(\frac{\gamma}{(1-\gamma)^2} \right)}{\partial \gamma} &= \frac{(1-\gamma)^2 + \gamma 2(1-\gamma)}{(1-\gamma)^4} \\
&= \frac{1-\gamma+2\gamma}{(1-\gamma)^3} \\
&= \frac{1+\gamma}{(1-\gamma)^3} > 0 \quad \text{always.}
\end{aligned}$$

Hence, in this equilibrium $\frac{\partial \bar{t}}{\partial \gamma} > 0$ for all parameter values in the region where equilibrium exists.

■

Table 12: **Alternative estimation methods: ISS**

This table shows average parameter estimates by proposal type, where the estimation was made for proposal type×ISS recommendation×γ decile×size decile.

ISS						
Proposal type	$v/(cn)$	t_L	t_R	Prob. misalignment $O_{\text{disc}}(p)$ vs $O_{\text{full}}(p)$	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$
Shareholder proposals						
Board	0.92	0.99	0.56	0.10	-0.23	0.09
Business	1.58	0.98	0.57	0.04	-0.25	0.18
CSR	1.98	0.99	0.59	0.02	-0.26	0.25
Compensation	1.13	0.98	0.56	0.10	-0.23	0.10
Defense	1.28	0.96	0.60	0.08	-0.19	0.11
Governance	0.89	0.95	0.60	0.14	-0.18	0.06
Payout	2.72	1.00	0.62	0.00	-0.24	0.30
Restructuring	1.39	1.00	0.57	0.01	-0.26	0.19
Total	1.37	0.98	0.58	0.07	-0.23	0.15
Management proposals						
Board	2.90	1.00	0.77	0.00	-0.16	0.44
Business	2.81	0.99	0.77	0.00	-0.18	0.47
CSR	3.30	1.00	0.87	0.00	-0.09	0.54
Compensation	2.09	0.98	0.69	0.04	-0.19	0.30
Defense	3.01	1.00	0.79	0.00	-0.13	0.43
Governance	2.57	0.97	0.80	0.02	-0.11	0.33
Merger	2.71	1.00	0.71	0.00	-0.22	0.46
Payout	2.80	1.00	0.74	0.00	-0.21	0.52
Restructuring	2.46	0.98	0.79	0.02	-0.14	0.39
SOP	2.47	1.00	0.71	0.01	-0.21	0.41
Total	2.29	0.98	0.72	0.03	-0.18	0.34