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Abstract

Further progress in digital money, electronically stored monetary value, may enable pricing in units of any currency in any country. This paper studies monetary policy in such a world, using a two-country open economy model with nominal rigidities. The findings are three-fold. First, domestic monetary policy becomes less effective as digital dollarization - pricing using digital money, denominated in and pegged to a foreign currency - deepens. Second, digital dollarization is more likely to occur in a smaller country that is more open to trade and has a greater tradable sector and stronger input-output linkages. Third, monetary policy can facilitate or discourage digital dollarization depending on its stance on the stabilization of macroeconomic variables.

Keywords: Digital money; monetary policy; dollarization
JEL classification: E52, F41

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1 Introduction

Technological innovation has created new forms of private digital money such as cryptographic digital tokens. These tokens include stablecoins (also referred to as fiat tokens), which aim to maintain the value of the token and are intended to function as payment systems.[1] Technological innovation has also led many central banks to explore central bank digital currencies. Further progress in these developments may enable pricing in units of any currency in any country. In such a world, dollarization may ensue, where domestic prices and wages are set in units of digital money that is denominated in and pegged to a foreign currency. This phenomenon is termed as *dollarization 2.0* by Lagarde (2017) and *digital dollarization* by Brunnermeier et al. (2019). Many policy makers have become increasingly aware of potential negative effects of digital dollarization (Jordan, 2019; Kuroda, 2019; G7 Working Group on Stablecoins, 2019). Among the many potential issues regarding digital money, digital dollarization poses specific questions for monetary policy. How does digital dollarization affect the effectiveness of monetary policy? Under what conditions does digital dollarization deepen? Can monetary policy block or facilitate digital dollarization?

This paper addresses these three questions using a cash-less open economy model with nominal rigidities, consisting of two countries: Home and Foreign. By considering a cash-less economy as in Woodford (2003), the paper focuses on the role of money as a unit of account. In the open economy, there are two units of account: the Home and the Foreign currencies. The presence of digital money, denominated in and pegged to the Foreign currency, in the open economy allows Home agents in the Home country to set prices and wages in units of the Foreign currency in as frictionless a manner as in units of the Home currency. With nominal rigidities, invoice-currency choices have consequences on the transmission of monetary policy and other shocks. New open economy macroeconomics (NOEM) literature, such as Devereux et al. (2004), Engel (2006), and Gopinath et al. (2010), has already explored currency choices for import and export goods pricing. This paper considers currency choices for domestic prices and wages and their effects on monetary policy in a world of digital money that makes such choices possible.

The paper's main findings are three-fold. First, the Home monetary policy becomes less effective as digital dollarization deepens. In the case of full digital dollarization in which all prices and wages are set in units of the Foreign currency, domestic monetary

[1] For a taxonomy of digital money, see Adrian and Mancini-Griffoli (2019).

policy becomes completely ineffective: it does not have any effects on the real economy. Second, digital dollarization is more likely to occur in a smaller country that is more open to trade and has a larger tradable sector and stronger input-output linkages. Third, monetary policy can discourage or facilitate digital dollarization, depending on its policy stance. In particular, the numerical analysis shows that a strong stance of the Home monetary policy on inflation stabilization can block digital dollarization, while its weak stance on inflation stabilization or a strong stance on real exchange rate and output stabilization facilitates digital dollarization.

This paper derives these implications for monetary policy both analytically and numerically. The analytical model is based on Corsetti and Pesenti (2009), extended to incorporate the economic size of a country, the degree of trade openness (home bias), price setting complementarities (real rigidities), a non-tradable sector, intermediate input (input-output linkages), and nominal wage rigidities. As stated in the main findings, these additional ingredients have effects on invoice-currency choices as well as on the effectiveness of monetary policy. In this analytical model, prices and wages are set one period in advance as in Obstfeld and Rogoff (1995), and monetary policy is assumed to affect the nominal aggregate in units of the Home currency as in Woodford (2003, Chapter 3). When prices and wages in the Home country are set in units of the Home currency as in the standard model such as Corsetti and Pesenti (2009), monetary policy has real effects because of nominal rigidities in the form of one-period-in-advance price and wage setting. Because current prices and wages are already set, a sudden increase in the nominal aggregate – an expansionary monetary policy shock – increases consumption and hours worked.

The first finding on the impact of digital dollarization on monetary policy can be understood intuitively through the lens of the analytical model. In the case of full dollarization, all prices in the Home country are set in units of the Foreign currency. Because these prices are set in one period in advance, the current prices in units of the Home currency move in tandem with the exchange rate in response to a change in monetary policy. In this analytical model with complete asset markets and log utility, the exchange rate moves one-to-one with a change in the monetary aggregate and so do the prices in units of the Home currency. The Home prices fluctuate as if they were flexible, and consequently, monetary policy has no real effects under full digital dollarization. In the case of partial digital dollarization in which some prices in the Home country are set in units of the Foreign currency, monetary policy has some real effects but the effects abate as dollarization deepens.

The second finding on conditions for digital dollarization can be rephrased intuitively as follows: the Home country that is more connected to and more affected by the Foreign country is more vulnerable to digital dollarization. Firms in such a country set prices by paying attention to the Foreign firms' prices through price setting complementarities that work through intermediate input and kinked demand curves. The effect of such complementarities becomes the smaller the country's economic size, the more open the country is to trade, and the higher the share of its tradable sector. With the effect strong enough, the firms' flexible prices – newly set prices if prices were flexible – become heavily dependent on the Foreign firms' prices, which are set in units of the Foreign currency. As a result, choosing the Foreign currency as an invoice currency becomes more profitable than choosing the Home currency. In such a country, digital dollarization is more likely to occur.

These two results are derived analytically using the model with some simplifying assumptions such as independent and identically distributed (i.i.d.) shocks, linear disutility of labor, the nominal aggregate as a monetary policy instrument, and subsidies that eliminate markup distortions. For robustness, the paper extends the model by relaxing these assumptions and confirms numerically that these results continue to hold.

The extended model is also used to shed light on the role of monetary policy in digital dollarization. In the Home country, a Taylor-type monetary policy rule that puts less weight on inflation stabilization, more weight on output stabilization, or too much weight on real exchange rate stabilization facilitates digital dollarization. Such a monetary policy rule makes inflation in units of the Home currency volatile, which in turn makes the Home currency less attractive as an invoice currency. The Foreign monetary policy can also influence digital dollarization in the Home country. Its effects mirror those of the Home monetary policy upside down: a strong stance on inflation stabilization facilitates dollarization, while a strong stance on output and real exchange rate stabilization discourages dollarization in the Home country.

In this paper, the term "digital dollarization" is used against the background of digital innovation and digital money, but it is essentially equivalent to dollarization for price and wage setting. However, this paper differs from the traditional literature on dollarization and currency substitution (Calvo and Végh, 1992; Végh, 2013).[2] While the literature has focused on developing economies in which currency substitution has occurred under high inflation and fiscal imbalances, this paper mainly focuses on developed economies with

[2]Calvo and Végh (1992) and Végh (2013) define *currency substitution* as the use of a foreign currency as a medium of exchange and define *dollarization* as the use of a foreign currency in any of the three traditional functions of money including a unit of account. This paper follows their terminology.

stable inflation, under an independent central bank, and no fiscal imbalances.

Related literature This paper contributes to the emerging literature on digital money and monetary policy. Most of the papers in this literature have focused on a medium of exchange (or liquidity services) as the role of money. Fernández-Villaverde and Sanches (2019), building on the model of Lagos and Wright (2005), argue that currency competition between private and official currencies creates problems for monetary policy implementation. Schilling and Uhlig (2019) study an endowment economy with private and official currencies and show that the value of the private currency forms a martingale, so that it can be highly volatile. Baughman and Flemming (2020) study a basket-backed stablecoin and show that overall demand for such a currency is small in their two-country open economy model. Benigno (2019) studies a world of competing multiple currencies issued by private agents using the Lucas and Stokey (1987) environment and shows that the nominal interest rate and inflation are both determined by structural factors in such a world. In a similar setup, Benigno et al. (2019) consider the global currency in a two-country model and show that the national interest rates must be equalized.

This paper is also related to the literature on currency substitution – the use of a foreign currency as a medium of exchange. Considering both domestic and foreign money in utility functions or shopping costs, Felices and Tuesta (2007) argue that a high degree of currency substitution makes the domestic economy more vulnerable to foreign monetary policy, while Batini et al. (2010) and Kumamoto and Kumamoto (2014) argue that a high degree of currency substitution does not affect the effectiveness of domestic monetary policy.

Differing from the two strands of literature, this paper exclusively focuses on a unit of account for pricing as the role of money. The model economy is cashless and thus abstracts away a medium of exchange, but a unit of account for pricing matters because of nominal rigidities and price setting complementarities.[3] The model also abstracts away credit and its related roles for a unit of account. Such roles are studied by Doepke and Schneider (2017) and Bahaj and Reis (2020), who focus on the roles of relative price risk and interest rate risk, respectively.

This paper builds on the NOEM pioneered by Obstfeld and Rogoff (1995). The framework of the analytical model in Section 2 draws on a two-country NOEM model, as studied

[3]Because the model is cashless, it is free from the issue of exchange rate indeterminacy à la Kareken and Wallace (1981). The dynamics of exchange rates is determined by monetary policy as in the standard cashless NOEM models. See, e.g. Benigno and Benigno (2008) and Corsetti and Pesenti (2009).

by Corsetti and Pesenti (2009). The analysis on invoice-currency choices for domestic prices and wages and resulting possible digital dollarization in Section 3 closely follows that for import and export prices, as in Devereux et al. (2004), Engel (2006), Gopinath et al. (2010), and Mukhin (2018). Adopting a similar approach to domestic prices, Castillo (2006) shows that partial dollarization occurs if there are sector-specific shocks;[4] but, without such shocks, it never arises, which is a special case of this paper's model. Unlike Castillo (2006), however, this paper incorporates ingredients that link firms across countries such as price setting complementarities and input-output linkages, and derives rich implications for monetary policy and dollarization both analytically and numerically.

Following the literature on dominant currency pricing, including Goldberg and Tille (2016) and Gopinath et al. (2020), this paper assumes dominant currency pricing, in which the Foreign currency is dominantly used for import and export prices, as a starting point for considering digital dollarization in the Home country. Specifically, for the Home country, import prices are already set in units of the Foreign currency.

The rest of the paper is organized as follows. Section 2 presents the tractable NOEM model. Section 3 studies the model analytically and derives implications for the effects of digital dollarization on monetary policy and conditions for digital dollarization. Section 4 presents the extended model that relaxes key simplifying assumptions of the tractable model and numerically studies the implications obtained in Section 3 and the role of monetary policy in digital dollarization. Section 5 concludes.

2 Model

This section presents a cash-less two-country open economy model. Time is discrete, $t = 0, 1, 2, ...$, and the time horizon is infinite. The world economy, with its population normalized to unity, consists of Home and Foreign countries with each country populated by a continuum of households, with population size given by n and $1-n$, respectively. Home and Foreign countries and their own currencies are referred to as H and F, respectively.

The model assumes the availability of digital money denominated in and pegged to currency F, which allows firms and households in country H to set prices and wages in units of currency F as well as in their own country's currency H. Digital money is assumed

[4] Relatedly, Loyo (2002) studies multiple monies, including 'imaginary money,' as a unit of account in a closed economy.

to be frictionless, so that price and wage setters do not incur any costs by using digital money as the invoice currency. The model thus exclusively focuses on the unit of account as a role of digital money in the cash-less economy. In addition, the model assumes the dominant currency paradigm (Gopinath et al., 2020). Without loss of generality, country F is assumed to be the country with a dominant currency, so that export and import prices are set in units of currency F.

The model features nominal rigidities in both prices and wages. For analytical tractability, one-period-in-advance price and wage setting is assumed. In addition, all shocks are assumed to be i.i.d. The extended model, modified to incorporate the Calvo (1983)-type price and wage setting and persistent shocks among others, will be studied in Section 4. Because of symmetry between countries H and F, the model is presented for country H only. Variables in country F are denoted with superscript * following the convention of the literature. For simple exposition, the complete set of equilibrium conditions is relegated to Appendix A.

2.1 Households

Households consume a consumption bundle consisting of non-tradable goods and tradable goods. Tradable goods, produced in countries H and F, are aggregated into a bundle by a Kimball (1995) aggregator, which gives rise to price setting complementarities among tradable goods. Households have monopolistic power over their specialized labor and set wages one period in advance.

Consumption and saving A household $j \in (0, n)$ in country H has preferences over a sequence of consumption bundle $C_t(j)$ and hours worked $L_t(j)$:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log C_t(j) - \frac{\psi}{1+\frac{1}{\nu}} L_t(j)^{1+\frac{1}{\nu}} \right), \tag{1}$$

where $0 < \beta < 1$ is a preference discount factor, ν is the Frisch labor supply elasticity, and E_0 is an expectation operator conditional on information available in period $t = 0$.

The household j earns wage income $W_t(j) L_t(j)$, receives net transfers $\Theta_t(j)$, which consist of firms' profits and government lump-sum transfers net of taxes, and spends $P_t C_t(j)$ on consumption, where $W_t(j)$ is the nominal wage for specialized labor $L_t(j)$ and P_t is the price level. The asset market is complete and the household can trade Arrow securities.

Let $Q(s_{t+1}|s_t)$ denote the price of an Arrow security that pays one unit of Home currency in state s_{t+1} tomorrow, conditional on being in the state s_t today. Then, the household's flow budget constraint is written as

$$P_t C_t(j) + \sum_{s_{t+1}} Q(s_{t+1}|s_t) B_t(s_{t+1}; j) = W_t(j) L_t(j) + B_{t-1}(s_t; j) + \Theta_t(j), \tag{2}$$

where $B_t(s_{t+1}; j)$ denotes the holdings of the state-s_{t+1} Arrow security. The securities allow households to insure against the risk of wage income fluctuations due to nominal wage rigidities, so that they can enjoy the same level of consumption across households: $C_t(j) = \bar{C}_t$ for all j in equilibrium. The price of the risk-free bond is given by $Q_t = \sum_{s_{t+1}} Q(s_{t+1}|s_t)$ and the risk-free interest rate is given by $R_t = Q_t^{-1}$.

Consumption bundles and expenditure The consumption bundle $C_t(j)$ consists of the non-tradable consumption bundle $C_{Nt}(j)$ and the tradable consumption bundle $C_{Tt}(j)$, given by a Cobb-Douglas aggregator as

$$C_t(j) = \left(\frac{C_{Nt}(j)}{\gamma_n}\right)^{\gamma_n} \left(\frac{C_{Tt}(j)}{1-\gamma_n}\right)^{1-\gamma_n}, \quad 0 \leq \gamma_n \leq 1. \tag{3}$$

The corresponding consumption expenditure is given by $P_t C_t(j) = P_{Nt} C_{Nt}(j) + P_{Tt} C_{Tt}(j)$ where P_{Nt} and P_{Tt} denote corresponding price indices.

In the non-tradable sector, the non-tradable consumption bundle $C_{Nt}(j)$ consists of a continuum of non-tradable consumption goods $\{C_{Nt}(j,i)\}_{i\in(0,1)}$, given by a constant elasticity of substitution (CES) aggregator as

$$C_{Nt}(j) = \left[\int_0^1 C_{Nt}(j,i)^{\frac{\theta_n-1}{\theta_n}} di\right]^{\frac{\theta_n}{\theta_n-1}}, \tag{4}$$

where $\theta_n > 1$ is the elasticity of substitution among non-tradable consumption goods. The corresponding consumption expenditure is given by $P_{Nt} C_{Nt}(j) = \int_0^1 P_{Nt}(i) C_{Nt}(j,i) di$, where $P_{Nt}(i)$ is the price of the non-tradable consumption good i.

In the tradable sector, the tradable consumption bundle C_{Tt} consists of a continuum of Home goods $\{C_{Ht}(j,i)\}_{i\in(0,n)}$ and a continuum of the Foreign goods $\{C_{Ft}(j,i)\}_{i\in[0,1-n)}$, defined implicitly by a Kimball (1995) aggregator as

$$\frac{1-\gamma_{\tau}}{n}\int_{0}^{n}G\left(\frac{nC_{Ht}\left(j,i\right)}{\left(1-\gamma_{\tau}\right)C_{Tt}\left(j\right)}\right)di+\frac{\gamma_{\tau}}{1-n}\int_{0}^{1-n}G\left(\frac{\left(1-n\right)C_{Ft}\left(j,i\right)}{\gamma_{\tau}C_{Tt}\left(j\right)}\right)di=1, \tag{5}$$

where $\gamma_{\tau}=\left(1-n\right)\bar{\gamma}$ and $\bar{\gamma}\in\left[0,1\right]$ is the degree of trade openness. Specifically, $\bar{\gamma}=0$ corresponds to autarky and $\bar{\gamma}=1$ corresponds to perfect integration. As long as the trade openness lies between the two extremes, i.e. $0<\bar{\gamma}<1$, the case of $n\rightarrow 0$ corresponds to country H as a small open economy. The aggregator function $G\left(\cdot\right)$ has the following properties: $G'\left(\cdot\right)>0$, $G''\left(\cdot\right)<0$, $-G''\left(1\right)\in\left(0,1\right)$, and $G\left(1\right)=G'\left(1\right)=1$. As will be studied later, this aggregator gives rise to kinked demand curves and price setting complementarities for tradable goods in countries H and F. This type of aggregators is also used in the open economy models studied by Mukhin (2018) and Itskhoki and Mukhin (2019). The expenditure for the tradable consumption bundle is written as $P_{Tt}C_{Tt}\left(j\right)=\int_{0}^{n}P_{Ht}\left(i\right)C_{Ht}\left(j,i\right)di+\int_{0}^{1-n}P_{Ft}\left(i\right)C_{Ft}\left(j,i\right)di$, where $P_{Ht}\left(i\right)$ and $P_{Ft}\left(i\right)$ are the prices of the i-th Home tradable good and the i-th Foreign tradable good, respectively.

Employment agency The employment agency transforms specialized labor $\left\{L_{t}\left(j\right)\right\}_{j\in\left(0,n\right)}$ into the labor package L_t using the CES aggregator

$$L_{t}=\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta_{w}}}\int_{0}^{n}L_{t}\left(j\right)^{\frac{\theta_{w}-1}{\theta_{w}}}dj\right]^{\frac{\theta_{w}}{\theta_{w}-1}}. \tag{6}$$

The employment agency chooses the amount of all types of specialized labor $\left\{L_{t}\left(j\right)\right\}_{j\in\left(0,n\right)}$ so as to maximize profits $W_{t}L_{t}-\int_{0}^{n}W_{t}\left(j\right)L_{t}\left(j\right)dj$ subject to the aggregator (6), given W_t and $\left\{W_{t}\left(j\right)\right\}_{j\in\left(0,n\right)}$. The first-order condition yields the demand curve for each type of specialized labor

$$L_{t}\left(j\right)=\frac{1}{n}\left(\frac{W_{t}\left(j\right)}{W_{t}}\right)^{-\theta_{w}}L_{t}. \tag{7}$$

Substituting this curve into the aggregator (6) leads to the nominal wage W_t as

$$W_{t}=\left[\frac{1}{n}\int_{0}^{n}W_{t}\left(j\right)^{1-\theta_{w}}dj\right]^{\frac{1}{1-\theta_{w}}}. \tag{8}$$

Wage setting The household j sets the wage one period in advance in units of either currency H or F. On the one hand, if currency H is chosen, the household chooses $\bar{W}_{t}\left(j\right)$

to maximize the relevant expected utility:

$$\max_{\{\bar{W}_t(j)\}} E_{t-1}\left[\Lambda_t(j)\,\bar{W}_t(j)\,L_t(j) - \frac{\psi}{1+\frac{1}{\nu}} L_t(j)^{1+\frac{1}{\nu}}\right],$$

subject to the demand curve (7), where $\Lambda_t(j) = (P_t C_t(j))^{-1}$ is the Lagrange multiplier on the budget constraint (2). On the other hand, if currency F is chosen, the household chooses $\bar{W}_t^F(j)$ to solve

$$\max_{\{\bar{W}_t^F(j)\}} E_{t-1}\left[\Lambda_t(j)\,\mathcal{E}_t\bar{W}_t^F(j)\,L_t(j) - \frac{\psi}{1+\frac{1}{\nu}} L_t(j)^{1+\frac{1}{\nu}}\right]$$

subject to the demand curve (7), where $\mathcal{E}_t$ is the nominal exchange rate – the relative price of currency F in units of currency H. An increase in $\mathcal{E}_t$ means depreciation of currency H.

Let a small letter denote a variable of corresponding capital letter in log-deviations from its steady state. Then, the wage chosen by the household j is given by

$$w_t(j) = \begin{cases} \bar{w}_t = E_{t-1}(\tilde{w}_t) & \text{if currency } H \text{ is chosen} \\ \bar{w}_t^F = \bar{w}_t - E_{t-1}(e_t) & \text{if currency } F \text{ is chosen} \end{cases} \tag{9}$$

where $\tilde{w}_t$ is the flexible wage in units of currency H – the wage set by the household if the wage were flexible, given by:

$$\tilde{w}_t = \frac{1}{\nu} l_t + c_t + p_t. \tag{10}$$

In units of currency F, the flexible wage is given by $\tilde{w}_t^F = \tilde{w}_t - e_t$. With nominal rigidities, the wage is set to the expected flexible wage: $\bar{w}_t = E_{t-1}(\tilde{w}_t)$ and $\bar{w}_t^F = E_{t-1}\left(\tilde{w}_t^F\right)$.

Let $\lambda_w \in [0,1]$ denote a fraction of the Home households that set wages in units of currency H. The remaining fraction $1-\lambda_w$ of the households set wages in units of currency F. This fraction λ_w can be endogenized by solving a currency choice problem as will be studied in the next section, but this section treats λ_w as being exogenous. Then, the wage index is given by

$$w_t = E_{t-1}(\tilde{w}_t) + (1-\lambda_w)(e_t - E_{t-1}e_t). \tag{11}$$

Because a fraction $1-\lambda_w$ of wages is set in units of currency F, the wage index (in units of currency H) is affected by an actual change in the exchange rate. Because all shocks are assumed to be i.i.d., a surprise change in the exchange rate, $(1-\lambda_w)\,e_t$, is the only factor that affects the wage index.

2.2 Firms

Firms in both the non-tradable and the tradable sectors set prices one period in advance. An invoice currency – a currency used as a unit of account for price setting – is either currency H or F. This analytical model assumes subsidies that eliminate markup distortions in steady state. For simplicity, an invoice currency is exogenously given in this section, but it will be endogenized in the next section.

Production, costs, and profits The production technologies in the non-tradable and the tradable sectors are given by a Cobb-Douglas technology that combines labor $L_{\cdot t}(i)$ and intermediate input $X_{\cdot t}(i)$:

$$Y_{Nt}(i) = A_t \left(\frac{L_{Nt}(i)}{1-\phi_n}\right)^{1-\phi_n} \left(\frac{X_{Nt}(i)}{\phi_n}\right)^{\phi_n}, \quad 0 < \phi_n < 1, \tag{12}$$

$$Y_{Tt}(i) = A_t \left(\frac{L_{Tt}(i)}{1-\phi_\tau}\right)^{1-\phi_\tau} \left(\frac{X_{Tt}(i)}{\phi_\tau}\right)^{\phi_\tau}, \quad 0 < \phi_\tau < 1, \tag{13}$$

where $Y_{Nt}(i)$ is the output of non-tradable good $i \in (0,1)$, $Y_{Tt}(i)$ is the output of tradable good $i \in (0,n)$, and $a_t \equiv \log(A_t)$ is the common neutral technology, which is i.i.d. with standard deviation σ_a. The tradable goods bundle, given by equation (5), is used as the intermediate input.[5]

The non-tradable goods firm i sells its product $Y_{Nt}(i)$ only in country H. The profits in period t, taking into account the households' demand for the non-tradable good i, are given by

$$\Pi_{Nt}(i) = (P_{Nt}(i) - MC_{Nt}) \left(\frac{P_{Nt}(i)}{P_{Nt}}\right)^{-\theta_n} C_{Nt}, \tag{14}$$

where MC_{Nt} is the marginal cost faced by the non-tradable goods firm and $C_{Nt} = \int_0^n C_{Nt}(j)dj$ is the aggregate non-tradable consumption.

The tradable consumption goods firm i sells its product $Y_{Tt}(i)$ in both countries H and F. In each country, the product is sold as consumption and intermediate input. The profits $\Pi_{Tt}(i)$ consist of those made in countries H and F, $\Pi_{Ht}(i)$ and $\Pi^*_{Ht}(i)$, respectively, given

[5] For example, to use intermediate input $X_{Tt}(i)$, the tradable goods firm i needs to purchase varieties $\{X_{Ht}(i,i')\}_{i' \in (0,n)}$ from tradable goods firms in country H and varieties $\{X_{Ft}(i,i')\}_{i' \in (0,1-n)}$ from those in country F.

by

$$\Pi_{Tt}(i) = \underbrace{(P_{Ht}(i) - MC_{Tt})\, Y_{Ht}(i)}_{\Pi_{Ht}(i)} + \underbrace{(P^*_{Ht}(i)\,\mathcal{E}_t - MC_{Tt})\, Y^*_{Ht}(i)}_{\Pi^*_{Ht}(i)}, \tag{15}$$

where $Y_{Ht}(i)$ and $Y^*_{Ht}(i)$ are the amounts of the tradable good i sold in countries H and F, respectively, and MC_{Tt} is the marginal cost facing the tradable goods firm. The total demand for the tradable good i in countries H and F is in turn given by[6]

$$\begin{aligned} Y_{Ht}(i) &= g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right)\frac{1-\gamma_\tau}{n}\left(C_{Tt} + X_{Tt} + X_{Nt}\right), &(16)\\ Y^*_{Ht}(i) &= g\left(\frac{P^*_{Ht}(i)}{\mathcal{P}^*_{Tt}}\right)\frac{\gamma^*_\tau}{n}\left(C^*_{Tt} + X^*_{Tt} + X^*_{Nt}\right), &(17)\end{aligned}$$

where $\mathcal{P}_{Tt}$ is an auxiliary variable for the price index of the tradable goods and $C_{Tt} \equiv \int_0^n C_{Tt}(j)\,dj$, $X_{Tt} \equiv \int_0^n X_{Tt}(i)\,di$, and $X_{Nt} \equiv n\int_0^1 X_{Nt}(i)di$ are the aggregate consumption, the aggregate intermediate input used by the tradable firms, and the aggregate intermediate input used by the non-tradable firms, respectively.

Flexible price benchmark In a hypothetical economy with flexible prices, the non-tradable goods firm i sets the price $P_{Nt}(i)$ to maximize the profits (14). The optimality condition is simply $P_{Nt}(i) = \tilde{P}_{Nt}(i) = MC_{Nt}$. The tradable goods firm i sets prices $P_{Ht}(i)$ and $P^*_{Ht}(i)$ to maximize the profits (15) subject to the demand curves (16) and (17). Log-linearizing the first-order conditions of the problem around a symmetric steady state yields the flexible prices $\tilde{p}_{Ht}(i)$ and $\tilde{p}^*_{Ht}(i)$, given by

$$\begin{aligned} \tilde{p}_{Ht}(i) &= (1-\alpha)\, mc_{Tt} + \alpha p_{Tt}, &(18)\\ \tilde{p}^*_{Ht}(i) &= (1-\alpha)\,(mc_{Tt} - e_t) + \alpha p^*_{Tt}, &(19)\end{aligned}$$

where $\alpha \equiv \Gamma/(1+\Gamma) \in [0,1]$, $\Gamma \equiv -\frac{g''(1)}{\theta_\tau(\theta_\tau - 1)} + \frac{\theta_\tau + 1}{\theta_\tau - 1}$, and $\theta_\tau \equiv -g'(1)/g(1)$ is the demand elasticity at the symmetric steady state.[7] Parameter α governs the degree of price setting

[6]Equations (16) and (17) are derived from these two equations, respectively:

$$\begin{aligned} Y_{Ht}(i) &= \int_0^n C_{Ht}(j,i)\,dj + \int_0^n X_{Ht}(i',i)\,di' + n\int_0^1 X_{Nt}(i',i)di',\\ Y^*_{Ht}(i) &= \int_0^{1-n} C^*_{Ht}(j,i)\,dj + \int_0^{1-n} X^*_{Ht}(i',i)\,di' + (1-n)\int_0^1 X^*_{Nt}(i',i)di'.\end{aligned}$$

[7]For the derivation of the steady state, see Appendix A.4.

complementarities. In the case of a constant demand elasticity, there are no complementarities, $\alpha = 0$, so that the optimal price depends only on the marginal cost. But if the elasticity is increasing in the relative price around the steady state, the demand becomes more sensitive to a price increase than in the case of a constant demand elasticity. Facing this increasing demand elasticity, the tradable goods firms become more sensitive to changes in the prices of other goods, and thereby the flexible prices depend not only on the marginal cost but also on the tradable goods price indices as shown in equations (18) and (19).

Nominal rigidities Both the non-tradable and the tradable goods firms set prices one period in advance using either currency H or F as the invoice currency. First, consider the price setting problem of the non-tradable goods firms. The log-linearized price set by such a firm i in terms of currency H is given by

$$p_{Nt}(i) = \begin{cases} E_{t-1}\left(\tilde{p}_{Nt}(i)\right) & \text{if currency } H \text{ is chosen} \\ E_{t-1}(\tilde{p}_{Nt}(i) - e_t) + e_t & \text{if currency } F \text{ is chosen} \end{cases} \tag{20}$$

where $\tilde{p}_{Nt}(i) = mc_{Nt}$ is the flexible price – the price set by the firm in a fully flexible environment. In the case of currency H pricing, the price p_{Nt} is preset at the expected flexible price. In the case of currency F pricing, the price is preset in terms of currency F at $E_{t-1}(\tilde{p}_{Nt}(i) - e_t)$, so that in terms of currency H the price is changed one-to-one with the exchange rate. In other words, an exchange rate path-through – the effect of a change in the exchange rate on the non-tradable goods price $p_{Nt}(i)$ – is zero under currency H pricing, but it is unity under currency F pricing.

Next, consider the price setting problem of the tradable goods firm i that sets prices $P_{Ht}(i)$ and $P^*_{Ht}(i)$ in countries H and F, respectively. Similar to the case of the non-tradable goods firms, the log-linearized price in country H in terms of currency H is given by

$$p_{Ht}(i) = \begin{cases} E_{t-1}\left(\tilde{p}_{Ht}(i)\right) & \text{if currency } H \text{ is chosen} \\ E_{t-1}(\tilde{p}_{Ht}(i) - e_t) + e_t & \text{if currency } F \text{ is chosen} \end{cases} \tag{21}$$

where $\tilde{p}_{Ht}(i)$, given by equation (18), is the corresponding flexible price. The log-linearized price of the tradable good i sold in country F in terms of currency F is given by

$$p^*_{Ht}(i) = \begin{cases} E_{t-1}\left(\tilde{p}^*_{Ht}(i) + e_t\right) - e_t & \text{if currency } H \text{ is chosen} \\ E_{t-1}(\tilde{p}^*_{Ht}(i)) & \text{if currency } F \text{ is chosen} \end{cases} \tag{22}$$

where $\tilde{p}^*_{Ht}(i)$, given by equation (19), is the corresponding flexible price in country F. Under the assumption of dominant currency pricing in this model, all prices are set in units of currency F in country F. Hence, the exchange rate path-through to the flexible price $\tilde{p}^*_{Ht}(i)$ is always zero in country F.

2.3 Price Indices

Let λ_h and λ^*_h denote a fraction of firms that employ currency H pricing in countries H and F, respectively, for non-tradable goods firms ($h = N$), tradable Home goods firms ($h = H$), and tradable Foreign goods firms ($h = F$).

The price index P_t for the consumption bundle is given by the weighted sum of the non-tradable goods prices (20), the tradable Home goods prices (21), and the tradable Foreign goods prices in country H. Because of the assumption of i.i.d. shocks, the prices set one period in advance are at their steady state values. Hence, only a surprise change in the exchange rate appears in the log-deviation of the price index, given by

$$p_t = \left\{\gamma_n(1 - \lambda_N) + (1 - \gamma_n)\left[(1 - \gamma_\tau)(1 - \lambda_H) + \gamma_\tau (1 - \lambda_F)\right]\right\} e_t. \tag{23}$$

If the prices are set in currency F, a depreciation of currency H – an increase in e_t – increases the price index p_t. The coefficient on e_t in equation (23) captures the degree of an exchange rate pass-through. The more the Home firms or Foreign firms set prices in units of currency F in country H, the more the exchange rate affects the price index. Symmetrically, the price index for the Foreign consumption bundle is given by

$$p^*_t = -\left\{\gamma_n \lambda^*_N + (1 - \gamma_n)\left[(1 - \gamma_\tau)\lambda^*_F + \gamma_\tau \lambda^*_H\right]\right\} e_t.$$

Under the assumption of dominant currency pricing, no firms use currency H as an invoice currency in country F, i.e., $\lambda^*_N = \lambda^*_F = \lambda^*_H = 0$, which yields $p^*_t = 0$: a change in the exchange rate does not have any impact on the Foreign price level in units of currency F.

2.4 Equilibrium

Monetary policy For analytical tractability, as in Corsetti and Pesenti (2009), a simple monetary policy that targets the per capita nominal aggregate is considered:

$$M_t = P_t \bar{C}_t. \tag{24}$$

An unexpected change in M_t can be interpreted as a monetary policy shock. The log of M_t, $m_t \equiv \log(M_t)$, is assumed to be i.i.d. The monetary policy in country F is symmetric, given by $M_t^* = P_t^* \bar{C}_t^*$.

Exchange rate The combination of the complete asset market, log-utility, and the monetary policy rule (24) gives rise to the exchange rate, given by

$$\mathcal{E}_t = \frac{M_t}{M_t^*} e^{z_{et}}, \tag{25}$$

where z_{et} is an i.i.d. shock to the exchange rate with standard deviation σ_e. A monetary expansion in country H – an increase in M_t – depreciates the exchange rate, while a monetary expansion in country F – an increase in M_t^* – appreciates the exchange rate from the viewpoint of country H.

Market clearing In the labor market, the labor supply L_t is equated to the labor demand $\int_0^n L_{Nt}(i)\, di + \int_0^n L_{Tt}(i) di$. In the non-tradable goods market, the demand $\int_0^n C_{Nt}(j) dj$ is equated to its supply $n \int_0^1 Y_{Nt}(i) di$. In the tradable goods market, from equations (16) and (17), the total demand is given by aggregating the good i sold in both countries over i as

$$Y_{Tt} = \frac{1-\gamma}{n} (C_{Tt} + X_t) \int_0^n g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right) di + \frac{\gamma^*}{n} (C_{Tt}^* + X_t^*) \int_0^n g\left(\frac{P_{Ht}^*(i)}{\mathcal{P}_{Tt}^*}\right) di, \tag{26}$$

where $X_t = X_{Tt} + X_{Nt}$ and $X_t^* = X_{Tt}^* + X_{Nt}^*$. This total demand is equated to the aggregate supply $\int_0^n Y_{Tt}(i) di$. Finally, the aggregate output Y_t is defined as $Y_t = (P_{Tt} Y_{Tt} + P_{Nt} Y_{Nt})/P_t$.

Competitive equilibrium A competitive equilibrium for this economy consists of allocations for aggregate variables $\left\{C_t, \{C_{st}, Y_{st}, X_{st}, L_{st}\}_{s\in\{N,T\}}\right\}$ for country H, corresponding variables for individual households and firms for country H, and corresponding aggregate and individual variables for country F such that, given prices, wages, interest rates, exogenous shocks, and monetary policy (24), for both countries, (i) the allocations solve the household problem presented in Section 2.1, (ii) the allocations solve the firms' problems presented in Section 2.2, and (iii) markets for labor, non-tradable goods, and tradable goods clear.

3 Analytical Implications

This section derives analytical implications for digital dollarization and monetary policy. Specifically, it first studies the impact of digital dollarization – the use of digital money, denominated in and pegged to currency F, in country H– on the transmission of monetary policy in the country. Next, it analyzes under what conditions digital dollarization can deepen. All proofs and derivations of equations can be found in Appendix A.

3.1 Monetary Policy Transmission

The use of digital money (i.e. currency F) in country H can affect monetary policy transmission, and its effect on consumption in particular. The simple monetary policy (24) suggests that the impact of monetary policy on consumption, $c_t = m_t - p_t$, depends on how the price level responds to the monetary policy, where m_t is a monetary policy shock in country H. By using equations (23), (25), and $\gamma_\tau = (1-n)\bar{\gamma}$, the consumption response can be written as

$$c_t = (1-\chi_p)\, m_t + \chi_p m_t^*, \tag{27}$$

where

$$\chi_p \equiv \gamma_n(1-\lambda_N) + (1-\gamma_n)\left[(1-(1-n)\bar{\gamma})\,(1-\lambda_H) + (1-n)\bar{\gamma}\,(1-\lambda_F)\right]$$

Equation (27) shows that an increasing use of currency F as an invoice currency – decreases in the ratios λ_N, λ_H, and λ_F – weakens the impact of the Home monetary policy m_t on consumption, while it strengthens the impact of the Foreign monetary policy m_t^*. As more prices are set in units of currency F, the exchange rate path-through becomes stronger and thereby monetary policy has a greater impact on the price level, resulting in a weaker response of consumption.

Under the paradigm of currency F as a dominant currency, imported goods are priced in units of currency F: $\lambda_F = 0$. In this case, the impact of the Home monetary policy on consumption becomes smaller as (i) trade openness $\bar{\gamma} \in [0,1]$ increases, and (ii) given $\bar{\gamma} > 0$, the economic size of country H, n, becomes smaller.[8] This would also be the case in a milder environment where some imported goods, though a smaller fraction than that of domestic tradable goods, are priced in units of currency H, i.e. $\lambda_F < \lambda_H$. In addition, given that the exchange rate path-through is greater for tradable goods prices than non-tradable goods prices, i.e. $1-\lambda_N < (1-(1-n)\bar{\gamma})(1-\lambda_H)+(1-n)\bar{\gamma}(1-\lambda_F)$, an increase in the size of the tradable sector relative to the non-tradable sector – an increase in $1-\gamma_n$ – decreases the impact of the Home monetary policy on consumption.

The impact of the Foreign monetary policy is exactly the opposite. Whereas the impact of the Home monetary policy is summarized by $1-\chi_p$, that of the Foreign monetary policy is summarized by χ_p. Under the assumptions made above, as more firms use currency F as an invoice currency, as the Home bias decreases so that trade openness increases, as the economic size of the Home country becomes smaller relative to the Foreign country, and as the relative size of the tradable sector becomes larger, the Home monetary policy will become less influential and the Home consumption will be more affected by the Foreign monetary policy.

The effects on monetary policy can be summarized in the following proposition.

Proposition 1 [Monetary policy transmission] *Consider the model presented in Section 2. The impact of Home monetary policy m_t on Home consumption c_t decreases and that of Foreign monetary policy m^* increases as:*

a) More firms use currency F as an invoice currency (as λ_N, λ_H, and λ_F decrease);

b) The economic size of country H becomes smaller (as n decreases) if $\lambda_F < \lambda_H$;

c) Country H becomes more open to trade (as $\bar{\gamma}$ increases) if $\lambda_F < \lambda_H$;

d) The tradable sector becomes greater (as γ_n decreases) if $\lambda_N > (1-(1-n)\bar{\gamma})\lambda_H + (1-n)\bar{\gamma}\lambda_F$

3.2 Invoice-Currency Choices

The analysis in Section 3.1 has shown that an increasing use of digital money, denominated in and pegged to currency F, in country H attenuates the impact of Home monetary

[8] Although parameter n denotes the relative size of population in country H, it also captures the relative economic size of the country since production technologies are symmetric between countries H and F.

policy on consumption. Then, what determines the use of such digital money as an invoice currency? This section considers the issues involved for firms or households when they make the choice between currency H or F for price and wage setting – an invoice-currency choice problem. By doing so, light is shed on the conditions under which private agents are more likely to use currency F as a unit of account.

For analytical tractability, the linear disutility in labor, $\nu = \infty$, is assumed. In addition, following Mukhin (2019), an exogenous i.i.d. shock to the exchange rate is considered as a main shock while monetary policy shocks are shut down.[9] In this case, the volatility of the exchange rate is exogenous, denoted as σ_e^2.

Approximated problem Consider the model presented in Section 2. Let $\Pi_{st}^h(i)$ denote profits when the Home firm i, which is in either the non-tradable sector ($s = N$) or the tradable sector in the Home market ($s = H$), chooses currency $h \in \{H, F\}$ as the invoice currency. Then the firm's currency choice problem is formulated as

$$\max_{h \in \{H,F\}} E_{t-1} Q_{t-1,t} \Pi_{st}^h(i).$$

In the case of a tie, the firm is assumed to choose its own country's currency. To the first-order approximation, the pre-set price is given by equations (20) and (21) for $s = N$ and H, respectively. In the tradable sector in the Home market ($s = H$), for example, in period t, the deviation of the preset price from the flexible price in terms of currency H is $E_{t-1}(\tilde{p}_{Ht}(i)) - \tilde{p}_{Ht}(i)$ if currency H was chosen and $E_{t-1}(\tilde{p}_{Ht}(i) - e_t) - (\tilde{p}_{Ht}(i) - e_t)$ if currency F was chosen. In period $t-1$ when the firm chose the invoice currency, the firm would prefer a currency with which the second moment (or volatility) of the deviation from the flexible price is smaller. This intuition can be formalized in the following lemma.

Lemma 1 [Invoice-currency choice problem] *Consider the model, presented in Section 2, with $\nu = \infty$. To the second-order approximation, an invoice-currency choice problem for firm i in sector $s \in \{N, H\}$, $\max_{h \in \{H,F\}} E_{t-1} Q_{t-1,t} \Pi_{st}^h(i)$, is equivalent to $\min_{h \in \{H,F\}} V_s^h(i)$, where $V_s^H(i) \equiv V(\tilde{p}_{st}(i))$, $V_s^F(i) \equiv V(\tilde{p}_{st}(i) - e_t)$, $V(\cdot)$ is a variance operator, and $\tilde{p}_{st}$ is the firm's flexible price.*

[9] Appendix B discusses the case of monetary policy shocks instead of an exchange rate shock and shows that with nominal wage rigidity the two cases lead to the same results regarding currency choices for price setting.

No complementarities and no intermediate input The approximated invoice-currency choice problem can be solved analytically. As a benchmark, consider the case where there are no price setting complementarities, $\alpha = 0$, and no intermediate input, $\phi_n = \phi_\tau = 0$. In this case, the flexible price for the Home tradable goods firm i is given by $\tilde{p}_{Ht}(i) = mc_{Tt}$ from equation (18), and the marginal cost is given by $mc_{Tt} = w_t - a_t$. Without nominal wage rigidity, the wage is given by equation (10) with $\nu = \infty$ as $w_t = p_t + c_t = m_t$, where the last equality holds from the monetary policy (24). Then, under the assumption of no monetary policy shocks and an exogenous volatility of the exchange rate, the approximated problem is reduced to $\min\{0, \sigma_e^2\}$, where the former and latter correspond to the expected profits under the choices of currencies H and F, respectively. Hence, currency H is chosen as the invoice currency. This result holds irrespective of other firms' invoice currencies, i.e. independent of the fractions, λ_H, λ_F, and λ_N. In other words, in the case of $\alpha = \phi_n = \phi_\tau = 0$, invoice-currency choices exhibit no complementarities.

With nominal wage rigidity, the invoice currency for wages affects invoice-currency choices for prices. Since the wage is given by equation (11) as $w_t = (1 - \lambda_w)\, e_t$, the approximated problem is reduced to $\min\{(1-\lambda_w)^2\sigma_e^2,\, \lambda_w^2\sigma_e^{*2}\}$. This problem implies that currency H is chosen as long as $\lambda_w \geq 1/2$, i.e., no less than half of wages are set in units of currency H. This result underlines the effect of the invoice currency for wages on invoice-currency choices for prices. Such an effect is exerted when wages are sticky.

Under the assumptions of $\alpha = \phi_n = \phi_\tau = 0$, a currency choice problem is exactly the same for the Home non-tradable goods firm i as it is for the Home tradable goods firm. Hence, currency H is chosen without nominal wage rigidity; with such rigidity, currency H is chosen if no less than half of wages are set in units of the same currency.

Complementarities and intermediate input With either price setting complementarities, $\alpha > 0$, or intermediate input, $\phi_\tau, \phi_n > 0$, the approximated problems of tradable and non-tradable goods firms are changed dramatically. Now their flexible prices $\tilde{p}_{Ht}(i)$ and $\tilde{p}_{Nt}(i)$ depend on the tradable price index p_{Tt} because strategic complementarities make tradable goods firms sensitive to the tradable price index as shown in equation (18), or because both tradable and non-tradable goods firms use the tradable consumption goods bundle as an intermediate input.

Let us consider the case of nominal wage rigidity, because it includes the case of no nominal wage rigidity as a special case of $\lambda_w = 1$ as shown in Appendix B. Solving the approximated problem for the Home tradable goods firm i shows that the firm chooses currency F in the Home market if and only if

$$\zeta_H \equiv (1-\alpha)(1-\phi_\tau)(1-\lambda_w) + ((1-\alpha)\phi_\tau + \alpha)(1-(1-\gamma_\tau)\lambda_H - \gamma_\tau\lambda_F) > \frac{1}{2}, \quad (28)$$

where $\gamma_\tau = (1-n)\bar{\gamma}$. The left-hand-side of inequality (28), denoted as ζ_H, corresponds to the exchange rate pass-through to the flexible price $\tilde{p}_{Ht}$, consisting of the weighted average of the pass-through to the wage, $1-\lambda_w$, and the path-through to the tradable goods price index, $1-(1-\gamma_\tau)\lambda_H - \gamma_\tau\lambda_F$, with weights given by $(1-\alpha)(1-\phi_\tau)$ and $(1-\alpha)\phi_\tau + \alpha = 1-(1-\alpha)(1-\phi_\tau)$, respectively. Condition (28) is equivalent to saying that a 1 percent depreciation (appreciation) in the exchange rate causes the flexible price to increase (decrease) by more than 0.5 percent.

Condition (28) clarifies the effects on the invoice currency choice of invoice-currency complementarities for price and wage setting, price setting complementarities and intermediate input, trade openness, and the economic size. First, as more tradable goods firms, both Home and Foreign, set prices in units of currency F, i.e. as λ_H and λ_F become lower, condition (28) becomes more likely to hold. Due to price setting complementarities and/or tradable goods as an intermediate input, a single firm's currency choice is affected by that of other firms in a similar way to the effect of other firms' price setting decisions on the single firm's price setting. This invoice-currency complementarity implies that all tradable goods firms will choose currency F as an invoice currency if condition (28) is satisfied for some λ_H. In short, condition (28) is satisfied for $\lambda_H = 0$ (full dollarization in the tradable sector) if it is satisfied for some $\lambda_H > 0$ (partial or no dollarization in the sector). In addition, as in the case of no price setting complementarities and no intermediate input, with nominal wage rigidity the Home tradable goods firms are more likely to set prices in units of currency F as more wages are set in units of currency F, i.e., as λ_w becomes lower.

Second, an increase in the degree of price setting complementarities, α, or an increase in the intermediate input share, ϕ_τ, makes condition (28) more likely to hold so that tradable firms become more likely to set prices in units of currency F if the pass-through to the flexible price $\tilde{p}_{Ht}$, exerted through the tradable goods price index p_{Tt}, is greater than that exerted through the wage w_t, i.e., if $1-[(1-\gamma_\tau)\lambda_H + \gamma_\tau\lambda_F] > 1-\lambda_w$ holds. As is clear from condition (28), an increase in α or an increase in ϕ_τ puts more weight on the pass-through to the tradable goods price index, as both the price setting complementarities and the intermediate input share make the flexible price sensitive to other tradable goods prices. Given that $1-[(1-\gamma_\tau)\lambda_H + \gamma_\tau\lambda_F] > 1-\lambda_w$ holds, an increase in the weight on the pass-through to the tradable goods price index increases the overall pass-through to the flexible price, making condition (28) more likely to hold.

A third situation that makes condition (28) more likely to hold is an increase in trade openness $\bar{\gamma}$ or a decrease in the Home country's economic size n, if the Foreign tradable goods firms set prices in units of currency F more often than the Home tradable goods firms, i.e., $\lambda_F < \lambda_H$. In other words, in a small open economy $n = 0$ with no home bias $\bar{\gamma} = 1$ and with dominant currency pricing $\lambda_F = 0$, the Home firms are more likely to set prices in units of currency F.

Similar to the tradable goods firms, the non-tradable goods firms set prices in units of currency F if and only if the pass-through to the corresponding flexible price $\tilde{p}_{Nt}$ is greater than 1/2:

$$\zeta_N \equiv (1 - \phi_n)(1 - \lambda_w) + \phi_n \left(1 - (1 - \gamma_\tau)\lambda_H - \gamma_\tau \lambda_F\right) > \frac{1}{2}. \tag{29}$$

This condition coincides with condition (28), with $\alpha = 0$ and ϕ_τ replaced by ϕ_n. Hence, the three implications about an invoice currency choice for the tradable goods firms, mentioned above, also apply to the non-tradable goods firms, except for that regarding price setting complementarities α. Because of the CES aggregator for the non-tradable goods bundle (4), there are no price setting complementarities in the non-tradable sector.

The main results on Home firms' invoice-currency choices for price setting are summarized in the following proposition.

Proposition 2 [Invoice-currency choices for price setting] *Consider the model, presented in Section 2, with no monetary policy shocks and $\nu = \infty$. Both the Home tradable and the non-tradable goods firms choose currency F as an invoice currency if and only if the exchange rate pass-through to their corresponding flexible price is greater than 1/2, where the pass-through is given by ζ_H in equation (28) and ζ_N in equation (29) for the tradable and the non-tradable firms, respectively. These firms are more likely to set prices in units of currency F as:*

a) More tradable goods prices are set in units of currency F (as λ_H and λ_F decrease);

b) The economic size of country H becomes smaller (as n decreases) if $\lambda_F < \lambda_H$;

c) Country H becomes more open to trade (as $\bar{\gamma}$ increases) if $\lambda_F < \lambda_H$;

d) More wages are set in units of currency F (as λ_w decreases);

e) The intermediate input shares ϕ_τ and ϕ_n become higher;

f) The degree of complementarities α increases (for the tradable goods firms only).

Conditions a)-c) in Proposition 2 are broadly consistent with conditions a)-c) in Proposition 1 regarding the impact of monetary policy. Hence, the Home firms are more likely to start using the digital money (i.e. currency F) in an environment where the impact on the Home consumption of the Home monetary policy becomes smaller and that of the Foreign monetary policy becomes greater.

Invoice-currency choices for wage setting As shown in Proposition 2-d), an invoice currency for wages has complementarities for an invoice currency for prices, and therefore it is also worth studying an invoice currency choice for wage setting. Similar in spirit to Lemma 1, the individual wage $w_t(j)$ is set in units of currency H if and only if $V(\tilde{w}_t) \leq V(\tilde{w}_t - e_t)$, where $\tilde{w}_t$ is the flexible wage, given by equation (10), and $V(\cdot)$ is a variance operator. Under the continued assumption of $\nu = \infty$ and no monetary policy shocks, the variance of the flexible wage is zero, $V(\tilde{w}_t) = 0$, so that the wage is set in units of currency H irrespective of the country's economic size and the degree of trade openenss.

To derive richer implications, let us drop the assumption of $\nu = \infty$. But, for analytical tractability, let us consider the economy with no non-tradable sector. In this environment, as shown in Appendix B.2, given currency F as a dominant currency so that prices and wages are set in units of currency F in country F, the Home wage is set in units of currency F if and only if the path-through to the flexible wage, ζ_w, is greater than 1/2:

$$\zeta_w \equiv \frac{(1-\phi_\tau)\left[(\gamma_\tau + \phi_\tau(1-\gamma_\tau^*) - 1)\left(1-(1-\gamma_\tau)\lambda_H - \gamma_\tau\lambda_F\right) - \gamma_\tau\phi_\tau(1-\lambda_w)\right]}{\nu\left[1-(1-\gamma_\tau)\phi_\tau - (1-\gamma_\tau^*)\phi_\tau + (1-\gamma_\tau-\gamma_\tau^*)\phi_\tau^2\right]} > \frac{1}{2}, \quad (30)$$

where the trade openness is assumed to be symmetric between countries H and F so that $\gamma_\tau = (1-n)\bar{\gamma}$ and $\gamma_\tau^* = n\bar{\gamma}$. Because the denominator of the left-hand-side of the inequality (30) is positive, a necessary condition for satisfying the inequality (30) is $\gamma_\tau + \phi_\tau(1-\gamma_\tau^*) > 1$, that is, $\bar{\gamma}(1-n(1+\phi_\tau)) + \phi_\tau > 1$. As a special case, in a small open economy ($n = 0$) with no use of currency F for wage setting ($\lambda_w = 1$) and with more use of currency F for import goods than domestic tradable goods ($\lambda_H > \lambda_F$), condition (30) is more likely to hold as the trade openness $\bar{\gamma}$ increases. Intuitively, in the small open economy with a high degree of trade openness, a depreciation in currency H (an increase in e_t) stimulates labor supply l_t and positively affects the flexible wage $\tilde{w}_t = l_t/\nu$, so that the path-through ζ_w becomes high.

Trade openness brings similar implications for invoice-currency choices, for price setting and for wage setting. But, unlike an invoice currency for prices, that for wages does not feature complementarities. Contrarily, an increasing use of currency F for wages (i.e. a decrease in λ_w) makes condition (30) less likely to hold. Intuitively, as more wages are set in units of currency F, a depreciation in currency H raises the nominal wage and dampens labor supply, so that the pass-through becomes smaller, making condition (30) less likely to hold.

4 Numerical Analyses

This section presents the extended model, which relaxes four critical assumptions of the analytical model presented in Section 2 and examines the robustness of the model's implications for digital dollarization and monetary policy, especially Propositions 1 and 2. First, staggered prices and wages à la Calvo (1983) and Erceg et al. (2000) are introduced instead of one-period-in-advance price and wage setting. Second, shocks are extended to allow for persistence as opposed to i.i.d. shocks. Third, a monetary policy rule for the short-term interest rate is introduced in place of the nominal aggregate as a monetary policy instrument. Fourth, the labor supply elasticity of $\nu = \infty$ is relaxed to be $\nu < \infty$. In addition, as the extended model does not assume subsidies, markup distortions remain.

In what follows, Section 4.1 presents the detail of the first three extensions. Section 4.2 parameterizes the model to numerically analyze the extended model. Using the extended model, Section 4.3 examines the implications of digital dollarization on monetary policy transmission (Proposition 1). Section 4.4 examines under what conditions digital dollarization can deepen (Proposition 2). Section 4.5 analyzes whether monetary policy, Home and/or Foreign, can block or facilitate digital dollarization.

4.1 Extensions

Nominal rigidities Staggered prices and wages à la Calvo (1983) and Erceg et al. (2000) are introduced in place of one-period-in-advance price and wage setting. Specifically, in every period, firms can reset prices with probability $1 - \xi_p$ with $0 < \xi_p < 1$ and households can reset wages with probability $1 - \xi_w$ with $0 < \xi_w < 1$. In these staggered price and wage settings, either the Home or the Foreign currency is used as the invoice currency. As considered in Section 3.2, an invoice-currency choice problem is formulated such that a currency, either Home or Foreign, that delivers a higher expected discounted sum of profits or utilities is chosen. The price and wage settings in country F are symmetric to those in country H. The description of the price and wage setting problems and the derivation of the equilibrium conditions including those of the currency-choice problems can be found in Appendix C.

Persistent shocks As in the model presented in Section 2, there are three shocks: productivity, exchange rate, and monetary policy shocks. These shocks were assumed to be i.i.d. in Section 2, but in this section the first two shocks are extended to allow for persistence. Specifically, the productivity $a_t \equiv \log(A_t)$ is assumed to follow the AR(1) process:

$$a_t = \rho_a a_{t-1} + \epsilon_{at}, \quad \text{with} \quad 0 \leq \rho_a < 1,$$

where $\epsilon_{a,t} \sim$ i.i.d.$N(0, \sigma_a^2)$. Similarly, the exchange rate shock z_{et} is assumed to follow the AR(1) process:

$$z_{et} = \rho_e z_{et-1} + \epsilon_{et}, \quad \text{with} \quad 0 \leq \rho_e < 1,$$

where $\epsilon_{et} \sim$ i.i.d.$N(0, \sigma_e^2)$. The shock processes for country F are symmetric to those of country H.

Monetary policy rules The monetary policy for the nominal aggregate, (24), is replaced by a simple interest rate rule that responds to the current inflation and the lagged interest rate, given by

$$\log\left(\frac{R_t}{R}\right) = \rho_r \log\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r) r_\pi \log(\pi_t) + \epsilon_{rt}, \text{ with } 0 \leq \rho_r < 1, \tag{31}$$

where R is the interest rate in steady state and $\epsilon_{rt} \sim$ i.i.d.$N(0, \sigma_r^2)$ is a monetary policy shock. A monetary policy rule for country F is symmetric to that of country H.

4.2 Baseline Parameterization

The objective of baseline parameterization is to numerically examine how robust the digital dollarization and monetary policy implications, namely Propositions 1 and 2, are in the extended model. To parameterize the model, the functional form of the consumption aggregator $G(\cdot)$ is specified by following Dotsey and King (2005) as

$$G(x) = \frac{\theta_\tau}{\theta_\tau - 1 - \varepsilon}\left[\left(1 - \frac{\varepsilon}{\theta_\tau}\right)x + \frac{\varepsilon}{\theta_\tau}\right]^{\frac{\theta_\tau - 1 - \varepsilon}{\theta_\tau - \varepsilon}} - \frac{\theta_\tau}{\theta_\tau - 1 - \varepsilon} + 1. \tag{32}$$

In this aggregator, parameter $\theta_\tau = -g'(x)x/g(x)|_{x=1}$ corresponds to the elasticity of substitution among tradable goods, evaluated at $x = 1$, where $g(x) = G'(x)$; parameter $\varepsilon = \theta_\tau'(x)x/\theta_\tau(x)|_{x=1}$ corresponds to the superelasticity evaluated at $x = 1$, where $\theta_\tau(x) = -g'(x)x/g(x)$. If the superelasticity is zero, $\varepsilon = 0$, the aggregator $G(\cdot)$ is collapsed to a CES aggregator.

There are three sets of parameters: standard parameters and shock parameters, which are fixed in the numerical analysis, and key parameters, which are varied in the analysis. Although baseline values are set for key parameters, the central focus of the numerical analysis is how implications for digital dollarization and monetary policy change, if at all, in response to changes in these parameter values. Hence, for these key parameters, baseline values should be interpreted as reference values. Table 1 summarizes all parameter values.

The unit of time in the model is quarterly. Regarding standard parameters, the preference discount factor is set at $\beta = 0.98^{1/4}$, implying an annual real interest rate of two percent in steady state. The Frisch labor supply elasticity is set at $\nu = 1$, which is within standard parameter values used in the business cycle literature. The parameter governing the disutility of labor, ψ, is set in such a way that the labor is normalized to unity, $L = 1$, in steady state. The elasticities of substitution for non-tradable goods and specialized labor are set at $\theta_n = \theta_w = 11$, implying a corresponding markup of ten percent. The elasticity of substitution for tradable goods is set at $\theta_\tau = 3$, which is consistent with the empirical evidence of Feenstra et al. (2018). The degree of nominal rigidities for prices and wages is set at $\xi_p = \xi_w = 0.75$, implying an average duration of prices/wages of one year. In the monetary policy rule, the coefficient on inflation and the persistence parameter are set at $\phi_\pi = 2$ and $\rho_r = 0.8$, respectively, which are within standard values estimated in the literature such as Justiniano et al. (2010). To focus on the use of digital money (i.e. currency F) in country H and to reflect the empirical fact reported by Goldberg and Tille (2016) and Gopinath et al. (2020), the dominant currency pricing is assumed so that currency F is exclusively used for exports and imports as well as in country F: $\lambda_N^* = \lambda_H^* = \lambda_F^* = \lambda_w^* = \lambda_F = 0$.

Regarding shock parameters, the persistence of shocks is assumed be relatively high: $\rho_a = \rho_e = 0.95$. The standard deviations of productivity, exchange rate, and monetary

Table 1: Parameters

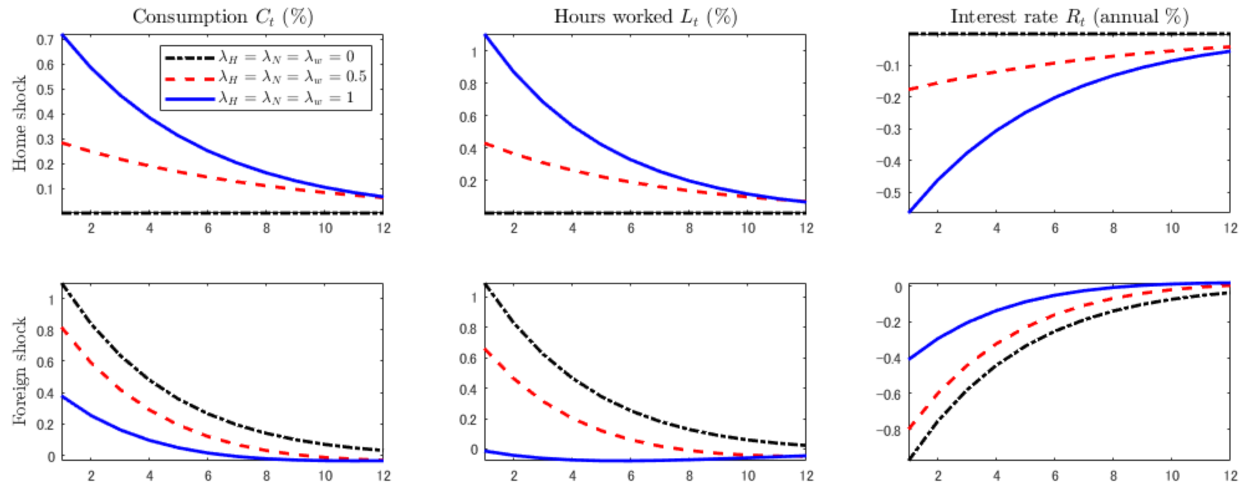
Parameter	Value	Parameter description	Value source/description
		Standard parameters	
β	$0.98^{1/4}$	Preference discount factor	Annual real interest rate of 2% in SS
ν	1	Frisch labor supply elasticity	Standard
ψ	0.424	Disutility of labor	$L = 1$ in SS
θ_n, θ_w	11	Non-tradable goods/labor elasticity	Markup of 10%
θ_τ	3	Tradable goods elasticity	Feenstra et al. (2018)
ξ_p, ξ_w	0.75	Nominal rigidities	Average duration of 1 year
ϕ_π	2	Monetary policy rule, inflation	Standard
ρ_r	0.8	Monetary policy rule, persistence	Standard
λ_N^*, λ_H^*, λ_F^*, λ_w^*, λ_F	0	Currency choice	Dominant currency pricing
		Shock parameters	
ρ_a, ρ_e	0.95	Shock persistence	High persistence
σ_a, σ_e	0.01	SD of productivity and exchange rate shocks	1% SD quarterly
σ_r	0.0025	SD of monetary policy shocks	0.25% SD quarterly
		Key parameters (baseline values)	
λ_N, λ_H, λ_w	1	Currency choice	Dominant currency pricing
n	0.01	Size of the Home country	Small country
γ_n	0.44	Share of non-tradable goods	Lombardo and Ravenna (2012)
ϕ_n, ϕ_τ	0.33	Share of intermediate input	Lombardo and Ravenna (2012)
$\bar{\gamma}$	0.433	Trade openness	Trade-to-output ratio of 0.5
α	0.6	Tradable goods pricing complementarities	Gopinath and Itskhoki (2011)

Note: SS and SD denote steady state and standard deviation, respectively.

policy shocks are set at 1%, 1%, and 0.25%, respectively. Although the three shocks are simultaneously considered in using the second-order approximation of the model, it is worth emphasizing that the main results of such an analysis, to be presented in Sections 4.4 and 4.5, are driven mainly by the productivity shocks, which are also main shocks in the standard NOEM literature such as Obstfeld and Rogoff (1995) and Corsetti and Pesenti (2009).

Regarding key parameters, as a baseline, the Home currency pricing is assumed in country H: $\lambda_N = \lambda_H = \lambda_w = 1$. Given that most countries in the world are relatively small, with a few 'large' exceptions, the economic size of country H is set at $n = 0.01$ as the baseline. The share of non-tradable goods is set at $\gamma_n = 0.44$, the average value of OECD countries reported in Lombardo and Ravenna (2012). The share varies from 0.25 in relatively small countries to 0.8 in relatively large countries. Given that the intermediate input share is 0.54 on average over OECD countries and the average tradable input share

Figure 1: Impulse responses to monetary policy shocks



Note: Consumption and hours worked are measured in percentage deviation from the steady state. The interest rate is measured in annual percent difference from the steady state.

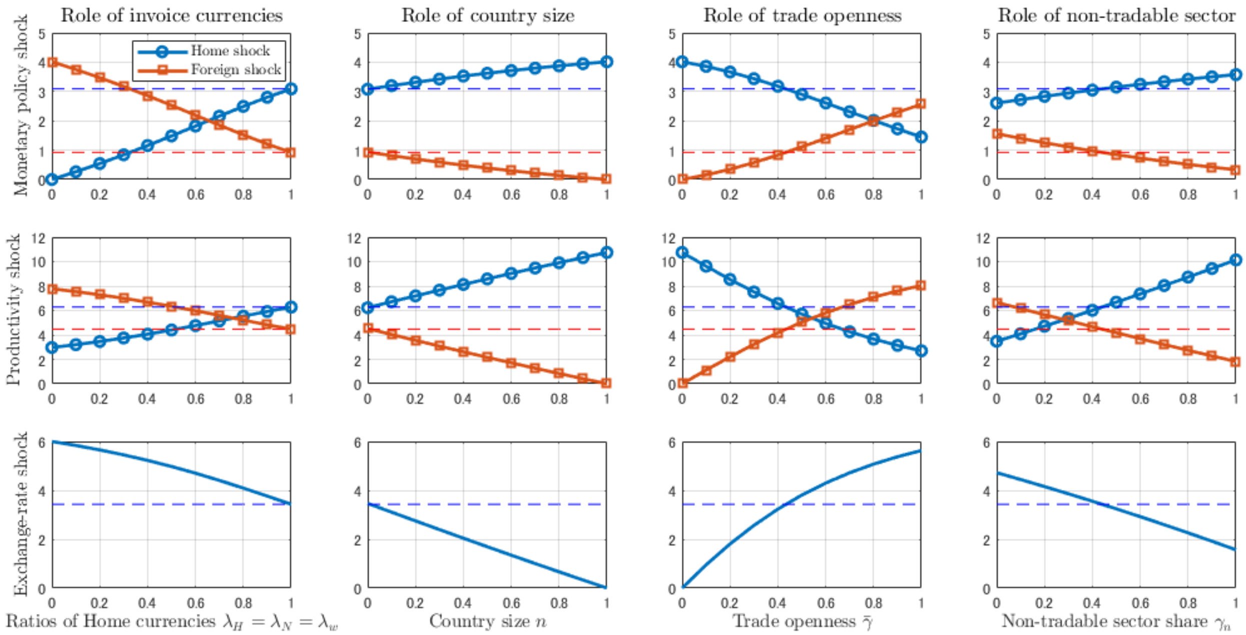
of consumption is 0.66 according to Lombardo and Ravenna (2012), the intermediate input share is set at $\phi_n = \phi_\tau = 0.36$ $(= 0.54 \times 0.66)$. The intermediate input share ranges from 0.41 to 0.67 according to the OECD input-output tables, while the tradable input share of consumption ranges from 0.3 to 0.8 according to Lombardo and Ravenna (2012). Trade openness is set at $\bar{\gamma} = 0.433$ in such a way that the trade-to-output ratio in steady state is equal to the average value of 0.5 among OECD countries, given other parameter values, where the trade-to-output ratio is defined by the sum of export and import divided by the total output. The OECD input-output tables show that the trade-to-output ratio ranges from less than 0.2 to over 1. Finally, instead of the superelasticity ε in the aggregator (32), a value is assigned to $\alpha = 0.6$, which is consistent with the evidence reported by Gopinath and Itskhoki (2011).[10]

4.3 Monetary Policy Transmission

How does digital dollarization – the use of currency F in country H – affect the effectiveness of the Home monetary policy? Figure 1 plots the impulse responses of the Home consumption C_t, inflation π_t, and the interest rate R_t to the expansionary monetary policy shocks of the magnitude of one standard deviation, originated in countries H (top panels) and F (bottom panels) for three cases: no dollarization ($\lambda_H = \lambda_N = \lambda_w = 1$); partial dollarization ($\lambda_H = \lambda_N = \lambda_w = 0.5$); and full dollarization ($\lambda_H = \lambda_N = \lambda_w = 0$). Consistent

[10]Parameter α can be derived as a function of ε as $\alpha = \varepsilon/(\theta_\tau - 1 + \varepsilon)$.

Figure 2: Cumulative effects on the Home consumption (%, over 8 quarters)



Note: The vertical axis of each panel denotes percentage deviation from the steady state. The two dashed lines in each panel represent the cumulative effects of the Home and Foreign shocks, respectively, under the baseline parameter values.

with Proposition 1 for the simple model, a deepening in dollarization weakens the effect of the Home monetary policy on the Home consumption and strengthens that of the Foreign monetary policy. In particular, when all agents set prices and wages using currency F, the Home monetary policy becomes completely ineffective, i.e., the responses of the Home consumption, hours worked, and the nominal rate become zero. The nominal rate does not move in spite of the Home monetary policy shock because the inflation response cancels out the effect of the policy shock. In this case, in the system of the equilibrium conditions, the Home inflation π_t appears only in the consumption Euler equation and the monetary policy rule in country H. The Home inflation becomes de-facto flexible so that the Home monetary policy has no real effects. The inflation is determined simply because the central bank continues to be able to control the short-term interest rate in units of currency H.

The effects of the other shocks are also affected by the degree of dollarization in country H. The left three panels of Figure 2 plot the cumulative responses over the first eight quarters of the Home consumption to monetary policy shocks (top), productivity shocks (middle), and an exchange rate shock (bottom), as a function of the degree of dollarization ($\lambda_H = \lambda_N = \lambda_w$). The shock magnitude is one standard deviation for all shocks. Similar to the effects of monetary policy shocks, as dollarization deepens (as $\lambda_H = \lambda_N = \lambda_w$ decreases), the effects of the Home productivity shock is weakened and those of the Foreign productivity shock are strengthened (left middle panel); and the effects of the exchange rate shock is strengthened (left bottom panel). In other words, country H becomes more vulnerable to the Foreign shocks, including the exchange rate shock, as dollarization deepens.

Figure 2 also shows the roles played by the country's economic size (n), the trade openness ($\bar{\gamma}$), and the share of the non-tradable sector (γ_n) in the effects of the three shocks. Consistent with Proposition 1, the effect of the Home monetary policy shock becomes smaller and that of the Foreign monetary policy shock becomes larger as the country's economic size becomes smaller (second-left top panel), the country becomes more open to trade (second-right top panel), and the share of the non-tradable sector gets smaller (right top panel). These implications also hold for the Home and Foreign productivity shocks (middle panels) and the exchange rate shock (bottom panels). To summarize, the analysis implies that a smaller and more open country is more susceptible to shocks originating from foreign countries and foreign exchange markets.

4.4 Invoice-Currency Choices

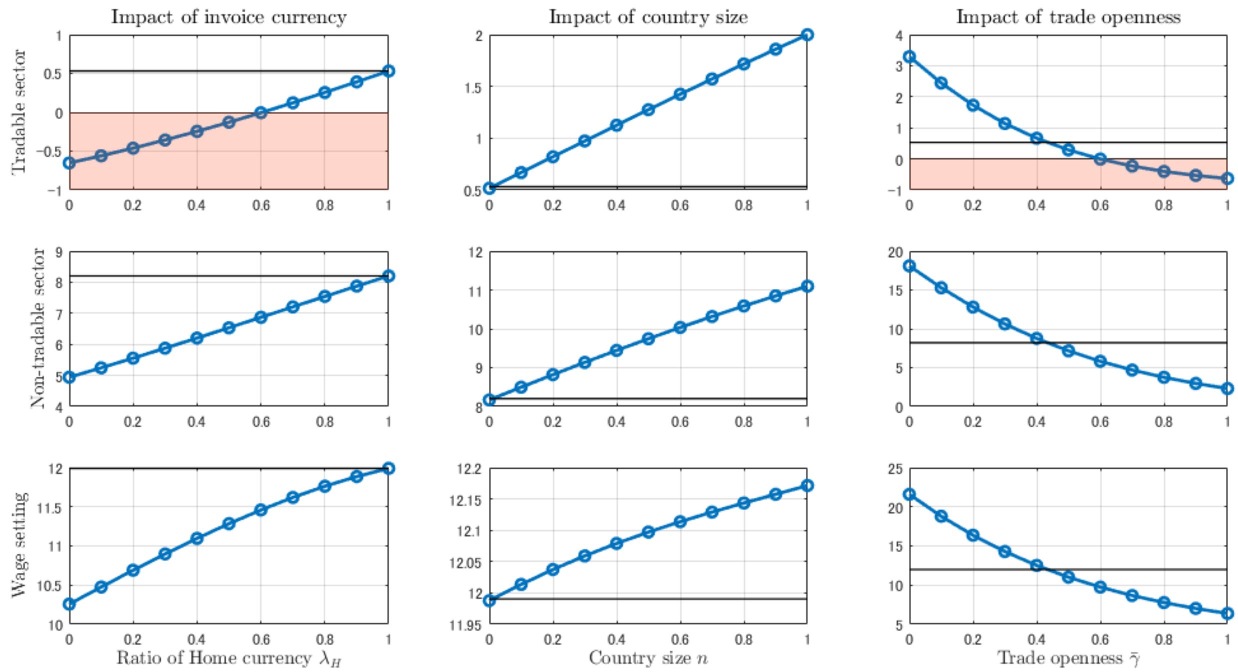
The previous subsection showed that the effects of the Home monetary policy weaken as dollarization deepens in country H. Then, under what circumstances do the firms and households choose currency F as an invoice currency? This subsection addresses the question numerically using the extended model.

An invoice-currency choice problem is formulated for the extended model as for the simple analytical model studied in Section 3.2. Let Π_{s0}^{h} denote the expected present discounted value of choosing currency $h = H$ or currency $h = F$, conditional on the initial state being the steady state, for the Home tradable goods firm ($s = H$), the Home non-tradable goods firm ($s = N$), and for the Home household ($s = w$). Then, the invoice-currency choice problem is formulated as $\max_{h\in\{H,F\}} \Pi_{s0}^{h}$. In solving this problem, the value Π_{s0}^{h} is approximated around the steady state by using the second-order approximation as in Schmitt-Grohé and Uribe (2007). The derivation of equations for Π_{s0}^{h} can be found in Appendix C.

Dollarization in the tradable sector Figure 3 plots the net benefits of choosing currency H, $(\Pi_{s0}^{H} - \Pi_{s0}^{F})/\Pi_{s}^{H} \times 100$, where Π_{s}^{h} is the corresponding value in steady state.[11] By

[11] The value of choosing currency H coincides with that of choosing currency F in steady state: $\Pi_{s}^{H} = \Pi_{s}^{F}$.

Figure 3: Net benefits of choosing currency H



Note: In each panel, a vertical axis denotes a percentage difference between the value of choosing currency H and that of choosing currency F, divided by the value of choosing currency H in steady state. A solid horizontal line denotes the net benefits of choosing currency H under the baseline parameter values.

construction, the positive (negative) value means that currency H (F) is preferred by firms or households.

Figure 3 provides four implications for dollarization. First, the left three panels of the figure show the roles of invoice-currency complementarities for invoice-currency choices and possible dollarization. The panels plot the net benefits of choosing currency H as a function of λ_H – the share of currency H pricing in the Home tradable sector – for three types of private agents: the tradable goods firms (top), the non-tradable goods firms (middle), and the wage-setting households (bottom). For all these panels, the net benefits are increasing in λ_H, implying that as more tradable firms start using currency F it becomes less attractive to choose currency H. Such complementarities affect the tradable sector particularly strongly: currency F would be chosen, i.e., the net benefits sink into negative territory (shaded area in the top left panel) if more than 40 percent of the tradable goods firms use currency F.

Second, the country's economic size n (middle panels) and degree of trade openness $\bar{\gamma}$ (left panels) also affect the invoice-currency choices and possible dollarization. For all three types of agent, the net benefits of choosing currency H decrease the smaller the country's economic size and the greater the degree of trade openness. In a smaller country that is more open to trade and has more tradable goods firms that set prices in units of currency F, the firms and households have a higher incentive to choose currency F as an invoice currency. This observation is consistent with Proposition 2 for the simple model presented in Section 2.

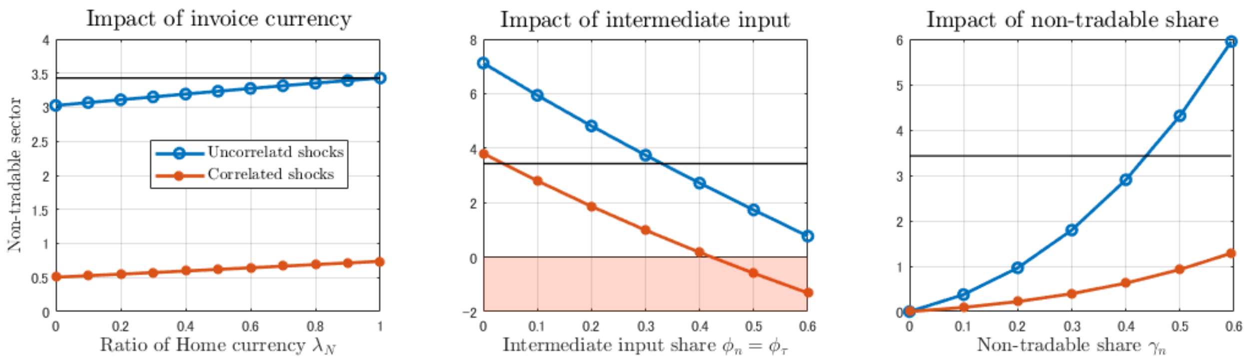
Third, dollarization in the tradable sector may not be far from reality. The top right panel of Figure 3 shows that even when all the other tradable goods firms use the Home currency, every single Home tradable goods firm has an incentive to switch from currency H to F if the trade openness is higher than $\bar{\gamma} = 0.6$. Such a relatively high trade openness is not unrealistic: in the calibration presented in Section 4.2, $\bar{\gamma} = 0.6$ corresponds to the trade-output ratio of 0.69. Such a level is not uncommon for small open real-world economies. In addition, because of the currency choice complementarities mentioned above, as more tradable goods firms start using currency F, currency F would be preferred increasingly. Then, as indicated by the negative net benefits at $\lambda_H = 0$ in the top left panel, once dollarization occurs in the tradable sector, it is stable, as all the tradable goods firms continue to prefer currency F.

Fourth, although dollarization can occur in the tradable sector, under the parameter values considered, it is unlikely in the non-tradable sector or the labor market. As shown in the middle and bottom panels of Figure 3, the net benefits are all well above zero. As discussed in Section 3, the tradable goods prices are affected by price setting complementarities (α) and input-output linkages (ϕ_τ), while such complementarities are absent in the non-tradable sector and both the complementarities and the input-output linkages are absent in the labor market. In other words, because the Home tradable goods prices are heavily affected by the prices of imported Foreign tradable goods under the complementarities and input-output linkages, the Home tradable sector is more vulnerable to dollarization than the non-tradable sector and the labor market.

Dollarization in the non-tradable sector Consider the case of high trade openness of $\bar{\gamma} = 0.7$ where full dollarization occurs in the tradable sector ($\lambda_H = 0$). In this situation, can dollarization ensue in the non-tradable sector as well?

Figure 4 shows the net benefits of choosing the Home currency in the non-tradable sector in the case of $\lambda_H = 0$ and $\bar{\gamma} = 0.7$. In the case of baseline parameter values, denoted as "Uncorrelated shocks," the net benefits of choosing H decline as more non-tradable goods firms start using currency F (as λ_N decreases), the intermediate input shares, ϕ_n and ϕ_τ, increase, and the non-tradable share γ_n decreases. These results are consistent with

Figure 4: Net benefits of choosing currency H when $\lambda_H = 0$ and $\bar{\gamma} = 0.7$



Note: "Uncorrelated shocks" corresponds to the model presented in Section 4.1 and "Correlated shocks" corresponds to the same model but with the Home productivity shock given by $(\epsilon_{at} + \epsilon^*_{at})/2$. A solid horizontal line denotes the net benefits of choosing currency H under the baseline parameter values but with $\lambda_H = 0$ and $\bar{\gamma} = 0.7$.

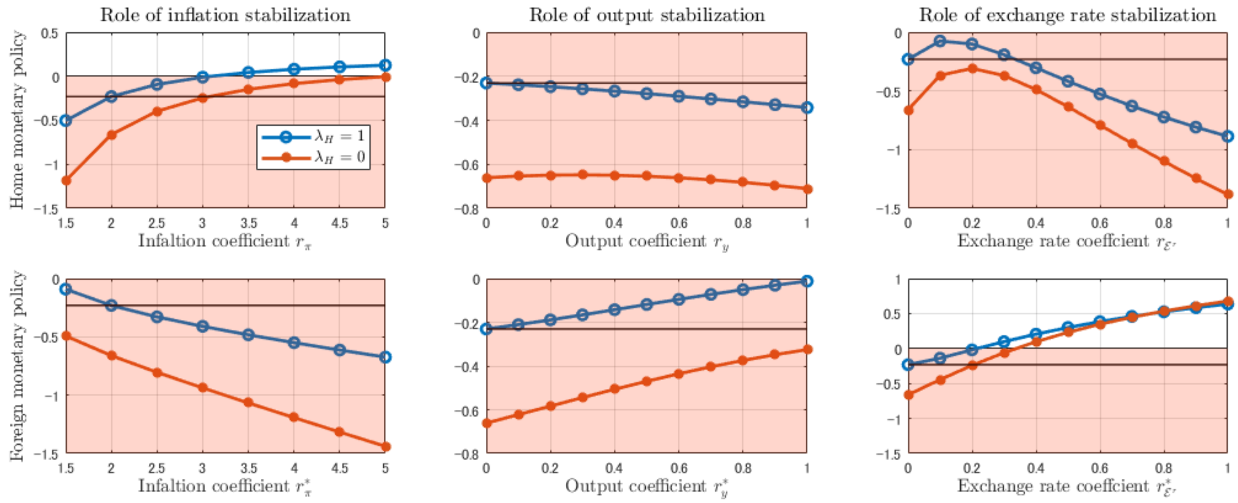
Proposition 2 for the non-tradable sector. Still, the net benefits are all positive, implying that the non-tradable goods firms continue to prefer currency H. However, if the Home productivity shock is correlated with the Foreign productivity shock, given by $(\epsilon_{at} + \epsilon^*_{at})/2$ in place of ϵ_{at}, the net benefits shift down, and they become negative when the intermediate input shares are relatively high (middle panel of Figure 4). By making the Home economy co-move with the Foreign economy, such a correlated productivity shock makes currency H pricing less attractive and can induce a shift to currency F pricing. It is worth noting that in all cases shown in Figure 4, currency F continues to be preferred in the tradable sector (so that $\lambda_H = 0$ is rationalized) but not in the labor market.

To summarize, the numerical analysis in this subsection suggests that in a smaller country that is more open to trade and has a greater tradable sector and stronger input-output linkages, dollarization is more likely to occur in the tradable and the non-tradable sectors, if not in the labor market. In addition, if the Foreign productivity shock has a global impact so that it also drives the Home productivity to some degree, the likelihood of dollarization increases further.

4.5 Roles of Monetary Policy in Dollarization

Thus far, the analysis on dollarization has assumed a fixed monetary policy rule, but in practice, a central bank may consider trying to counter dollarization. Depending on the monetary policy stance on inflation and other target variables, monetary policy may be able to block dollarization. To illustrate the role of monetary policy in dollarization, the

Figure 5: Roles of monetary policy in dollarization:
net benefits of choosing currency H in the tradable sector when $\bar{\gamma} = 0.7$



Note: In each panel, a solid horizontal line denotes the net benefits of choosing currency H under the baseline parameter values but with $\bar{\gamma} = 0.7$.

Home monetary policy (31) is modified to incorporate output growth and the real exchange rate:

$$\log\left(\frac{R_t}{R}\right) = \rho_r \left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left[r_\pi \log(\pi_t) + r_y \log\left(\frac{Y_t}{Y_{t-1}}\right) + r_{\mathcal{E}^r} \log\left(\frac{\mathcal{E}_t^r}{\mathcal{E}^r}\right)\right] + \epsilon_{rt}, \quad (33)$$

where parameters r_π, r_y, and $r_{\mathcal{E}^r}$ govern the central bank's stance on the stabilization of inflation, output, and the real exchange rate, respectively. The rule (33) is classified as a simple and implementable monetary policy rule (Schmitt-Grohé and Uribe (2007)) as it depends only on a few variables that are observable in practice.

Consider the model economy with the monetary policy rule (33) and baseline parameter values with $r_\pi = 2$ and $r_y = r_{\mathcal{E}^r} = 0$ except that the trade openness is set at $\bar{\gamma} = 0.7$. In this economy, the net benefits of choosing currency H are negative in the tradable sector as shown in the top right panel of Figure 3, so that the tradable goods firms prefer to switch to currency F. A question is how the monetary policy stance on inflation, output, and exchange rate stabilization affects the firms' incentive to use currency F as an invoice currency.

The upper panels of Figure 5 show the effects of the Home monetary policy on the incentive of using currency H in the tradable sector in the cases of no dollarization ($\lambda_H = 1$) and full dollarization ($\lambda_H = 0$) in the sector, respectively. As the coefficient on inflation, r_π, increases, the incentive to use currency H increases (the upper left panel). At

$r_\pi = 5$, the net benefits become positive in both cases of $\lambda_H = 1$ and $\lambda_H = 0$, implying that dollarization can be ruled out by such a monetary policy rule. However, a positive stance on output stabilization ($r_y > 0$) and a strong positive stance on real exchange rate stabilization ($r_{\mathcal{E}^r} > 0.3$) are counterproductive: such policy rules facilitate dollarization instead (the upper middle and right panels). Intuitively, monetary policy that strongly focuses on inflation stabilization restrains nominal disruptions in units of currency H by stabilizing the Home inflation rate, which makes setting prices in units of currency H more attractive.[12] Stabilizing the real exchange rate contributes to stabilizing import prices in units of currency H, which *ceteris paribus* makes currency H more attractive. But, a too strong stance toward real exchange rate stabilization undermines inflation stabilization and thereby facilitates dollarization. Actually, when the stance is strong enough ($r_{\mathcal{E}^r} > 0.5$), the incentive to use currency H becomes negative for price setting in the non-tradable sector and wage setting as well, so that the whole economy would switch to full dollarization.

The result of the Home monetary policy suggests that the Foreign monetary policy can also affect dollarization in country H. The lower panels of Figure 5 show such effects in the cases of $\lambda_H = 1$ and $\lambda_H = 0$, respectively. Overall, the effects of the Foreign monetary policy mirror those of the Home monetary policy upside down: a strong stance on inflation stabilization facilitates dollarization in country H, but a strong stance on output and real exchange rate stabilization discourages it. In particular, a strong stance on real exchange rate stabilization of $r_{\mathcal{E}^r} > 0.3$ completely blocks dollarization in country H.

The monetary policy analysis in this subsection suggests that monetary policies, both Home and Foreign, can have significant impacts on dollarization in country H. Although optimal monetary policy and welfare implications are beyond the scope of this paper, the incentive to use currency H in the tradable sector continues to be negative even under the cooperative Ramsey policies with the policy instruments being the interest rates R_t and R_t^* in the economy with $\bar{\gamma} = 0.7$.[13] This observation suggests that in spite of the potential capacity of monetary policy to block dollarization, such a policy may conflict with other objectives from a social welfare perspective.

[12] A similar implication is derived by Ize and Yeyati (2003), who consider financial dollarization in which domestic agents hold foreign assets and show that inflation targeting helps reduce financial dollarization.

[13] The Ramsey policy chooses R_t and R_t^* to maximize the weighted sum of the average household utilities of countries H and F with equal weights, subject to the competitive equilibrium conditions except for the monetary policy rules. The problem is solved by the matlab toolbox developed by Bodenstein et al. (2019).

5 Conclusion

Against the background of digital innovations and possible further progress in digital money, this paper considers monetary policy in a world economy in which prices and wages can be set in units of any currency in any country. Building a two-country open economy model with nominal rigidities, the paper exclusively focuses on a unit of account as the role of money by considering a cash-less economy. It then explores, both analytically and numerically, the implications of digital dollarization – price and wage setting in units of digital money, denominated in and pegged to a foreign currency – for monetary policy and under what conditions digital dollarization can occur. The findings are three-fold. First, the capacity of monetary policy to affect the real economy weakens as digital dollarization deepens in the country. Second, digital dollarization is more likely to occur in a country with a smaller economic size, with less home bias, and with a greater tradable sector and stronger input-output linkages. Third, depending on their stance on the stabilization of inflation, output, and the real exchange rate, monetary policies, both domestic and foreign, can have significant impacts on digital dollarization.

The paper is concluded with some caveats regarding the three findings. First, the paper has exclusively focused on a unit of account as the role of money, in line with the New Keynesian literature such as Woodford (2003), and has abstracted away other roles of money such as a medium of exchange and a store of value. In this respect, this paper complements the emerging literature on digital money as a medium of exchange, including Fernández-Villaverde and Sanches (2019), Schilling and Uhlig (2019), and Benigno (2019). Second, this paper has considered frictionless costless digital money without specifying its detail. For example, if digital money takes the form of stablecoins, an increase in the circulation of such digital money would increase the demand for safe assets, including government bonds and central bank digital currencies, if any, that back up the value of the digital money, which may have some consequences for monetary policy. Finally, this paper has considered digital money in units of a single currency. However, in practice, digital money can be denominated in and pegged to a synthesized currency, backed up by safe assets in units of multiple currencies. The creation of a synthetic currency could affect the international monetary system, as argued by Carney (2019), and taking a synthetic currency into account in currency choice problems may carry different implications for monetary policy.

References

Adrian, T. and T. Mancini-Griffoli (2019). The rise of digital money. Fintech notes, International Monetary Fund.

Bahaj, S. and R. Reis (2020). Jumpstarting an international currency. Bank of England Staff Working Paper No.874.

Batini, N., P. Levine, and J. Pearlman (2010). Monetary rules in emerging economies with financial market imperfections. In J. Gali and M. J. Gertler (Eds.), *International Dimensions of Monetary Policy*, pp. 251–311. University of Chicago Press.

Baughman, G. and J. Flemming (2020). Global demand for basket-backed stablecoins. Finance and Eeconomics Discussion Series, Division of Research & Statistics and Monetary Affairs, Federal Reserve Board 2020-048.

Benigno, G. and P. Benigno (2008). Exchange rate determination under interest rate rules. *Journal of International Money and Finance 27*, 971–993.

Benigno, P. (2019). Monetary policy in a world of cryptocurrencies. mimeo.

Benigno, P., L. M. Schilling, and H. Uhlig (2019). Cryptocurrencies, currency competition, and the impossible trinity. mimeo.

Bodenstein, M., L. Guerrieri, and J. LaBriola (2019). Macroeconomic policy games. *Journal of Monetary Economics 101*, 64–81.

Brunnermeier, M. K., H. James, and J.-P. Landau (2019). The digitalization of money. mimeo.

Calvo, G. A. (1983). Staggered prices in a utility maximizing framework. *Journal of Monetary Economics 12*, 383–398.

Calvo, G. A. and C. A. Végh (1992). Currency substitution in developing countries: An introduction. IMF Wokring Paper.

Carney, M. (2019). The growing challenges for monetary policy in the current international monetary and financial system. Speech at Jackson Hole Symposium 2019.

Castillo, P. B. (2006). Optimal monetary policy and endogenous price dollarization. mimeo, Central Reserve Bank of Peru.

Corsetti, G. and P. Pesenti (2009). The simple geometry of transmission and stabilization in closed and open economies. In R. Clarida and F. Giavazzi (Eds.), *NBER International Seminar on Macroeconomics 2007*, pp. 65–116. University of Chicago Press.

Devereux, M. B., C. Engel, and P. E. Storgaard (2004). Endogenous exchange rate pass-through when nominal prices are set in advance. *Journal of International Economics 63*, 263–291.

Doepke, M. and M. Schneider (2017). Money as a unit of account. *Econometrica 85*(5), 1537–1574.

Dotsey, M. and R. G. King (2005). Implications of state-dependent pricing for dynamic macroeconomic models. *Journal of Monetary Economics 52*(1), 213–242.

Engel, C. (2006). Equivalence results for optimal pass-through, optimal indexing to exchange rates, and optimal choice of currency for exporting pricing. *Journal of European Economic Association 4*(6), 1249–1260.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000). Optimal monetary policy with staggered wage and price contracts. *Journal of Monetary Economics 46*(2), 281–313.

Feenstra, R. C., P. Luck, M. Obstfeld, and K. N. Russ (2018). In search of the armington elasticity. *Review of Economics and Statistics 100*(1), 135–150.

Felices, G. and V. Tuesta (2007). Monetary policy in a dual currency environment. Banco Central de Reserva del Perú, Working Paper Series 2007-006.

Fernández-Villaverde, J. and D. Sanches (2019). Can currency competition work? *Journal of Monetary Economics 106*, 1–15.

G7 Working Group on Stablecoins (2019). Investigating the impact of global stablecoins. A report by the G7 Working Group on Stablecoins.

Goldberg, L. S. and C. Tille (2016). Micro, macro, and strategic forces in international trade invoicing: Synthesis and novel patterns. *Journal of International Economics 102*(C), 173–187.

Gopinath, G., E. Boz, C. Casas, F. J. Díez, P.-O. Gourinchas, and M. Plagborg-Møller (2020). Dominant currency paradigm. *American Economic Review 110*(3), 677–719.

Gopinath, G. and O. Itskhoki (2011). In search of real rigidities. In D. Acemoglu and M. Woodford (Eds.), *NBER Macroeconomics Annual 2010, Volume 25*, pp. 261–309. University of Chicago Press.

Gopinath, G., O. Itskhoki, and R. Rigobon (2010). Currency choice and exchange rate path-through. *American Economic Review 100*(1), 304–336.

Itskhoki, O. and D. Mukhin (2019). Exchange rate disconnect in general equilibrium. mimeo.

Ize, A. and E. L. Yeyati (2003). Financial dollarization. *Journal of International Economics 59*, 323–347.

Jordan, T. J. (2019). Currencies, money and digital tokens. Speech at 30th anniversary of the WWZ and VB̈O, University of Basel.

Justiniano, A., G. Primiceri, and A. Tambalotti (2010). Investment shocks and business cycles. *Journal of Monetary Economics 57*(2), 132–145.

Kareken, J. and N. Wallace (1981). On the indeterminacy of equilibrium exchange rates. *Quarterly Journal of Economics 96*, 207–222.

Kimball, M. S. (1995). The quantitative analytics of the basic neomonetarist model. *Journal of Money, Credit and Banking 17*(4), 1241–1277.

Kumamoto, M. and H. Kumamoto (2014). Currency substitution and monetary policy under the incomplete financial market. *Japanese Journal of Monetary and Financial Economics 2*(2), 16–45.

Kuroda, H. (2019). Payments innovations and the role of central banks: Addressing challenges posed by stablecoins. Speech at the Symposium for the 35th Anniversary of the Center for Financial Industry Information Systems.

Lagarde, C. (2017). Central banking and fintech – a brave new world? Speech at the Bank of England conference, London.

Lagos, R. and R. Wright (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(3), 463–484.

Lombardo, G. and F. Ravenna (2012). The size of the tradable and non-tradable sectors: Evidence from input-output tables for 25 countries. *Economic Letters 116*, 558–561.

Loyo, E. (2002). Imaginary money against sticky relative prices. *European Economic Review 46*, 1073–1092.

Lucas, R. E. J. and N. L. Stokey (1987). Money and interest in a cash-in-advance economy. *Econometrica 55*(3), 491–513.

Mukhin, D. (2018). An equilibirum model of the international price system. mimeo.

Obstfeld, M. and K. Rogoff (1995). Exchange rate dynamics redux. *Journal of Political Economy 103*(3), 624–660.

Schilling, L. and H. Uhlig (2019). Some simple bitcoin economics. *Journal of Monetary Economics 106*, 16–26.

Schmitt-Grohé, S. and M. Uribe (2007). Optimal simple and implementable monetary and fiscal rules. *Journal of Monetary Economics 54*, 1702–1725.

Végh, C. A. (2013). *Open Economy Macroeconomics in Developing Countries*, Chapter 15. The MIT Press.

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Appendix

A Model and Analytical Results

This appendix presents the analytical model discussed in Section 2.

A.1 Households

Consumption and saving problem The Lagrangean representation of the household problem – maximizing the utility (1) subject to the budget constraint (2) – can be written as

$$\begin{aligned}\mathcal{L} &= E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\frac{1}{\sigma}} C_t(j)^{1-\frac{1}{\sigma}} - \frac{\psi}{1+\frac{1}{\nu}} L_t(j)^{1+\frac{1}{\nu}} \right. \\ &\left. + \Lambda_t(j) \left(W_t(j) L_t(j) + B_{t-1}(s_t; j) + \Theta_t(j) - P_t C_t(j) - \sum_{s_{t+1}} Q(s_{t+1}|s_t) B_{t,h}(s_{t+1}; j) \right) \right],\end{aligned}$$

where $\Lambda_t(j)$ is the Lagrange multiplier on the budget constraint. Here the assumption of log-utility is related. The log utility in (1) corresponds to the case of $\sigma = 1$. The first-order conditions with respect to $C_t(j)$ and $B_t(s_{t+1}; j)$ are given by

$$\Lambda_t(j) P_t = C_t(j)^{-\frac{1}{\sigma}}, \tag{A.1}$$

$$\Lambda_t(j) Q(s_{t+1}|s_t) = \beta \Pr(s_{t+1}|s_t) \Lambda_{t+1}(j), \tag{A.2}$$

where $\Pr(s_{t+1}|s_t)$ denotes the probability of the state s_{t+1} in the next period conditional on the current state s_t. Combining the first-order conditions yields

$$Q(s_{t+1}|s_t) = \beta \Pr(s_{t+1}|s_t) \left(\frac{C_t(j)}{C_{t+1}(j)} \right)^{\frac{1}{\sigma}} \frac{P_t}{P_{t+1}}. \tag{A.3}$$

A similar equation holds for country F:

$$Q^*(s_{t+1}|s_t) = \beta \Pr(s_{t+1}|s_t) \left(\frac{C_t^*(j^*)}{C_{t+1}^*(j^*)} \right)^{\frac{1}{\sigma}} \frac{P_t^*}{P_{t+1}^*}. \tag{A.4}$$

Because the asset market is complete, the following arbitrage condition holds:

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} Q(s_{t+1}|s_t) = Q^*(s_{t+1}|s_t). \tag{A.5}$$

Combining equations (A.3)-(A.5) yields

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \left(\frac{C_t(j)}{C_{t+1}(j)} \right)^{\frac{1}{\sigma}} \frac{P_t}{P_{t+1}} = \left(\frac{C_t^*(j^*)}{C_{t+1}^*(j^*)} \right)^{\frac{1}{\sigma}} \frac{P_t^*}{P_{t+1}^*}. \tag{A.6}$$

Let Q_t denote the price of the risk-free bond that pays one unit of the Home currency in the next period. The price of such a bond is given by $Q_t = \sum_{s_{t+1}} Q(s_{t+1}|s_t)$, so that the following Euler equation holds:

$$1 = E_t \left[\beta \left(\frac{C_t(j)}{C_{t+1}(j)} \right)^{\frac{1}{\sigma}} \frac{P_t}{P_{t+1}} R_t \right], \tag{A.7}$$

where $R_t = Q_t^{-1}$ is the risk-free gross nominal interest rate.

Demand and price indices There are three types of consumption bundle: the total consumption bundle $C_t(j)$, the non-tradable consumption bundle $C_{Nt}(j)$, and the tradable consumption bundle $C_{Tt}(j)$, where index j denotes the j-th household. First, consider the choice of the composition of $C_t(j)$. The household j chooses $C_{Nt}(j)$ and $C_{Tt}(j)$ to minimize the expenditure $P_tC_t(j) = P_{Nt}C_{Nt}(j) + P_{Tt}C_{Tt}(j)$ subject to the consumption aggregator (3). The first-order conditions and the envelop theorem lead to the demand functions for $C_{Nt}(j)$ and $C_{Tt}(j)$ as

$$C_{Nt}(j) = \left(\frac{P_{Nt}}{P_t} \right)^{-1} \gamma_n C_t(j), \tag{A.8}$$

$$C_{Tt}(j) = \left(\frac{P_{Tt}}{P_t} \right)^{-1} (1-\gamma_n) C_t(j). \tag{A.9}$$

The expenditure shares of non-tradables and tradables are constant and given by γ_n and $1-\gamma_n$, respectively. Substituting equations (A.8) and (A.9) into the aggregator (3) leads to the price index as

$$P_t = P_{Nt}^{\gamma_n} P_{Tt}^{1-\gamma_n}. \tag{A.10}$$

Next, consider the choice of the composition of the non-tradable consumption bundle $C_{Nt}(j)$. The household j chooses $\{C_{Nt}(j,i)\}_{i\in(0,1)}$ to minimize the expenditure $P_{Nt}C_{Nt}(j) = \int_0^1 P_{Nt}(i)C_{Nt}(j,i)di$ subject to the non-tradable consumption aggregator (4). The first-order conditions and the envelop theorem lead to a demand function for $C_{Nt}(j,i)$ as

$$C_{Nt}(j,i) = \left(\frac{P_{Nt}(i)}{P_{Nt}} \right)^{-\theta_n} C_{Nt}(j). \tag{A.11}$$

Substituting equation (A.11) into the CES aggregator (4) yields the non-tradable goods price index as

$$P_{Nt} = \left(\int_0^1 P_{Nt}(i)^{1-\theta_n} di \right)^{\frac{1}{1-\theta_n}}. \tag{A.12}$$

Finally, consider the choice of the composition of the tradable consumption bundle $C_{Tt}(j)$. The household j chooses consumption varieties $\{C_{H,t}(j,i)\}_{i\in(0,n)}$ and $\{C_{F,t}(j,i)\}_{i\in[0,1-n)}$ to minimize the expenditure

$$P_{Tt}C_{Tt}(j) = \int_0^n P_{Ht}(i) C_{Ht}(j,i) di + \int_0^{1-n} P_{Ft}(i) C_{Ft}(j,i) di,$$

subject to the consumption aggregator (5). Let $\mu_t(j)$ denote a Lagrange multiplier on the constraint (5).

Then, the expenditure minimization problem can be written in a Lagrangean form as

$$\begin{aligned} P_{Tt}C_{Tt}(j) &= \min \int_0^n P_{Ht}(i)C_{Ht}(j,i)di + \int_0^{1-n} P_{Ft}(i)C_{Ft}(j,i)di \\ &\quad - \mu_t(j)\left[\frac{1-\gamma_\tau}{n}\int_0^n G\left(\frac{nC_{Ht}(j,i)}{(1-\gamma_\tau)C_{Tt}(j)}\right)di + \frac{\gamma_\tau}{1-n}\int_0^{1-n} G\left(\frac{(1-n)\,C_{Ft}(j,i)}{\gamma_\tau C_{Tt}(j)}\right)di - 1\right]. \end{aligned}$$

The envelop condition is given by

$$\mu_t\,(j) = P_{Tt}\left[\int_0^n G'\left(\frac{nC_{Ht}(j,i)}{(1-\gamma_\tau)C_{Tt}(j)}\right)\left(\frac{C_{Ht}(j,i)}{C_{Tt}(j)^2}\right)di + \int_0^{1-n} G'\left(\frac{(1-n)\,C_{Ft}(j,i)}{\gamma_\tau C_{Tt}(j)}\right)\left(\frac{C_{Ft}(j,i)}{C_{Tt}(j)^2}\right)di\right]^{-1}.$$

Define $\mathcal{P}_{Tt} \equiv \mu_t(j)/C_{Tt}(j)$. Then, the above condition can be written as

$$\begin{aligned} &\frac{\mathcal{P}_{Tt}}{P_{Tt}} \\ &\equiv \left[\int_0^n G'\left(\frac{nC_{Ht}(j,i)}{(1-\gamma_\tau)C_{Tt}(j)}\right)\left(\frac{C_{Ht}(j,i)}{C_{Tt}(j)}\right)di + \int_0^{1-n} G'\left(\frac{(1-n)C_{Ft}(j,i)}{\gamma_\tau C_{Tt}(j)}\right)\left(\frac{C_{Ft}(j,i)}{C_{Tt}(j)}\right)di\right]^{-1}. \end{aligned} \tag{A.13}$$

The first-order conditions with respect to $C_{Ht}(j,i)$ and $C_{Ft}(j,i)$ are

$$\begin{aligned} P_{Ht}(i) &= \frac{\mu_t\,(j)}{C_{Tt}(j)}G'\left(\frac{nC_{Ht}(j,i)}{(1-\gamma_\tau)C_{Tt}(j)}\right), \\ P_{Ft}(i) &= \frac{\mu_t(j)}{C_{Tt}(j)}G'\left(\frac{(1-n)C_{Ft}(j,i)}{\gamma_\tau C_{Tt}(j)}\right). \end{aligned}$$

By using $\mathcal{P}_{Tt} \equiv \mu_t(j)/C_{Tt}(j)$, these conditions can be rewritten as

$$C_{Ht}(j,i) = g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right)\frac{1-\gamma_\tau}{n}C_{Tt}(j), \tag{A.14}$$

$$C_{Ft}(j,i) = g\left(\frac{P_{Ft}(i)}{\mathcal{P}_{Tt}}\right)\frac{\gamma_\tau}{1-n}C_{Tt}(j), \tag{A.15}$$

where $g\,(\cdot) \equiv G'^{-1}\,(\cdot)$ satisfies $g\,(1) = 1$ and $g'\,(\cdot) < 0$. Substituting these demand curves into the aggregator (5) yields

$$\frac{1-\gamma_\tau}{n}\int_0^n G\left(g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right)\right)di + \frac{\gamma_\tau}{1-n}\int_0^{1-n} G\left(g\left(\frac{P_{Ft}(i)}{\mathcal{P}_{Tt}}\right)\right)di = 1. \tag{A.16}$$

This equation implicitly defines the auxiliary price index $\mathcal{P}_{Tt}$. In a symmetric steady state where individual prices are identical, the auxiliary price index becomes equal to the price index for the tradable consumption bundle: $\mathcal{P}_{Tt} = P_{Tt}$. The ratio of expenditure for Home goods is calculated as

$$\frac{\int_0^n P_{Ht}(i)C_{Ht}(j,i)di}{P_{Tt}C_{Tt}(j)} = (1-\gamma_\tau)\,\frac{1}{n}\int_0^n \frac{P_{Ht}(i)}{P_{Tt}}g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right)di.$$

The ratio is reduced to a constant, $1-\gamma_\tau$, in a symmetric steady state. Substituting equations (A.14) and (A.15) into the expenditure identity yields

$$P_{Tt} = \frac{1-\gamma_\tau}{n}\int_0^n P_{Ht}(i)g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right)di + \frac{\gamma_\tau}{1-n}\int_0^{1-n} P_{Ft}(i)g\left(\frac{P_{Ft}(i)}{\mathcal{P}_{Tt}}\right)di. \tag{A.17}$$

Let a small letter variable denote a log deviation of the corresponding capital letter variable from its steady state value. Log-linearizing equation (A.16) around a symmetric steady state yields

$$0=\frac{1-\gamma_{\mathcal{T}}}{n}\int_0^n G'\left(g\left(1\right)\right)g'\left(1\right)\left(p_{Ht}\left(i\right)-d\log\mathcal{P}_{Tt}\right)di+\frac{\gamma_{\mathcal{T}}}{1-n}\int_0^{1-n} G'\left(g\left(1\right)\right)g'\left(1\right)\left(p_{Ft}\left(i\right)-d\log\mathcal{P}_{Tt}\right)di,$$

or

$$d\log\mathcal{P}_{Tt}=\frac{1-\gamma_{\mathcal{T}}}{n}\int_0^n p_{Ht}\left(i\right)di+\frac{\gamma_{\mathcal{T}}}{1-n}\int_0^{1-n} p_{Ft}\left(i\right)di. \tag{A.18}$$

Log-linearizing equation (A.17) around a symmetric steady state yields

$$\begin{aligned} p_{Tt} &= \frac{1-\gamma_{\mathcal{T}}}{n}\int_0^n \left[g\left(1\right)p_{Ht}\left(i\right)+g'\left(1\right)\left(p_{Ht}\left(i\right)-d\log\mathcal{P}_{Tt}\right)\right]di \\ &\quad +\frac{\gamma}{1-n}\int_0^{1-n}\left[g\left(1\right)p_{Ft}\left(i\right)+g'\left(1\right)\left(p_{Ft}\left(i\right)-d\log\mathcal{P}_{Tt}\right)\right]di, \end{aligned}$$

Let $\theta_{\mathcal{T}}$ denote the demand elasticity at a symmetric steady state, that is, $\theta_{\mathcal{T}}=-g'(x)x/g(x)|_{x=1}$. Then, the above equation can be written as

$$\theta_{\mathcal{T}}d\log\mathcal{P}_{Tt}-p_{Tt}=\left(\theta_{\mathcal{T}}-1\right)\left[\frac{1-\gamma_{\mathcal{T}}}{n}\int_0^n p_{Ht}\left(i\right)di+\frac{\gamma_{\mathcal{T}}}{1-n}\int_0^{1-n} p_{Ft}\left(i\right)di\right]. \tag{A.19}$$

Substituting equation (A.18) into equation (A.19) leads to

$$p_{Tt}=d\log\mathcal{P}_{Tt}=\frac{1-\gamma_{\mathcal{T}}}{n}\int_0^n p_{Ht}\left(i\right)di+\frac{\gamma_{\mathcal{T}}}{1-n}\int_0^{1-n} p_{Ft}\left(i\right)di. \tag{A.20}$$

Wage setting problem It is useful to derive the CES aggregator (6). Consider the following CES aggregator with parameter ϑ.

$$L_t=\left[\vartheta\int_0^n L_t\left(j\right)^{\frac{\theta_w-1}{\theta_w}}dj\right]^{\frac{\theta_w}{\theta_w-1}}.$$

In a symmetric equilibrium where $L_t\left(j\right)$ is identical for all $j\in(0,n)$, this equation is reduced to

$$L_t=\left(\vartheta n\right)^{\frac{\theta_w}{\theta_w-1}}L_t\left(j\right).$$

Here, $L_t\left(j\right)$ is the per capita labor supply, while L_t is the aggregate labor supply. Since the population is $n\in(0,1)$, the aggregate labor has to be $L_t=nL_t\left(j\right)$. Hence, parameter ϑ is pinned down as $\vartheta=(1/n)^{1/\theta_w}$.

The profit maximization problem of the employment agency is

$$\max_{\{L_t(j)\}_{j\in(0,n)}} W_t\left[\left(\frac{1}{n}\right)^{\frac{1}{\theta_w}}\int_0^n L_t\left(j\right)^{\frac{\theta_w-1}{\theta_w}}dj\right]^{\frac{\theta_w}{\theta_w-1}}-\int_0^n W_t\left(j\right)L_t\left(j\right)dj.$$

The first-order condition leads to the labor demand curve

$$L_t\left(j\right)=\frac{1}{n}\left(\frac{W_t\left(j\right)}{W_t}\right)^{-\theta_w}L_t.$$

In the case of the H-currency wage setting, the problem is

$$\max_{\{W_t(j)\}} E_{t-1}\left(\Lambda_t(j)\, W_t(j)\, L_t(j) - \frac{\psi}{1+\frac{1}{\nu}} L_t(j)^{1+\frac{1}{\nu}}\right),$$

subject to the demand curve. The first-order condition leads to

$$W_t(j) = \frac{\theta_w \psi}{\theta_w - 1} \frac{E_{t-1}\left(L_t(j)^{1+\frac{1}{\nu}}\right)}{E_{t-1}\left(\Lambda_t(j)\, L_t(j)\right)}.$$

Substituting the labor demand curve into this condition yields

$$W_t(j)^{1+\frac{\theta_w}{\nu}} = \frac{\theta_w \psi n^{-\frac{1}{\nu}}}{\theta_w - 1} \frac{E_{t-1}\left(W_t^{\theta_w\left(1+\frac{1}{\nu}\right)} L_t^{1+\frac{1}{\nu}}\right)}{E_{t-1}\left(\Lambda_t(j)\, W_t^{\theta_w} L_t\right)}. \tag{A.21}$$

Keep in mind that $\Lambda_t(j)$ is given by equation (A.1) and $C_t = nC_t(j)$. Then, log-linearizing the above equation yields

$$w_t(j) = \left(1+\frac{\theta_w}{\nu}\right)^{-1} E_{t-1}\left(\frac{\theta_w}{\nu} w_t + \frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right). \tag{A.22}$$

In the case of the F-currency wage setting, the problem is

$$\max_{\{W_t^{\mathcal{E}}(j)\}} E_{t-1}\left[\Lambda_t(j)\, \mathcal{E}_t W_t^{\mathcal{E}}(j)\, L_t(j) - \frac{\psi}{1+\frac{1}{\nu}} L_t(j)^{1+\frac{1}{\nu}}\right],$$

subject to the labor demand curve

$$L_t(j) = \frac{1}{n}\left(\frac{W_t^F(j)}{\mathcal{E}_t^{-1} W_t}\right)^{-\theta_w} L_t,$$

where $W_t^F(j) = \mathcal{E}_t^{-1} W_t(j)$. The first-order condition leads to

$$W_t^F(j) = \frac{\theta_w \psi}{\theta_w - 1} \frac{E_{t-1}\left(L_t(j)^{1+\frac{1}{\nu}}\right)}{E_{t-1}\left(\Lambda_t(j)\, \mathcal{E}_t L_t(j)\right)}.$$

Substituting the labor demand curve into this condition yields

$$W_t^F(j)^{1+\frac{\theta_w}{\nu}} = \frac{\theta_w \psi n^{-\frac{1}{\nu}}}{\theta_w - 1} \frac{E_{t-1}\left(\mathcal{E}_t^{-\theta_w\left(1+\frac{1}{\nu}\right)} W_t^{\theta_w\left(1+\frac{1}{\nu}\right)} L_t^{1+\frac{1}{\nu}}\right)}{E_{t-1}\left(\Lambda_t(j)\, \mathcal{E}_t^{1-\theta_w} W_t^{\theta_w} L_t\right)}. \tag{A.23}$$

Log-linearizing this equation yields

$$w_t^F(j) = \left(1+\frac{\theta_w}{\nu}\right)^{-1} E_{t-1}\left(\frac{\theta_w}{\nu} w_t + \frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right) - E_{t-1} e_t \tag{A.24}$$

If there were no nominal wage rigidity, the optimal wage would be given by

$$\tilde{W}_t(j) = \frac{\theta_w \psi}{\theta_w - 1} \frac{L_t(j)^{\frac{1}{\nu}}}{\Lambda_t(j)}.$$

Substituting the labor demand curve into this equation yields

$$\tilde{W}_t(j)^{1+\frac{\theta_w}{\nu}} = \frac{\theta_w \psi n^{-\frac{1}{\nu}}}{\theta_w - 1} \frac{W_t^{\frac{\theta_w}{\nu}} L_t^{\frac{1}{\nu}}}{\Lambda_t(j)}.$$

Log-linearizing this equation yields

$$\tilde{w}_t(j) = \left(1 + \frac{\theta_w}{\nu}\right)^{-1} \left(\frac{\theta_w}{\nu} w_t + \frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right).$$

Because $\tilde{w}_t(j) = w_t$ in the flexible-price economy, the optimal price $\tilde{w}_t(j)$ can be written as

$$\tilde{w}_t(j) = \frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t. \tag{A.25}$$

A fraction λ_w of households set wages in units of currency H and the rest of the households set wages in units of currency F. Then, the wage index (8) can be written as

$$W_t = \left[\lambda_w W_t(j)^{1-\theta_w} + (1-\lambda_w)\left(\mathcal{E}_t W_t^F(j)\right)^{1-\theta_w}\right]^{\frac{1}{1-\theta_w}},$$

where $W_t(j)$ is given by (A.21) and $W_t^F(j)$ is given by (A.23). Log-linearizing this equation yields

$$w_t = \lambda_w w_t(j) + (1-\lambda_w)\left(e_t + w_t^F(j)\right). \tag{A.26}$$

Substituting equations (A.22) and (A.24) into equation (A.26) yields

$$\begin{aligned}\left(1 + \frac{\theta_w}{\nu}\right) w_t &= \lambda_w E_{t-1}\left(\frac{\theta_w}{\nu} w_t + \frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right) \\ &\quad + (1-\lambda_w)\left(\left(1 + \frac{\theta_w}{\nu}\right)(e_t - E_{t-1} e_t) + E_{t-1}\left(\frac{\theta_w}{\nu} w_t + \frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right)\right).\end{aligned}$$

By taking expectation in period $t-1$, this equation can be written as

$$E_{t-1} w_t = E_{t-1}\left(\frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right).$$

Substituting this into equations (A.22) and (A.24), respectively, yields

$$\begin{aligned} w_t(j) &= E_{t-1}\left(\frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right), \\ w_t^F(j) &= E_{t-1}\left(\frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right) - E_{t-1} e_t. \end{aligned}$$

Substituting these two equations into equation (A.26) yields equation (11) as

$$w_t = E_{t-1}\left(\frac{1}{\nu} l_t + \frac{1}{\sigma} c_t + p_t\right) + (1-\lambda_w)(e_t - E_{t-1} e_t).$$

A.2 Firms

Factor prices and marginal costs Cost minimization in the non-tradable and the tradable sectors yields factor prices W_t and P_{Tt} as

$$W_t = MC_{Nt}(1-\phi_n)Y_{Nt}(i)/L_{Nt}(i), \tag{A.27}$$

$$W_t = MC_{Tt}(1-\phi_\tau)Y_{Tt}(i)/L_{Tt}(i), \tag{A.28}$$

$$P_{Tt} = MC_{Nt}\phi_n Y_{Nt}(i)/X_{Nt}(i), \tag{A.29}$$

$$P_{Tt} = MC_{Tt}\phi_\tau Y_{Tt}(i)/X_{Tt}(i), \tag{A.30}$$

where marginal costs are given by

$$MC_{Nt} = \frac{1}{A_t} W_t^{1-\phi_n} P_{Tt}^{\phi_n}, \tag{A.31}$$

$$MC_{Tt} = \frac{1}{A_t} W_t^{1-\phi_\tau} P_{Tt}^{\phi_\tau}. \tag{A.32}$$

Flexible price benchmark There are two types of firms: non-tradable goods firms and tradable goods firms. First, consider the flexible-price setting problem of non-tradable goods firms. Such a frim i chooses $P_{Nt}(i)$ to maximize its period-t profits (14). The first-order condition is

$$P_{Nt}(i) = \tilde{P}_{Nt} = \frac{\theta_n}{\theta_n - 1} MC_{Nt}. \tag{A.33}$$

Because this pricing holds for all i, the price index of the non-tradable bundle is given by $P_{Nt} = \tilde{P}_{Nt}$.

Next, consider the flexible-price setting problem of the tradable goods firm i. It maximizes its period-t profits (15) subject to the demand curves (16) and (17). The first-order condition with respect to $P_{Ht}(i)$ leads to

$$P_{Ht}(i) = \tilde{P}_{Ht}(i) = \frac{\theta_\tau(x_{Ht}(i))}{\theta_\tau(x_{Ht}(i)) - 1} MC_{Tt}, \tag{A.34}$$

where $x_{Ht}(i) \equiv P_{Ht}(i)/\mathcal{P}_{Tt}$ is the relative price and $\theta_\tau(x_{Ht}(i)) = -g'(x_{Ht}(i))x_{Ht}(i)/g(x_{Ht}) > 1$ is the demand elasticity. Similarly, the corresponding price set in country F is given by

$$P^*_{Ht}(i) = \tilde{P}^*_{Ht}(i) = \frac{\theta_\tau(x^*_{Ht}(i))}{\theta_\tau(x^*_{Ht}(i)) - 1} \frac{MC_{Tt}}{\mathcal{E}_t},$$

where $x^*_{Ht}(i) \equiv P^*_{Ht}(i)/\mathcal{P}^*_{Tt}$.

Keeping in mind that $d\log \mathcal{P}_{Tt} = p_{Tt}$, log-linearizing equation (A.34) around a symmetric steady state yields

$$p_{Ht}(i) = mc_t - \Gamma(p_{Ht}(i) - p_{Tt}), \tag{A.35}$$

where $\Gamma \equiv \theta'_\tau(1) / (\theta_\tau(\theta_\tau - 1))$, and $\theta'_\tau(1)$ is given by

$$\begin{aligned} \theta'_\tau(1) &= \left. \frac{d\left[-g'(x)\,x/g(x)\right]}{dx} \right|_{x=1} = \left. \frac{\left[-g''(x)\,x - g'(x)\right] g(x) + g'(x)^2 x}{g(x)^2} \right|_{x=1} \\ &= -\left(g''(1) + g'(1)\right) g(1) + g'(1)^2 \\ &= -\left(g''(1) - \theta_\tau\right) + \theta_\tau^2 \\ &= -g''(1) + \theta_\tau(1 + \theta_\tau). \end{aligned}$$

Solving equation (A.35) for $p_{Ht}(i)$ yields

$$p_{Ht}(i) = \tilde{p}_{Ht}(i) = (1 - \alpha)\, mc_{Tt} + \alpha p_{Tt}, \tag{A.36}$$

where $\alpha \equiv \Gamma / (1 + \Gamma) \in (0, 1)$. A similar equation can be derived for the price of the Home goods in country F:

$$p^*_{Ht}(i) = \tilde{p}^*_{Ht}(i) = (1 - \alpha)\left(mc_{Tt} - e_t\right) + \alpha p^*_{Tt}. \tag{A.37}$$

In the case of the CES aggregator, $G(x) = 1 + \frac{\theta_\tau}{\theta_\tau - 1}\left(x^{\frac{\theta_\tau - 1}{\theta_\tau}} - 1\right)$, the derivatives are given as $g(x) \equiv G'^{-1}(x) = x^{-\theta_\tau}$, $g'(x) = -\theta_\tau x^{-(1+\theta_\tau)}$, and $g''(x) = \theta_\tau(1 + \theta_\tau)\, x^{-(2+\theta_\tau)}$. Hence, $\Gamma \propto \theta'_\tau(1) = -g''(1) + \theta_\tau(1 + \theta_\tau) = 0$ so that $\alpha = 0$ under the CES aggregator.

Nominal rigidities: $P_{Nt}(i)$ and $P^*_{Nt}(i)$ Consider a price setting problem of the non-tradable goods firm i in country H. It sets the price one period in advance by using either currency H or F as the invoice currency. First, consider the case of currency H as the invoice currency. In this case, the firm sets the price $P_{Nt}(i) = \bar{P}_{Nt}(i)$ to solve the following profit maximization problem:

$$\max_{\{\bar{P}_{Nt}(i)\}} E_{t-1}\left[Q_{t-1,t}\left(\bar{P}_{Nt}(i) - MC_{Nt}\right)\left(\frac{\bar{P}_{Nt}(i)}{P_{Nt}}\right)^{-\theta_n} C_{Nt}\right].$$

where $Q_{t-1,t}$ is the stochastic discount factor. The first-order condition is

$$E_{t-1}\left[Q_{t-1,t}(1 - \theta_n)C_{Nt} + Q_{t-1,t}\theta_n MC_{Nt}C_{Nt}/\bar{P}_{Nt}(i)\right] = 0,$$

where $P_{Nt} = \bar{P}_{Nt}(i)$ is imposed. Log-linearizing this condition around a symmetric steady state yields

$$\bar{p}_{Nt}(i) = E_{t-1}(mc_{Nt}) = E_{t-1}(\tilde{p}_{Nt}(i)), \tag{A.38}$$

where $\tilde{p}_{Nt}(i) = mc_{Nt}$ is the optimal price in a flexible-price environment as implied by equation (A.33).

Next, consider the case in which the non-tradable goods firm i sets its price $\bar{P}^F_{Nt}(i)$ using currency F. In this case, the price in units of currency H is given by $P_{Nt}(i) = \bar{P}^F_{Nt}(i)\mathcal{E}_t$, where the price $\bar{P}^F_{Nt}(i)$ is set in one period in advance. The firm's profit maximization problem is

$$\max_{\{\bar{P}^F_{Nt}(i)\}} E_{t-1}\left[Q_{t-1,t}\left(\bar{P}^F_{Nt}(i)\mathcal{E}_t - MC_{Nt}\right)\left(\frac{\bar{P}^F_{Nt}(i)\mathcal{E}_t}{P_{Nt}}\right)^{-\theta_n} C_{Nt}\right].$$

The first-order condition is

$$E_{t-1}\left[Q_{t-1,t}(\theta_n-1)\mathcal{E}_tC_{Nt}+Q_{t-1,t}\theta_nMC_{Nt}C_{Nt}/\bar{P}^F_{Nt}(i)\right]=0,$$

where $\mathcal{E}_t\bar{P}^F_{Nt}(i)=P_{Nt}$ is imposed. Log-linearizing this condition yields

$$\bar{p}^F_{Nt}(i)=E_{t-1}(mc_{Nt}-e_t)=E_{t-1}(\tilde{p}_{Nt}(i)-e_t). \tag{A.39}$$

A price setting problem of Foreign non-tradable goods firms is symmetric to that of the Home non-tradable goods firms, so that the log-linearized prices are given by

$$\bar{p}^*_{Nt}(i)=E_{t-1}(mc^*_{Nt}), \tag{A.40}$$

$$\bar{p}^{*H}_{Nt}(i)=E_{t-1}(mc^*_{Nt}+e_t). \tag{A.41}$$

Nominal rigidities: $P_{Ht}(i)$ **and** $P^*_{Ft}(i)$ Consider the tradable goods firm i that sets prices in the Home market. First, consider the case of currency H as the invoice currency. In this case, the tradable goods firm i sets the price $\bar{P}^H_{Ht}(i)$ to solve the following profit maximization problem:

$$\max_{\left\{\bar{P}^H_{Ht}(i)\right\}} E_{t-1}\left[Q_{t-1,t}\left(\bar{P}^H_{Ht}(i)-MC_t\right)g\left(\frac{\bar{P}^H_{Ht}(i)}{\mathcal{P}_{Tt}}\right)(C_{Tt}+X_t)\right].$$

where $X_t=X_{Tt}+X_{Nt}$ is the aggregate intermediate input. The first-order condition is

$$E_{t-1}\left\{Q_{t-1,t}\left(C_{Tt}+X_t\right)\left[g\left(\frac{\bar{P}^H_{Ht}(i)}{\mathcal{P}_{Tt}}\right)+\left(\bar{P}^H_{Ht}(i)-MC_{Tt}\right)g'\left(\frac{\bar{P}^H_{Ht}(i)}{\mathcal{P}_{Tt}}\right)\frac{1}{\mathcal{P}_{Tt}}\right]\right\}=0. \tag{A.42}$$

Log-linearizing this condition around a symmetric steady state ($\bar{P}_H(i)=P=[\theta_\tau/(\theta_\tau-1)]MC_T$) yields

$$\bar{p}^H_{Ht}(i)=E_{t-1}\left[-\Gamma\left(\bar{p}^H_{Ht}(i)-d\log\mathcal{P}_{Tt}\right)+mc_{Tt}\right],$$

where $\Gamma\equiv-\frac{g''(1)}{\theta_\tau(\theta_\tau-1)}+\frac{\theta_\tau+1}{\theta_\tau-1}$, and $\theta_\tau\equiv-g'(1)/g(1)$. From equation (A.20), $d\log\mathcal{P}_{Tt}=p_{Tt}$. Then, the log-linearized equation can be written as

$$\bar{p}^H_{Ht}(i)=E_{t-1}\left[(1-\alpha)mc_{Tt}+\alpha p_{Tt}\right]=E_{t-1}\left(\tilde{p}_{Ht}(i)\right), \tag{A.43}$$

where $\alpha\equiv\Gamma/(1+\Gamma)$ and $\tilde{p}_{Ht}(i)$ is given by equation (A.36).

Next, consider the case of currency F as the invoice currency. In this case, the firm i sets the price $\bar{P}^F_{Ht}(i)$ in units of currency F to solve the following problem:

$$\max_{\left\{\bar{P}^F_{Ht}(i)\right\}} E_{t-1}\left[Q_{t-1,t}\left(\bar{P}^F_{Ht}(i)\mathcal{E}_t-MC_{Tt}\right)g\left(\frac{\bar{P}^F_{Ht}(i)\mathcal{E}_t}{\mathcal{P}_{Tt}}\right)(C_{Tt}+X_t)\right].$$

The first-order condition is

$$E_{t-1}\left\{Q_{t,t-1}\left(C_{Tt}+X_t\right)\mathcal{E}_t\left[g\left(\frac{\bar{P}^F_{Ht}(i)\mathcal{E}_t}{\mathcal{P}_{Tt}}\right)+\left(\bar{P}^F_{Ht}(i)\mathcal{E}_t-MC_{Tt}\right)g'\left(\frac{\bar{P}^F_{Ht}(i)\mathcal{E}_t}{\mathcal{P}_{Tt}}\right)\frac{1}{\mathcal{P}_{Tt}}\right]\right\}=0. \tag{A.44}$$

Log-linearizing this condition yields

$$\bar{p}_{Ht}^{F}(i) = E_{t-1}\left[-\Gamma\left(\bar{p}_{Ht}^{F}(i) + e_t - d\log\mathcal{P}_{Tt}\right) + mc_{Tt} - e_t\right].$$

Because $d\log\mathcal{P}_{Tt} = p_{Tt}$, this equation can be written as

$$\bar{p}_{Ht}^{F}(i) = E_{t-1}\left[(1-\alpha)\,mc_{Tt} + \alpha p_{Tt} - e_t\right] \equiv E_{t-1}\left(\tilde{p}_{Ht}(i) - e_t\right). \tag{A.45}$$

A price setting problem of the corresponding firm i^* in country F is symmetric to that in country H. In the case of its own currency as the invoice currency, the log-linearized price $\bar{p}_{Ft}^{*}(i)$ is given by

$$\bar{p}_{Ft}^{*}(i) = E_{t-1}\left[(1-\alpha)\,mc_{Tt}^{*} + \alpha p_{Tt}^{*}\right] = E_{t-1}\left(\tilde{p}_{Ft}^{*}(i)\right). \tag{A.46}$$

In the case of the H-currency pricing, the log-linearized price for $p_{Ft}^{*H}(i) \equiv p_{Ft}^{*}(i) + e_t$ is given by

$$\bar{p}_{Ft}^{*H}(i) = E_{t-1}\left[(1-\alpha)\,mc_{Tt}^{*} + \alpha p_{Tt}^{*} + e_t\right] = E_{t-1}\left(\tilde{p}_{Ft}^{*}(i) + e_t\right). \tag{A.47}$$

Nominal rigidities: $P_{Ht}^{*}(i)$ and $P_{Ft}(i)$ Next, import (export) prices are considered. In terms of an invoice currency, there are three cases: producer currency pricing, local currency pricing, and dominant currency pricing. Dominant currency pricing is given by the combination of the other two types of pricing. In the case of producer currency pricing, the Home firm i sets the price $P_{Ht}^{*H}(i) \equiv P_{Ht}^{*}(i)\,\mathcal{E}_t$ in units of currency H to solve the following problem:

$$\max_{\left\{\bar{P}_{Ht}^{*H}(i)\right\}} E_{t-1}\left[Q_{t-1,t}\left(\bar{P}_{Ht}^{*H}(i) - MC_t\right) g\left(\frac{\bar{P}_{Ht}^{*H}(i)}{\mathcal{P}_{Tt}^{*}\mathcal{E}_t}\right)\left(C_t^{*} + X_t^{*}\right)\right].$$

The first-order condition is

$$E_{t-1}\left\{Q_{t-1,t}\left(C_{Tt}^{*} + X_t^{*}\right)\left[g\left(\frac{\bar{P}_{Ht}^{*H}(i)}{\mathcal{P}_{Tt}^{*}\mathcal{E}_t}\right) + \left(\bar{P}_{Ht}^{*H}(i) - MC_t\right) g'\left(\frac{\bar{P}_{Ht}^{*H}(i)}{\mathcal{P}_{Tt}^{*}\mathcal{E}_t}\right)\frac{1}{\mathcal{P}_{Tt}^{*}\mathcal{E}_t}\right]\right\} = 0.$$

Log-linearizing this condition yields

$$\bar{p}_{Ht}^{*H}(i) = E_{t-1}\left[(1-\alpha)\,mc_{Tt} + \alpha\left(p_{Tt}^{*} + e_t\right)\right] = E_{t-1}\left(\tilde{p}_{Ht}^{*}(i) + e_t\right), \tag{A.48}$$

where $\tilde{p}_{Ht}^{*}(i)$ is given by equation (19). Symmetrically, the price set by the Foreign firm i^* in the case of producer currency pricing is given by

$$\bar{p}_{Ft}^{F}(i) = E_{t-1}\left[(1-\alpha)\,mc_{Tt}^{*} + \alpha\left(p_{Tt} - e_t\right)\right] = E_{t-1}\left(\tilde{p}_{Ft}(i) - e_t\right). \tag{A.49}$$

In the case of local currency pricing, the Home firm i sets the price for $P_{Ht}^{*}(i)$ in units of currency F to solve the following problem:

$$\max_{\left\{\bar{P}_{Ht}^{*F}(i)\right\}} E_{t-1}\left[Q_{t-1,t}\left(\bar{P}_{Ht}^{*F}(i)\,\mathcal{E}_t - MC_{Tt}\right) g\left(\frac{\bar{P}_{Ht}^{*F}(i)}{\mathcal{P}_{Tt}^{*}}\right)\left(C_{Tt}^{*} + X_t^{*}\right)\right].$$

The first-order condition is

$$E_{t-1}\left\{Q_{t-1,t}\left(C^*_{Tt}+X^*_t\right)\left[\mathcal{E}_t g\left(\frac{\bar{P}^{*F}_{Ht}(i)}{\mathcal{P}^*_{Tt}}\right)+\left(\bar{P}^{*F}_{Ht}(i)\,\mathcal{E}_t-MC_{Tt}\right)g'\left(\frac{\bar{P}^{*F}_{Ht}(i)}{\mathcal{P}^*_t}\right)\frac{1}{\mathcal{P}^*_{Tt}}\right]\right\}=0.$$

Log-linearizing this condition yields

$$\bar{p}^{*F}_{Ht}(i)=E_{t-1}\left[(1-\alpha)\left(mc_{Tt}-e_t\right)+\alpha p^*_{Tt}\right]=E_{t-1}\left(\tilde{p}^*_{Ht}(i)\right). \tag{A.50}$$

Symmetrically, the price set by the Foreign firm i^* in the case of local currency pricing is given by

$$\bar{p}^{H}_{Ft}(i)=E_{t-1}\left[(1-\alpha)\left(mc^*_{Tt}+e_t\right)+\alpha p_{Tt}\right]=E_{t-1}\left(\tilde{p}_{Ft}(i)\right). \tag{A.51}$$

Price indices From equation (A.10), the price index for the consumption bundle in country H in units of currency H is given by $p_t=\gamma_n p_{Nt}+(1-\gamma_n)p_{Tt}$. Under the assumption that a fraction λ_N of the non-tradable goods firms set the prices using currency H, the price index of the non-tradable goods is given by

$$p_{Nt}=\lambda_N\bar{p}_{Nt}+(1-\lambda_N)(\bar{p}^F_{Nt}+e_t).$$

where $\bar{p}_{Nt}$ and $\bar{p}^F_{Nt}$ are given by equations (A.38) and (A.39), respectively. Similarly, given that a fraction λ_H (λ_F) of Home (Foreign) tradable goods firms set the prices using currency H, the price index of the tradable goods is given by

$$\begin{aligned} p_{Tt} &= \frac{1-\gamma_\tau}{n}\int_0^n p_{Ht}(i)\,di+\frac{\gamma_\tau}{1-n}\int_0^{1-n}p_{Ft}(i)\,di \\ &= (1-\gamma_\tau)\left[\lambda_H\bar{p}_{Ht}(i)+(1-\lambda_H)\left(\bar{p}^F_{Ht}(i)+e_t\right)\right]+\gamma_\tau\left[\lambda_F\bar{p}_{Ft}(i)+(1-\lambda_F)\left(\bar{p}^F_{Ft}(i)+e_t\right)\right], \end{aligned}$$

where $\bar{p}_{Ht}(i)$ is given by equation (A.43), $\bar{p}^F_{Ht}(i)$ is given by equation (A.45), $\bar{p}_{Ft}(i)$ is given by equation (A.51), and $\bar{p}^F_{Ft}(i)$ is given by equation (A.49). Under the assumption of i.i.d. shocks, the prices, $\bar{p}_{Nt}$, $\bar{p}^F_{Nt}$, $\bar{p}_{Ht}(i)$, $\bar{p}^F_{Ht}(i)$, $\bar{p}_{Ft}(i)$, and $\bar{p}^F_{Ft}(i)$, are equal to their respective steady state values. Then, the price indices can be written as

$$p_{Nt}=(1-\lambda_N)e_t, \tag{A.52}$$

$$p_{Tt}=\left[(1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)\right]e_t, \tag{A.53}$$

$$p_t=\left\{\gamma_n(1-\lambda_N)+(1-\gamma_n)\left[(1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)\right]\right\}e_t. \tag{A.54}$$

Similarly, the price index for the consumption bundle in country F in units of currency F is given by $p^*_t=\gamma_n p^*_{Nt}+(1-\gamma_n)p^*_{Tt}$. The price index of the non-tradable goods is given by

$$p^*_{Nt}=\lambda^*_N\left(\bar{p}^{*H}_{Nt}-e_t\right)+(1-\lambda_N)\bar{p}^*_{Nt},$$

where the prices $\bar{p}^{*H}_{Nt}$ and $\bar{p}^*_{Nt}$ are given by equations (A.41) and (A.40), respectively. The price index of the tradable goods is given by

$$
\begin{aligned}
p_{Tt}^{*} &= \frac{1-\gamma_\tau}{1-n}\int_0^{1-n} p_{Ft}^{*}(i)\,di + \frac{\gamma_\tau}{n}\int_0^n p_{Ht}^{*}(i)\,di \\
&= (1-\gamma_\tau)\left[\lambda_F^{*}\left(\bar{p}_{Ft}^{*H}(i)-e_t\right)+(1-\lambda_F^{*})\,\bar{p}_{Ft}^{*}(i)\right]+\gamma_\tau\left[\lambda_H^{*}\left(\bar{p}_{Ht}^{*H}(i)-e_t\right)+(1-\lambda_H^{*})\,\bar{p}_{Ht}^{*}(i)\right],
\end{aligned}
$$

where $\bar{p}_{Ft}^{*H}(i)$ is given by equation (A.47), $\bar{p}_{Ft}^{*}(i)$ is given by equation (A.46), $\bar{p}_{Ht}^{*H}(i)$ is given by equation (A.48), and $\bar{p}_{Ht}^{*}(i)$ is given by equation (A.50). Similar to the price index for country H, under the assumption of i.i.d. shocks, the price index for country F is written as

$$
p_t^{*} = -\left\{\gamma_n\lambda_N^{*} + (1-\gamma_n)\left[(1-\gamma_\tau)\,\lambda_F^{*} + \gamma_\tau\lambda_H^{*}\right]\right\} e_t.
$$

A.3 Equilibrium

Exchange rate Under the assumption of log utility of consumption (i.e. $\sigma = 1$) and per capita nominal spending as a monetary policy instrument, equation (A.6) can be written as

$$
\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\frac{M_t}{M_t^{*}} = \frac{M_{t+1}}{M_{t+1}^{*}}.
$$

From this equation, the exchange rate in the initial period $t = 0$ is given by

$$
\mathcal{E}_0 = \left(\frac{M_{-1}^{*}}{M_{-1}}\mathcal{E}_{-1}\right)\frac{M_0}{M_0^{*}},
$$

where $\left(M_{-1}^{*}/M_{-1}\right)E_{-1}$ is constant. Without loss of generality, the constant initial condition is set to unity, so that the exchange rate is given by.

$$
\mathcal{E}_t = \frac{M_t}{M_t^{*}}.
$$

Market clearing A labor market clearing condition is $L_t = L_{Nt} + L_{Tt}$, where $L_t = \int_0^n L_t(j)dj$ is the aggregate labor supply, $L_{Nt} = n\int_0^1 L_{Nt}(i)di$ is the aggregate labor demand in the non-tradable sector, and $L_{Tt} = \int_0^n L_{Tt}(i)di$ is the aggregate labor demand in the tradable sector. Because population in the Home country is n, the labor market clearing condition can be written in per capita terms as

$$
\bar{L}_t = \bar{L}_{Nt} + \bar{L}_{Tt},
$$

where $\bar{L}_t = (1/n)L_t$, $\bar{L}_{Nt} = (1/n)L_{Nt}$, and $\bar{L}_{Tt} = (1/n)L_{Tt}$.

The total quantities of tradable good i demanded in countries H and F, $Y_{Ht}(i)$ and $Y_{Ht}^{*}(i)$, are given by equations (16) and (17), respectively. The supply of tradable good i is given by the production function (13). Then, a market clearing condition for tradable good i is written as $Y_{Tt}(i) = Y_{Ht}(i) + Y_{Ht}^{*}(i)$. Aggregating this condition over i yields

$$
\begin{aligned}
Y_{Tt} =& A_t\left(\frac{L_{Tt}}{1-\phi_\tau}\right)^{1-\phi_\tau}\left(\frac{X_{Tt}}{\phi_\tau}\right)^{\phi_\tau} \\
=& \frac{1-\gamma_\tau}{n}\left(C_{Tt}+X_t\right)\int_0^n g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right)di + \frac{\gamma_\tau^{*}}{n}\left(C_{Tt}^{*}+X_t^{*}\right)\int_0^n g\left(\frac{P_{Ht}^{*}(i)}{\mathcal{P}_{Tt}^{*}}\right)di, \qquad \text{(A.55)}
\end{aligned}
$$

where $C_{Tt} \equiv \int_0^n C_{Tt}(j)\,dj$, $X_t \equiv X_{Nt} + X_{Tt} \equiv n\int_0^1 X_{Nt}(i)\,di + \int_0^n X_{Tt}(i)di$, $C_t^* \equiv \int_0^{1-n} C_t^*(j)\,dj$, and $X_t^* \equiv X_{Nt}^* + X_{Tt}^* \equiv (1-n)\int_0^1 X_{Nt}^*(i)\,di + \int_0^{1-n} X_{Tt}(i)di$. From equation (A.9), the quantities of the aggregate tradable goods consumed, C_{Tt} and C_{Tt}^*, are given, respectively, by

$$C_{Tt} = \left(\frac{P_{Tt}}{P_t}\right)^{-1}(1-\gamma_n)C_t, \qquad C_{Tt}^* = \left(\frac{P_{Tt}^*}{P_t^*}\right)^{-1}(1-\gamma_n)C_t^*, \tag{A.56}$$

where $C_t \equiv \int_0^n C_t(j)dj$ and $C_t^* \equiv \int_0^{1-n} C_t^*(j)dj$.

A market clearing condition for non-tradable good i is given by $Y_{Nt}(i) = \int_0^j C_{Nt}(j,i)dj$, where the supply $Y_{Nt}(i)$ is given by the production function (12) and the demand $C_{Nt}(j,i)$ by each household j is given by equation (A.11). Aggregating this condition over i yields

$$Y_{Nt} = A_t\left(\frac{L_{Nt}}{1-\phi_n}\right)^{1-\phi_n}\left(\frac{X_{Nt}}{\phi_n}\right)^{\phi_n} = \left[\int_0^n \left(\frac{P_{Nt}(i)}{P_{Nt}}\right)^{-\theta_n} di\right]C_{Nt}, \tag{A.57}$$

where $C_{Nt} = \int_0^n C_{Nt}(j)dj$ is given by equation (A.8) as

$$C_{Nt} = \left(\frac{P_{Nt}}{P_t}\right)^{-1}\gamma_n C_t.$$

A.4 Steady State

For simplicity, the following assumptions are imposed: log-utility ($\sigma = 1$); symmetric production technologies ($\phi_n = \phi_\tau$); subsidies that remove markup distortions in steady state. For normalization, per capita nominal spending, which is a monetary policy instrument, is set at unity: $M = M^* = 1$; the technology level is also set at unity: $A = A^* = 1$.

In steady state, the exchange rate is $\mathcal{E} = 1$. As shown below, the steady state is consistent with unitary prices: $P_t = P_{Nt} = P_{Nt}(i) = P_{Tt} = P_{Ht}(i) = P_{Ft}(i) = 1$. The corresponding Foreign prices are also unity. These unitary prices imply that per capita consumption is unity as well, $\bar{C} = 1$, because $\bar{C} = M/P$.

From equations (A.31) and (A.32), the marginal costs are given by

$$MC_N = W^{1-\phi_n}, \qquad MC_T = W^{1-\phi_\tau}.$$

Under the assumption that subsidies remove markup distortions, the marginal costs are unity: $MC_N = MC_T = 1$. Factor price equations (A.27) and (A.28) imply

$$\frac{\bar{X}_N}{\bar{L}_N} = \frac{\phi_n}{1-\phi_n}, \qquad \frac{\bar{X}_T}{\bar{L}_T} = \frac{\phi_\tau}{1-\phi_\tau}.$$

From the household problem, the labor supply is given by $\bar{L} = (1/\psi)^\nu \bar{C}^\nu = (1/\psi)^\nu$ under the assumption that subsidies remove markup distortions. Without loss of generality, ψ is set at unity so that $\bar{L} = 1$. The labor market clearing condition is $\bar{L} = 1 = \bar{L}_N + \bar{L}_T$. The total intermediate input in per capita terms is given by

$$\bar{X} = \frac{\phi_n}{1-\phi_n}(1-\bar{L}_T) + \frac{\phi_\tau}{1-\phi_\tau}\bar{L}_T = \frac{\phi}{1-\phi},$$

under the assumption of $\phi_n = \phi_\tau = \phi$. From equation (A.56), $C_T = 1-\gamma_n$ and $C_T^* = 1-\gamma_n$. Then, the market clearing condition for tradable goods (A.55) can be written in per capita terms as

$$\begin{aligned}\frac{\bar{L}_T}{1-\phi_\tau} =&(1-(1-n)\bar{\gamma})\left(1-\gamma_n+\bar{X}\right)+(1-n)\bar{\gamma}\left(1-\gamma_n+\bar{X}^*\right),\\ =&1-\gamma_n+\bar{X}=1-\gamma_n+\frac{\phi_n}{1-\phi_n}+\left(\frac{\phi_\tau}{1-\phi_\tau}-\frac{\phi_n}{1-\phi_n}\right)\bar{L}_T,\end{aligned}$$

where symmetry between countries H and F is imposed on the second equality. Then, under the assumption of $\phi_n=\phi_\tau=\phi$, the above equation can be solved for $\bar{L}_T$ as

$$\bar{L}_T=1-(1-\phi)\gamma_n.$$

The market clearing condition in the non-tradable sector (A.57) implies $\bar{L}_N=(1-\phi)\gamma_n$. Hence, these labor supplies satisfy the labor market clearing condition, $1=\bar{L}_N+\bar{L}_T$. The per capita output in the tradable sector is given by

$$\bar{Y}_T=\left(\frac{1}{1-\phi}\right)^{1-\phi}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{\bar{X}_T}{\bar{L}_T}\right)^{\phi}\bar{L}_T=\left(\frac{1}{1-\phi}\right)[1-(1-\phi)\gamma_n]$$

Similarly, the per capita output in the non-tradable sector is given by

$$\bar{Y}_N=\left(\frac{1}{1-\phi}\right)^{1-\phi}\left(\frac{1}{\phi}\right)^{\phi}\left(\frac{\bar{X}_N}{\bar{L}_N}\right)^{\phi}\bar{N}_T=\gamma_n.$$

B Invoice-Currency Choices

B.1 Invoice Currencies for Prices

For analytical tractability, this analysis assumes a log utility of consumption ($\sigma=1$) and a linear labor disutility ($\nu=\infty$). It considers two cases about exogenous shocks in solving invoice currency choice problems. The first case considers an exogenous shock to an exchange rate and no monetary policy shock, following Mukhin (2018). The second case considers the case of monetary policy shocks only. In the second case, standard deviations of monetary policy shocks are assumed to be identical between countries H and F.

B.1.1 No nominal wage rigidity

First, the cases of no nominal wage rigidity are considered.

Invoice currency for $P_{Ht}(i)$ Consider the Home tradable goods firm i, which chooses an invoice currency, either currency H or F, in setting the price in the Home market. Let $\Pi^H_{Ht}=(p_{Ht}(i))$ denote the profits made in the Home market by the Home firm i that sets the price in units of currency H, which is written as

$$\Pi^H_{Ht}(p_{Ht}(i))=\left(e^{p_{Ht}(i)}-MC_{Tt}\right)g\left(\frac{e^{p_{Ht}(i)}}{\mathcal{P}_{Tt}}\right)\frac{1-\gamma_\tau}{n}(C_{Tt}+X_t).$$

To the second-order approximation with respect to $p_{Ht}(i)$ around the flexible price $\tilde{p}_{Ht}(i)$, the profits can be written as

$$\Pi_{Ht}^{H}(p_{Ht}(i)) = \Pi_{Ht}(\tilde{p}_{Ht}(i)) + \Pi_{Ht}^{H\prime}(\tilde{p}_{Ht}(i))(p_{Ht}(i) - \tilde{p}_{Ht}(i)) + \frac{1}{2}\Pi_{Ht}^{H\prime\prime}(\tilde{p}_{Ht}(i))(p_{Ht}(i) - \tilde{p}_{Ht}(i))^2 .$$

Since $\Pi_{Ht}^{H\prime}(\tilde{p}_{Ht}(i)) = 0$, $\Pi_{Ht}^{H\prime\prime}(\tilde{p}_{Ht}(i)) < 0$, and $p_{Ht}(i) = E_{t-1}(\tilde{p}_{Ht}(i))$, the discounted expected profits are given by

$$E_{t-1}Q_{t-t,t}\Pi_{Ht}^{H}(p_{Ht}(i)) \propto -E_{t-1}(p_{Ht}(i) - \tilde{p}_{Ht}(i))^2 = -V(\tilde{p}_{Ht}(i)), \tag{B.1}$$

where $V(\tilde{p}_{Ht}(i))$ is the variance of $\tilde{p}_{Ht}(i)$.

Similarly, profits made in country H by the firm i that sets the price in units of currency F is written as

$$\Pi_{Ht}^{F}\left(p_{Ht}^{F}(i)\right) = \left(e^{p_{Ht}^{F}(i)}\mathcal{E}_t - MC_{Tt}\right) g\left(\frac{e^{p_{Ht}^{F}(i)}\mathcal{E}_t}{\mathcal{P}_{Tt}}\right)(C_{Tt} + X_t),$$

and the discounted expected profits up to the second-order approximation around $\tilde{p}_{Ht}(i) - e_t$ is given by

$$E_{t-1}Q_{t-1,t}\Pi_{Ht}^{F}\left(p_{Ht}^{F}(i)\right) \propto -E_{t-1}\left(p_{Ht}^{F}(i) - \tilde{p}_{Ht}(i) + e_t\right)^2 = -V(\tilde{p}_{Ht}(i) - e_t). \tag{B.2}$$

From equations (B.1) and (B.2), the tradable goods firm i chooses currency H as the invoice currency in the Home market if and only if $V(\tilde{p}_{Ht}(i)) \leq V(\tilde{p}_{Ht}(i) - e_t)$. From equations (A.32), (A.25), (A.36), and (A.53), the flexible price $\tilde{p}_{Ht}(i)$ can be written as

$$\begin{aligned}
\tilde{p}_{Ht}(i) &= (1-\alpha)\, mc_{Tt} + \alpha p_{Tt} \\
&= (1-\alpha)(-a_t + (1-\phi_\tau) w_t) + ((1-\alpha)\phi_\tau + \alpha) p_{Tt} \\
&= -(1-\alpha) a_t + (1-\alpha)(1-\phi_\tau) m_t + ((1-\alpha)\phi_\tau + \alpha)\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right] e_t.
\end{aligned} \tag{B.3}$$

In the case of a shock to an exchnage rate only, $V(\tilde{p}_{Ht}(i)) \leq V(\tilde{p}_{Ht}(i) - e_t)$ holds if and only if

$$(1-\gamma_\tau)\lambda_H + \gamma_\tau\lambda_F \geq 1 - \frac{1}{2((1-\alpha)\phi_\tau + \alpha)}. \tag{B.4}$$

In the case of monetary policy shocks only, the above equation for $\tilde{p}_{Ht}(i)$ can be written as

$$\begin{aligned}
\tilde{p}_{Ht}(i) \quad = \quad & -(1-\alpha) a_t + \{(1-\alpha)(1-\phi_\tau) + ((1-\alpha)\phi_\tau + \alpha)\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right]\} m_t \\
& -((1-\alpha)\phi_\tau + \alpha)\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right] m_t^*.
\end{aligned}$$

In this case, $V(\tilde{p}_{Ht}(i)) \leq V(\tilde{p}_{Ht}(i) - e_t)$ holds if and only if

$$(1-\gamma_\tau)\lambda_H + \gamma_\tau\lambda_F \geq \frac{1}{2}. \tag{B.5}$$

Invoice currency for $P_{Nt}(i)$ The Home non-tradable goods firm i sets the price using currency H if and only if $V(\tilde{p}_{Nt}(i)) \leq V(\tilde{p}_{Nt}(i) - e_t)$, where the flexible price $\tilde{p}_{Nt}(i)$ is given by

$$\begin{aligned}\tilde{p}_{Nt}(i) = mc_{Nt} &= -a_t + (1-\phi_n)w_t + \phi_n p_{Tt}, \\ &= -a_t + (1-\phi_n)m_t + \phi_n\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right]e_t.\end{aligned}$$

In the case of a shock to an exchange rate only, $V(\tilde{p}_{Nt}(i)) \leq V(\tilde{p}_{Nt}(i) - e_t)$ holds if and only if

$$(1-\gamma_\tau)\lambda_H + \gamma_\tau\lambda_F \leq 1 - \frac{1}{2\phi_n}. \tag{B.6}$$

In the case of monetary policy shocks only, the above equation for $\tilde{p}_{Nt}(i)$ is written as

$$\begin{aligned}\tilde{p}_{Nt}(i) = &-a_t + \{1 - \phi_n + \phi_n\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right]\}m_t \\ &- \phi_n\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right]m_t^*.\end{aligned}$$

Then, $V(\tilde{p}_{Nt}(i)) \leq V(\tilde{p}_{Nt}(i) - e_t)$ holds if and only if

$$(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F) \leq \frac{1}{2},$$

which is exactly the same as condition (B.5) for the Home tradable goods firm i.

Invoice currency for $P_{Ft}(i)$ Consider the Foreign tradable goods firm i, which chooses either currency H or F as the invoice currency in setting the price in the Home market. The firm i chooses currency F if and only if $V(\tilde{p}_{Ft}(i)) \geq V(\tilde{p}_{Ft}(i) - e_t)$, where the flexible price $\tilde{p}_{Ft}(i)$ is given by

$$\begin{aligned}\tilde{p}_{Ft}(i) &= (1-\alpha)(mc_{Tt}^* + e_t) + \alpha p_{Tt} \\ &= (1-\alpha)(-a_t^* + (1-\phi_\tau)w_t^* + \phi_\tau p_{Tt}^* + e_t) + \alpha p_{Tt} \\ &= -(1-\alpha)a_t^* + (1-\alpha)(1-\phi_\tau)m_t^* \\ &\quad + \{1-\alpha+\alpha\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right] - (1-\alpha)\phi_\tau\left[(1-\gamma_\tau^*)\lambda_F^* + \gamma_\tau^*\lambda_H^*\right]\}e_t.\end{aligned}$$

Assume that the Foreign firm i chooses its own currency as the invoice currency in its own market, i.e. $\lambda_F^* = 0$. In the case of a shock to an exchange rate only, $V(\tilde{p}_{Ft}(i)) \geq V(\tilde{p}_{Ft}(i) - e_t)$ holds if and only if

$$\alpha(\gamma_\tau(1-\lambda_H) + \gamma_\tau\lambda_F) \geq \alpha + (1-\alpha)(1-\phi_\tau\gamma_\tau^*\lambda_H^*) - \frac{1}{2}. \tag{B.7}$$

In the case of monetary policy shocks only, the optimal price $\tilde{p}_{Ft}(i)$ is written as

$$\begin{aligned}\tilde{p}_{Ft}(i) = &-(1-\alpha)a_t^* - \{\alpha\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right] + \phi_\tau(1-\alpha)(1-\gamma_\tau^*\lambda_H^*)\}m_t^* \\ &+ \{\alpha\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right] + (1-\alpha)(1-\phi_\tau\gamma_\tau^*\lambda_H^*)\}m_t.\end{aligned}$$

Then, $V(\tilde{p}_{Ft}(i)) \geq V(\tilde{p}_{Ft}(i) - e_t)$ holds if and only if

$$2\alpha\left[(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau(1-\lambda_F)\right] + (1-\alpha)(1+\phi_\tau - 2\phi_\tau\gamma_\tau^*\lambda_H^*) \geq 1. \tag{B.8}$$

Invoice currency for $P^*_{Ht}(i)$ The Home tradable goods firm i sets its export price in units of currency H if and only if $V(\tilde{p}^*_{Ht}(i)+e_t) \leq V(\tilde{p}^*_{Ht}(i))$, where the flexible price $\tilde{p}^*_{Ht}(i)$ is given by

$$\begin{aligned}
\tilde{p}^*_{Ht}(i) &= (1-\alpha)(mc_{Tt}-e_t)+\alpha p^*_{Tt} \\
&= (1-\alpha)(-a_t+(1-\phi_\tau)w_t+\phi_\tau p_{Tt}-e_t)+\alpha p^*_{Tt} \\
&= -(1-\alpha)a_t+(1-\alpha)(1-\phi_\tau)m_t \\
&\quad +\{(1-\alpha)[\phi_\tau(1-\gamma_\tau)(1-\lambda_H)+\phi_\tau\gamma_\tau(1-\lambda_F)-1]-\alpha[(1-\gamma^*_\tau)\lambda^*_F+\gamma^*_\tau\lambda^*_H]\}e_t.
\end{aligned}$$

Assume $\lambda^*_F=0$. In the case of an exchange rate shock only, $V(\tilde{p}^*_{Ht}(i)+e_t) \leq V(\tilde{p}^*_{Ht}(i))$ holds if and only if

$$\phi_\tau(1-(1-\gamma_\tau)\lambda_H-\gamma_\tau\lambda_F) \leq \frac{1-2\alpha}{2(1-\alpha)}+\frac{\alpha\gamma^*_\tau\lambda^*_H}{1-\alpha}. \tag{B.9}$$

In the case of monetary policy shocks only, the optimal price $\tilde{p}^*_{Ht}$ is written as

$$\begin{aligned}
\tilde{p}^*_{Ht} = &-(1-\alpha)a_t+\{(1-\alpha)[\phi_\tau(1-\gamma_\tau)(1-\lambda_H)+\phi_\tau\gamma_\tau(1-\lambda_F)-\phi_\tau]-\alpha[(1-\gamma^*_\tau)\lambda^*_F+\gamma^*_\tau\lambda^*_H]\}m_t \\
&-\{(1-\alpha)[\phi_\tau(1-\gamma_\tau)(1-\lambda_H)+\phi_\tau\gamma_\tau(1-\lambda_F)-1]-\alpha[(1-\gamma^*_\tau)\lambda^*_F+\gamma^*_\tau\lambda^*_H]\}m^*_t
\end{aligned}$$

Then, $V(\tilde{p}^*_{Ht}(i)+e_t) \leq V(\tilde{p}^*_{Ht}(i))$ holds if and only if

$$\phi_\tau(1-(1-\gamma_\tau)\lambda_H-\gamma_\tau\lambda_F) \leq \frac{\alpha\gamma^*_\tau\lambda^*_H}{1-\alpha}+\frac{1-2\alpha}{2(1-\alpha)}-\frac{1}{2}(1-\phi_\tau). \tag{B.10}$$

B.1.2 Nominal wage rigidity

Next, the cases of nominal wage rigidity are considered. In this case, as shown below, there is no difference between the case of an exchange rate shock only and the case of monetary policy shocks only.

Invoice currency for $P_{Ht}(i)$ With nominal wage rigidity the wage is given by equation (11). In this case, the flexible price $\tilde{p}_{Ht}(i)$ can be written as

$$\begin{aligned}
\tilde{p}_{Ht}(i) &= (1-\alpha)mc_{Tt}+\alpha p_{Tt} \\
&= (1-\alpha)(-a_t+(1-\phi_\tau)w_t)+((1-\alpha)\phi_\tau+\alpha)p_{Tt} \\
&= -(1-\alpha)a_t+\{(1-\alpha)(1-\phi_\tau)(1-\lambda_w)+((1-\alpha)\phi_\tau+\alpha)[(1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)]\}e_t.
\end{aligned}$$

Because the flexible price depends only on e_t, there is no difference between the case of an exchange rate shock only and the case of monetary policy shocks only. The firm i chooses its own currency as the invoice currency in the Home market if and only if $V(\tilde{p}_{Ht}(i)) \leq V(\tilde{p}_{Ht}(i)-e_t)$ holds or

$$(1-\alpha)(1-\phi_\tau)(1-\lambda_w)+((1-\alpha)\phi_\tau+\alpha)(1-(1-\gamma_\tau)\lambda_H-\gamma_\tau\lambda_F) \leq \frac{1}{2}.$$

If the Home wages are set in currency H, i.e., $\lambda_w=1$, the condition is reduced to

$$(1-\gamma_\tau)\lambda_H+\gamma_\tau\lambda_F \geq 1-\frac{1}{2((1-\alpha)\phi_\tau+\alpha)},$$

which is the same as condition (B.4) with no nominal wage rigidity in the case of an exchange rate shock only. This condition holds in the economy with no strategic complementarities $\alpha = 0$ and no intermediate input $\phi_\tau = 0$. In general, this condition is more likely to hold than condition (B.5); that is, in the case of monetary policy shocks only the firm i is more likely to set its price using currency H if the wages are set in units of currency H one period in advance.

Invoice currency for $P_{Nt}(i)$ With nominal wage rigidity, the flexible price $\tilde{p}_{Nt}(i)$ can be written as

$$\begin{aligned}\tilde{p}_{Nt}(i) &= -a_t + (1-\phi_n)w_t + \phi_n p_{Tt} \\ &= -a_t + \{(1-\phi_n)(1-\lambda_w) + \phi_n [(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau (1-\lambda_F)]\} e_t\end{aligned}$$

Then, the non-tradable goods firm i chooses the Home currency as the invoice currency if and only if $V(\tilde{p}_{Nt}(i)) \leq V(\tilde{p}_{Nt}(i) - e_t)$, or

$$(1-\phi_n)(1-\lambda_w) + \phi_n (1-(1-\gamma_\tau)\lambda_H - \gamma_\tau \lambda_F) \leq \frac{1}{2}$$

In the case of $\lambda_w = 1$, the condition is reduced to

$$(1-\gamma_\tau)\lambda_H + \gamma_\tau \lambda_F \geq 1 - \frac{1}{2\phi_n},$$

which is the same as the condition (B.6). This condition holds for any $\phi_n < 1/2$.

Invoice currency for $P_{Ft}(i^*)$ With nominal wage rigidity, the flexible price $\tilde{p}_{Ft}(i)$ can be written as

$$\begin{aligned}\tilde{p}_{Ft}(i) &= (1-\alpha)(-a_t^* + (1-\phi_\tau) w_t^* + \phi_\tau p_{Tt}^* + e_t) + \alpha p_{Tt} \\ &= -(1-\alpha) a_t^* + \{(1-\alpha)(1-(1-\phi_\tau)\lambda_w^*) \\ &\quad + \alpha [(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau (1-\lambda_F)] - (1-\alpha)\phi_\tau [(1-\gamma_\tau^*)\lambda_F^* + \gamma_\tau^* \lambda_H^*]\} e_t.\end{aligned}$$

Then, under the assumption that the Foreign firms set prices in country F using their own currency, i.e., $\lambda_F^* = 0$, the Foreign firm i chooses its own currency as the invoice currency in the Home market if and only if $V(\tilde{p}_{Ft}(i)) \geq V(\tilde{p}_{Ft}(i) - e_t)$, or

$$\alpha [(1-\gamma_\tau)(1-\lambda_H) + \gamma_\tau (1-\lambda_F)] + (1-\alpha)(1-(1-\phi_\tau)\lambda_w^* - \phi_\tau \gamma_\tau^* \lambda_H^*) \geq \frac{1}{2}.$$

If the nominal wage in country F is set in advance in terms of the country's currency, i.e., $\lambda_w^* = 0$, the above condition is reduced to condition (B.7). In the case of monetary policy shocks only, condition (B.7) with nominal wage rigidity is less tight than the corresponding condition (B.8) with no nominal wage rigidity.

Invoice currency for $P_{Ht}^{*}(i)$ With nominal wage rigidity, the flexible price $\tilde{p}_{Ht}^{*}(i)$ can be written as

$$\begin{aligned}\tilde{p}_{Ht}^{*}(i) &= (1-\alpha)(-a_t+(1-\phi_\tau)w_t+\phi_\tau p_{Tt}-e_t)+\alpha p_{Tt}^{*} \\ &= -(1-\alpha)a_t+\{(1-\alpha)[(1-\phi_\tau)(1-\lambda_w)+\phi_\tau[(1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)]-1] \\ &\quad -\alpha[(1-\gamma_\tau^{*})\lambda_F^{*}+\gamma_\tau^{*}\lambda_H^{*}]\}e_t.\end{aligned}$$

Then, the Home tradable goods firm i chooses its own currency as the invoice currency in country F if and only if $V(\tilde{p}_{Ht}^{*}(i)+e_t)\leq V(\tilde{p}_{Ht}^{*}(i))$ holds, or

$$\alpha[(1-\gamma_\tau^{*})\lambda_F^{*}+\gamma_\tau^{*}\lambda_H^{*}]+(1-\alpha)\{1-(1-\phi_\tau)(1-\lambda_w)-\phi_\tau[(1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)]\}\geq\frac{1}{2}.$$

Under the assumptions of $\lambda_F^{*}=0$ and $\lambda_w=1$, this condition is reduced to condition (B.9). In the case of monetary policy shocks only, this condition with nominal wage rigidity is less tight than the corresponding condition (B.10) with no nominal wage rigidity.

B.2 Invoice Currencies for Wages

Similar to the invoice-currency choices for price setting, a Home household chooses currency H as the invoice currency for wages if and only if $V(\tilde{w}_t)\leq V(\tilde{w}_t-e_t)$, where the flexible wage, $\tilde{w}_t$, is given by equation (A.25). Under the assumption of $\sigma=1$, the flexible wage is given by $\tilde{w}_t=(1/\nu)l_t+m_t$. If $\nu=\infty$ as assumed in the currency choice problems for price setting, the household would always choose currency H in the case of an exchange rate shock only because $0=V(\tilde{w}_t)<V(\tilde{w}_t-e_t)$. Hence, in this analysis, a general case of $\nu<\infty$ is considered. But, for analytical tractability, the model with no non-tradable sector is considered.

With no non-tradable sector, a goods market clearing condition is given by equation (A.55) with subscript T omitted. Up to the first-order approximation, the market clearing condition is written as:

$$a_t+\phi_\tau(x_t-l_t)+l_t=(1-\gamma_\tau)((1-\phi_\tau)c_t+\phi_\tau l_t)+\gamma_\tau((1-\phi_\tau)c_t^{*}+\phi_\tau l_t^{*}),$$

where the left-hand-side corresponds to the Cobb-Douglas production technology and the right-hand-side suppresses price dispersion terms because they are zero up to the first-order approximation. Equations (A.28) and (A.30), with subscript T omitted, imply that the ratio of intermediate input to labor is given by $x_t-l_t=w_t-p_t$ and similarly for country F. By using this ratio, the above equation can be written as

$$a_t+\gamma_\tau\phi_\tau[(w_t-p_t)-(w_t^{*}-p_t^{*})]+(1-(1-\gamma_\tau)\phi_\tau)l_t-\gamma_\tau\phi_\tau l_t^{*}=(1-\phi_\tau)[(1-\gamma_\tau)c_t+\gamma_\tau c_t^{*}].$$

For country F, a symmetric condition holds:

$$a_t^{*}+\gamma_\tau^{*}\phi_\tau[(w_t^{*}-p_t^{*})-(w_t-p_t)]+(1-(1-\gamma_\tau^{*})\phi_\tau)l_t^{*}-\gamma_\tau^{*}\phi_\tau l_t=(1-\phi_\tau)[(1-\gamma_\tau^{*})c_t^{*}+\gamma_\tau^{*}c_t].$$

Assume that prices and wages in country F are set in units currency F only, so that $p_t^{*}=w_t^{*}=0$. For country H, the price is given by equation (A.53), with subscript T suppressed, as $p_t=[(1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)]e_t$, and the wage is given by $w_t=(1-\lambda_w)e_t$. Substituting these equations into the above two goods market clearing conditions and solving for l_t yield

$$l_t = -\frac{(1-\phi_\tau)\left[(1-\gamma_\tau-\phi_\tau(1-\gamma_\tau^*))\left((1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)\right)+\gamma_\tau\phi_\tau(1-\lambda_w)\right]}{1-(1-\gamma_\tau)\phi_\tau-(1-\gamma_\tau^*)\phi_\tau+(1-\gamma_\tau-\gamma_\tau^*)\phi_\tau^2} e_t.$$

Given that $m_t = 0$ in the case of an exchange rate shock only, the flexible wage in units of currency H is given by $\tilde{w}_t = (1/\nu)l_t$ and that in units of currency F is given by $\tilde{w}_t^F = \tilde{w}_t - e_t$. Then, currency H is chosen if and only if $V(\tilde{w}_t) \leq V(\tilde{w}_t^F)$, or

$$\frac{(1-\phi_\tau)\left[(\gamma_\tau+\phi_\tau(1-\gamma_\tau^*)-1)\left((1-\gamma_\tau)(1-\lambda_H)+\gamma_\tau(1-\lambda_F)\right)-\gamma_\tau\phi_\tau(1-\lambda_w)\right]}{\nu\left[1-(1-\gamma_\tau)\phi_\tau-(1-\gamma_\tau^*)\phi_\tau+(1-\gamma_\tau-\gamma_\tau^*)\phi_\tau^2\right]} \leq \frac{1}{2}. \tag{B.11}$$

With $\gamma_\tau = (1-n)\bar{\gamma}$ and $\gamma_\tau^* = n\bar{\gamma}$ kept in mind, the denominator of the left-hand-side of equation (B.11) can be written as $1-\phi_\tau(2-\bar{\gamma}-\phi_\tau(1-\bar{\gamma})) > 0$ as long as $\phi_\tau < 1$. Then, in the case of $\lambda_w = 1$ (full H-currency wage setting), a sufficient condition for condition (B.11) is that the numerator of the left-hand-side is negative:

$$\bar{\gamma} + \phi_\tau - n\bar{\gamma}(1+\phi_\tau) < 1.$$

This condition is more likely to hold for a large country i.e. a large value of n. Conversely, this condition is more likely to be violated so that currency F is chosen as the invoice currency for wages in a smaller country with a higher degree of trade openness and a larger share of intermediate input. In a small open economy ($n = 0$) with $\lambda_w = 1$ and $\lambda_F < \lambda_H$, the left-hand-side of (B.11) is increasing in $\bar{\gamma}$.

C Extended Model

Consumption and saving A solution to the consumption and saving problems of the households in countries H and F is characterized by the Euler equations:

$$1 = E_t\left[\beta\left(\frac{\bar{C}_t}{\bar{C}_{t+1}}\right)^{\frac{1}{\sigma}}\frac{R_t}{\pi_{t+1}}\right], \tag{C.1}$$

$$1 = E_t\left[\beta\left(\frac{\bar{C}_t^*}{\bar{C}_{t+1}^*}\right)^{\frac{1}{\sigma}}\frac{R_t^*}{\pi_{t+1}^*}\right], \tag{C.2}$$

where $\bar{C}_t$ and $\bar{C}_t^*$ denote per capita consumption in countries H and F, respectively.

Wage setting First, consider the Home household j's wage setting problem with currency H as the invoice currency:

$$\Pi_{wt}^H = \max_{\{\bar{W}_t(j)\}} E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\left[\Lambda_{t+s}(j)\,\bar{W}_t(j)\,L_{t+s|t}(j) - \frac{\psi}{1+\frac{1}{\nu}}L_{t+s|t}(j)^{1+\frac{1}{\nu}}\right],$$

subject to the labor demand curve

$$L_{t+s|t}(j) = \frac{1}{n}\left(\frac{\bar{W}_t(j)}{W_{t+s}}\right)^{-\theta_w} L_{t+s}.$$

The first-order condition with respect to $\bar{W}_t(j)$, with index j omitted, is

$$\left(\frac{\bar{W}_t^r}{W_t^r}\right)^{1+\frac{\theta_w}{\nu}} = \frac{\theta_w \psi}{\theta_w - 1}\frac{V_{w1,t}}{V_{w2,t}}, \tag{C.3}$$

where

$$\begin{aligned}
V_{w1,t} &= E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \left(\frac{W_t^r}{W_{t+s}^r}\frac{P_t}{P_{t+s}}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} \bar{L}_{t+s}^{1+\frac{1}{\nu}}, \\
V_{w2,t} &= E_t \sum_{s=0}^{\infty} (\beta \xi_w)^s \left(\frac{W_t^r}{W_{t+s}^r}\frac{P_t}{P_{t+s}}\right)^{1-\theta_w} \Lambda_{t+s}(j)\, W_{t+s}^r \bar{L}_{t+s},
\end{aligned}$$

Here $\bar{W}_t^r = \bar{W}_t/P_t$ is the real wage set in period t, $W_t^r = W_t/P_t$ is the real wage, and $\bar{L}_{t+s} = n^{-1}L_{t+s}$ is the per capita labor supply. The auxiliary variables $V_{w1,t}$ and $V_{w2,t}$ can be written recursively as

$$V_{w1,t} = \bar{L}_t^{1+\frac{1}{\nu}} + \beta\xi_w \left[E_t \left(\frac{1}{\pi_{t+1}}\frac{W_t^r}{W_{t+1}^r}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} V_{w1,t+1}\right], \tag{C.4}$$

$$V_{w2,t} = \frac{W_t^r \bar{L}_t}{\bar{C}_t^{\frac{1}{\sigma}}} + \beta\xi_w E_t \left[\left(\frac{1}{\pi_{t+1}}\frac{W_t^r}{W_{t+1}^r}\right)^{1-\theta_w} V_{w2,t+1}\right]. \tag{C.5}$$

The discounted sum of utility by setting the wage in units of currency H is written as

$$\Pi_{wt}^H = \left(\frac{\bar{W}_t^r}{W_t^r}\right)^{1-\theta_w} V_{w2,t} - \frac{\psi}{1+1/\nu}\left(\frac{\bar{W}_t^r}{W_t^r}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} V_{w1,t}. \tag{C.6}$$

For country F, the optimality conditions are symmetric, given by

$$\left(\frac{\bar{W}_t^{r*}}{W_t^{r*}}\right)^{1+\frac{\theta_w}{\nu}} = \frac{\theta_w \psi}{\theta_w - 1}\frac{V_{w1,t}^*}{V_{w2,t}^*}, \tag{C.7}$$

where

$$V_{w1,t}^* = \bar{L}_t^{*1+\frac{1}{\nu}} + \beta\xi_w \left[E_t \left(\frac{1}{\pi_{t+1}^*}\frac{W_t^{r*}}{W_{t+1}^{r*}}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} V_{w1,t+1}^*\right], \tag{C.8}$$

$$V_{w2,t}^* = \frac{W_t^{r*} \bar{L}_t^*}{\bar{C}_t^{*\frac{1}{\sigma}}} + \beta\xi_w E_t \left[\left(\frac{1}{\pi_{t+1}^*}\frac{W_t^{r*}}{W_{t+1}^{r*}}\right)^{1-\theta_w} V_{w2,t+1}^*\right]. \tag{C.9}$$

Next, consider the same problem for the Home household but with the wage set in units of currency F:

$$\Pi_{wt}^F = \max_{\{\bar{W}_t^F(j)\}} E_t \sum_{s=0}^{\infty} (\beta\xi_w)^s \left[\Lambda_{t+s}(j)\, \mathcal{E}_{t+s} \bar{W}_t^F(j)\, L_{t+s|t}(j) - \frac{\psi}{1+\frac{1}{\nu}} L_{t+s|t}(j)^{1+\frac{1}{\nu}}\right],$$

subject to the labor demand curve

$$L_{t+s|t}(j) = \frac{1}{n}\left(\frac{\bar{W}_t^F(j)\, \mathcal{E}_{t+s}}{W_{t+s}}\right)^{-\theta_w} L_{t+s}.$$

The first-order condition with respect to $\bar{W}_t^F(j)$ is

$$\left(\frac{\bar{W}_t^{Fr}}{W_t^r}\mathcal{E}_t^r\right)^{1+\frac{\theta_w}{\nu}} = \frac{\theta_w\psi}{\theta_w-1}\frac{V_{w1,t}^F}{V_{w2,t}^F}, \tag{C.10}$$

where

$$\begin{aligned} V_{w1,t}^F &= E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\left(\frac{W_t^r}{W_{t+s}^r}\frac{\mathcal{E}_{t+s}^r}{\mathcal{E}_t^r}\frac{P_t^*}{P_{t+s}^*}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)}\bar{L}_{t+s}^{1+\frac{1}{\nu}}, \\ V_{w2,t}^F &= E_t\sum_{s=0}^{\infty}(\beta\xi_w)^s\Lambda_{t+s}(j)\left(\frac{W_t^r}{W_{t+s}^r}\frac{\mathcal{E}_{t+s}^r}{\mathcal{E}_t^r}\frac{P_t^*}{P_{t+s}^*}\right)^{1-\theta_w}P_{t+s}W_{t+s}^r\bar{L}_{t+s}, \end{aligned}$$

Here $\bar{W}_t^{Fr}=\bar{W}_t^F/P_t^*$, and $\mathcal{E}_{t+s}^r=\mathcal{E}_{t+s}P_{t+s}^*/P_{t+s}$ is the real exchange rate. The auxiliary variables $V_{w1,t}^F$ and $V_{w2,t}^F$ can be written recursively as

$$V_{w1,t}^F = \bar{L}_t^{1+\frac{1}{\nu}}+\beta\xi_w\left[E_t\left(\frac{1}{\pi_{t+1}^*}\frac{\mathcal{E}_{t+1}^r}{\mathcal{E}_t^r}\frac{W_t^r}{W_{t+1}^r}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)}V_{w1,t+1}^F\right], \tag{C.11}$$

$$V_{w2,t}^F = \frac{W_t^r\bar{L}_t}{\bar{C}_t^{\frac{1}{\sigma}}}+\beta\xi_w E_t\left[\left(\frac{1}{\pi_{t+1}^*}\frac{\mathcal{E}_{t+1}^r}{\mathcal{E}_t^r}\frac{W_t^r}{W_{t+1}^r}\right)^{1-\theta_w}V_{w2,t+1}^F\right]. \tag{C.12}$$

The discounted sum of utility by setting the wage in units of currency F is written as

$$\Pi_{wt}^F=\left(\frac{\bar{W}_t^{Fr}}{W_t^r}\mathcal{E}_t^r\right)^{1-\theta_w}V_{w2,t}^F-\frac{\psi}{1+1/\nu}\left(\frac{\bar{W}_t^{Fr}}{W_t^r}\mathcal{E}_t^r\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)}V_{w1,t}^F. \tag{C.13}$$

Wage indices The wage index $W_t=[(1/n)\int_0^n W_t(j)^{1-\theta_w}dj]^{1/(1-\theta_w)}$ can be decomposed as

$$W_t^{1-\theta_w}=\lambda_w\left(W_t^H\right)^{1-\theta_w}+(1-\lambda_w)\left(\mathcal{E}_tW_t^F\right)^{1-\theta_w},$$

where W_t^H and W_t^F denote indices of H-currency wages and F-currency wages, respectively. In real terms, the wage index is written as

$$(W_t^r)^{1-\theta_w}=\lambda_w\left(W_t^{Hr}\right)^{1-\theta_w}+(1-\lambda_w)\left(\mathcal{E}_t^rW_t^{Fr}\right)^{1-\theta_w}, \tag{C.14}$$

where $W_t^{Hr}=W_t^H/P_t$ and $W_t^{Fr}=W_t^F/P_t^*$. The indices of H-currency wages and F-currency wages are written recursively as

$$\left(W_t^{Hr}\right)^{1-\theta_w}=(1-\xi_w)\bar{W}_t^{1-\theta_w}+\xi_w\left(\frac{W_{t-1}^{Hr}}{\pi_t}\right)^{1-\theta_w} \tag{C.15}$$

$$\left(W_t^{Fr}\right)^{1-\theta_w}=(1-\xi_w)\left(\bar{W}_t^F\right)^{1-\theta_w}+\xi_w\left(\frac{W_{t-1}^{Fr}}{\pi_t^*}\right)^{1-\theta_w} \tag{C.16}$$

For country F, because all households are assumed to set wages in units of their own currency, the law of motion for the real wage is given by

$$\left(W_t^{r*}\right)^{1-\theta_w} = (1-\xi_w)\left(\bar{W}_t^{r*}\right)^{1-\theta_w} + \xi_w \left(\frac{W_{t-1}^{r*}}{\pi_t^*}\right)^{1-\theta_w} \tag{C.17}$$

Marginal costs and factor prices From equations (A.31) and (A.32), the real marginal costs MC_{Nt}^r and MC_{Tt}^r are given by

$$MC_{Nt}^r = \frac{1}{A_t}\left(W_t^r\right)^{1-\phi_n}\left(P_{Tt}^r\right)^{\phi_n}, \tag{C.18}$$

$$MC_{Tt}^r = \frac{1}{A_t}\left(W_t^r\right)^{1-\phi_\tau}\left(P_{Tt}^r\right)^{\phi_\tau}, \tag{C.19}$$

Equations (A.27)-(A.30) can be combined for each sector and can be written as

$$\frac{W_t^r}{P_{Tt}^r} = \frac{1-\phi_n}{\phi_n}\left(\frac{\bar{L}_{Nt}}{\bar{X}_{Nt}}\right)^{-1}, \tag{C.20}$$

$$\frac{W_t^r}{P_{Tt}^r} = \frac{1-\phi_\tau}{\phi_\tau}\left(\frac{\bar{L}_{Tt}}{\bar{X}_{Tt}}\right)^{-1}, \tag{C.21}$$

where variables with upper bar denote those in per capita terms. For country F, similar equations hold:

$$MC_{Nt}^{*r} = \frac{1}{A_t^*}\left(W_t^{*r}\right)^{1-\phi_n}\left(P_{Tt}^{*r}\right)^{\phi_n}, \tag{C.22}$$

$$MC_{Tt}^{*r} = \frac{1}{A_t^*}\left(W_t^{*r}\right)^{1-\phi_\tau}\left(P_{Tt}^{*r}\right)^{\phi_\tau}, \tag{C.23}$$

$$\frac{W_t^{*r}}{P_{Tt}^{*r}} = \frac{1-\phi_n}{\phi_n}\left(\frac{\bar{L}_{Nt}^*}{\bar{X}_{Nt}^*}\right)^{-1}, \tag{C.24}$$

$$\frac{W_t^{*r}}{P_{Tt}^{*r}} = \frac{1-\phi_\tau}{\phi_\tau}\left(\frac{\bar{L}_{Tt}^*}{\bar{X}_{Tt}^*}\right)^{-1}, \tag{C.25}$$

Non-tradable goods firms price setting The non-tradable goods firms set prices in units of either currency H or F. First, consider the case of currency H as the invoice currency. The price setting problem can be formulated as

$$\Pi_{Nt}^H = \max_{\{\bar{P}_{Nt}(i)\}} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t}\left(\bar{P}_{Nt}(i) - e^{v_{t+s}} MC_{Nt+s}\right)\left(\frac{\bar{P}_{Nt}(i)}{P_{Nt+s}}\right)^{-\theta_n} \bar{Y}_{Nt+s}^d,$$

where $\bar{Y}_{Nt+s}^d = \bar{C}_{Nt+s} + \bar{G}_{Nt+s}$ and $\Lambda_{t+s} = 1/(P_{t+s}\bar{C}_{t+s}^{1/\sigma})$. The first-order condition is

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t}\left(\frac{\bar{P}_{Nt}(i)}{P_{Nt+s}}\right)^{-\theta_n} \bar{Y}_{Nt+s}^d \left[-(\theta_n - 1) + \theta_n \frac{e^{v_{t+s}} MC_{Nt+s}}{\bar{P}_{Nt}(i)}\right],$$

or

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Nt+s}^d}{\bar{C}_{t+s}^{1/\sigma}}\left(\frac{P_t/P_{t+s}}{P_{Nt+s}^r}\right)^{-\theta_n}\left[-(\theta_n - 1)\frac{\bar{P}_{Nt}^r(i)}{P_{t+s}/P_t} + \theta_n e^{v_{t+s}} MC_{Nt+s}^r\right].$$

This equation can be solved for $\bar{P}^r_{Nt} = \bar{P}^r_{Nt}(i)$ as

$$\bar{P}^r_{Nt} = \frac{\theta_n}{\theta_n - 1} \frac{V_{n1,t}}{V_{n2,t}}, \tag{C.26}$$

where

$$\begin{aligned}
V_{n1,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Nt+s}}{\bar{C}^{1/\sigma}_{t+s}} \left(\frac{P_t/P_{t+s}}{P^r_{Nt+s}} \right)^{-\theta_n} e^{v_{t+s}} MC^r_{Nt+s}, \\
V_{n2,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Nt+s}}{\bar{C}^{1/\sigma}_{t+s}} \left(\frac{P_t/P_{t+s}}{P^r_{Nt+s}} \right)^{-\theta_n} \frac{P_t}{P_{t+s}}.
\end{aligned}$$

The auxiliary variables $V_{n1,t}$ and $V_{n2,t}$ can be written recursively as

$$V_{n1,t} = \frac{\bar{Y}^d_{Nt}}{\bar{C}^{1/\sigma}_t} \left(\frac{1}{P^r_{Nt}} \right)^{-\theta_n} e^{v_t} MC^r_{Nt} + \beta\xi_p E_t \left(\pi^{\theta_n}_{t+1} V_{n1,t+1} \right), \tag{C.27}$$

$$V_{n2,t} = \frac{\bar{Y}^d_{Nt}}{\bar{C}^{1/\sigma}_t} \left(\frac{1}{P^r_{Nt}} \right)^{-\theta_n} + \beta\xi_p E_t \left(\pi^{\theta_n - 1}_{t+1} V_{n2,t+1} \right). \tag{C.28}$$

The profits of the firm that has chosen currency H as the invoice currency are given by

$$\bar{\Pi}^H_{Nt} = \frac{\Pi^H_{Nt}}{P_t C^{1/\sigma}_t} = V_{n2,t} \left(\bar{P}^r_{Nt} \right)^{1-\theta_n} - V_{n1,t} \left(\bar{P}^r_{Nt} \right)^{-\theta_n}. \tag{C.29}$$

Next, consider the case of currency F as the invoice currency. The price setting problem can be formulated as

$$\Pi^F_{Nt} = \max_{\{\bar{P}^F_{Nt}(i)\}} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\bar{P}^F_{Nt}(i) \mathcal{E}_{t+s} - e^{v_{t+s}} MC_{Nt+s} \right) \left(\frac{\bar{P}^F_{Nt}(i)\mathcal{E}_{t+s}}{P_{Nt+s}} \right)^{-\theta_n} \bar{Y}^d_{Nt+s},$$

The first-order condition is

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{P_t \bar{C}^{1/\sigma}_t}{P_{t+s} \bar{C}^{1/\sigma}_{t+s}} \left(\frac{\bar{P}^F_{Nt}(i)\mathcal{E}_{t+s}}{P_{Nt+s}} \right)^{-\theta_n} \bar{Y}^d_{Nt+s} \left[(1-\theta_n)\mathcal{E}_{t+s} + e^{v_{t+s}} MC_{Nt+s}\theta_n / \bar{P}^F_{Nt}(i) \right],$$

or

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Nt+s}}{\bar{C}^{1/\sigma}_{t+s}} \left(\frac{\mathcal{E}^r_{t+s} P^*_t}{P^r_{Nt+s} P^*_{t+s}} \right)^{-\theta_n} \left[-(\theta_n - 1)\mathcal{E}^r_{t+s} \bar{P}^{Fr}_{Nt}(i) \frac{P^*_t}{P^*_{t+s}} + e^{v_{t+s}} MC^r_{Nt+s}\theta_n \right].$$

This equation can be solved for $\bar{P}^{Fr}_{Nt} = \bar{P}^{Fr}_{Nt}(i)$ as

$$\bar{P}^{Fr}_{Nt} = \frac{\theta_n}{\theta_n - 1} \frac{V^F_{n1,t}}{V^F_{n2,t}}, \tag{C.30}$$

where

$$V^F_{n1,t} = \frac{\bar{Y}^d_{Nt}}{\bar{C}^{1/\sigma}_t} \left(\frac{\mathcal{E}^r_t}{P^r_{Nt}} \right)^{-\theta_n} e^{v_t} MC^r_{Nt} + \beta\xi_p E_t \left(\left(\pi^*_{t+1} \right)^{\theta_n} V^F_{n1,t+1} \right), \tag{C.31}$$

$$V^F_{n2,t} = \frac{\bar{Y}^d_{Nt}}{\bar{C}^{1/\sigma}_t} \frac{\left(\mathcal{E}^r_t \right)^{1-\theta_n}}{\left(P^r_{Nt} \right)^{-\theta_n}} + \beta\xi_p E_t \left(\left(\pi^*_{t+1} \right)^{\theta_n - 1} V^F_{n2,t+1} \right). \tag{C.32}$$

The profits of the firm that has chosen currency F as the invoice currency is given by

$$\bar{\Pi}_{Nt}^{F}=\frac{\Pi_{Nt}^{F}}{P_t C_t^{1/\sigma}}=V_{n2,t}^{F}\left(\bar{P}_{Nt}^{Fr}\right)^{1-\theta_n}-V_{n1,t}^{F}\left(\bar{P}_{Nt}^{Fr}\right)^{-\theta_n}. \tag{C.33}$$

For country F, the non-tradable goods firms set the prices using currency F only. Their price setting is characterized by

$$\bar{P}_{Nt}^{*}=\frac{\theta_n}{\theta_n-1}\frac{V_{n1,t}^{*}}{V_{n2,t}^{*}}, \tag{C.34}$$

where

$$V_{n1,t}^{*} = \frac{\bar{Y}_{Nt}^{d*}}{\bar{C}_t^{*1/\sigma}}\left(\frac{1}{P_{Nt}^{*r}}\right)^{-\theta_n} e^{v_t^*} MC_{Nt}^{*r}+\beta\xi_p E_t\left(\left(\pi_{t+1}^{*}\right)^{\theta_n} V_{n1,t+1}^{*}\right), \tag{C.35}$$

$$V_{n2,t}^{*} = \frac{\bar{Y}_{Nt}^{d*}}{\bar{C}_t^{*1/\sigma}}\left(\frac{1}{P_{Nt}^{*r}}\right)^{-\theta_n} +\beta\xi_p E_t\left(\left(\pi_{t+1}^{*}\right)^{\theta_n-1} V_{n2,t+1}^{*}\right). \tag{C.36}$$

Tradable goods aggregator The tradable goods aggregator $G(\cdot)$ is given by

$$G(x)=\frac{\theta_\tau}{\theta_\tau-1-\epsilon}\left[\left(1-\frac{\epsilon}{\theta_\tau}\right)x+\frac{\epsilon}{\theta_\tau}\right]^{\frac{\theta_\tau-1-\epsilon}{\theta_\tau-\epsilon}}-\frac{\theta_\tau}{\theta_\tau-1-\epsilon}+1.$$

The function $g(\cdot)=G'^{-1}(\cdot)$ and its derivatives are computed as

$$\begin{aligned} g(x) &= \frac{\theta_\tau x^{-(\theta_\tau-\epsilon)}-\epsilon}{\theta_\tau-\epsilon}, \\ g'(x) &= -\theta_\tau x^{-(1+\theta_\tau-\epsilon)}, \\ g''(x) &= \theta_\tau\left(1+\theta_\tau-\epsilon\right)x^{-(2+\theta_\tau-\epsilon)} \end{aligned}$$

With this functional form, in steady state, the demand elasticity is given as $\theta_\tau=-g'(x)\,x/g(x)\,|_{x=1}$ and the superelasticity is given as $\epsilon=\theta_\tau'(x)\,x/\theta_\tau(x)\,|_{x=1}$. The parameter $\Gamma\equiv-g''(1)/\theta_\tau(1-\theta_\tau)+(1+\theta_\tau)/(1-\theta_\tau)$ can be written as

$$\Gamma=\frac{\epsilon}{1-\theta_\tau}.$$

Then the parameter of strategic complementarities $\alpha\equiv\Gamma/(1+\Gamma)$ is written as

$$\alpha=\frac{\epsilon}{\epsilon+\theta_\tau-1}.$$

Then, $\epsilon=\alpha(\theta_\tau-1)/(1-\alpha)$.

Tradable goods firms price setting: (Goods, Market, Currency) = (H,H,H) Consider a price setting problem in which the Home tradable goods firm i sets its price in the Home market using currency H:

$$\Pi_{Ht}^{H}=\max_{\left\{\bar{P}_{Ht}(i)\right\}} E_t\sum_{s=0}^{\infty}\left(\beta\xi_p\right)^s\frac{\Lambda_{t+s}}{\Lambda_t}\left(\bar{P}_{Ht}(i)-e^{v_{t+s}}MC_{Tt+s}\right)Y_{H,t+s|t}(i),$$

subject to the demand curve

$$Y_{H,t+s|t}(i) = g\left(\frac{\bar{P}_{Ht}(i)}{\mathcal{P}_{Tt+s}}\right)(1-\gamma_\tau)\bar{Y}^d_{Tt+s},$$

where $\bar{Y}^d_{Tt+s} = \bar{C}_{Tt+s} + \bar{X}_{t+s} + \bar{G}_{Tt+s}$. The first-order condition with respect to $\bar{P}_{Ht}(i)$ is

$$\begin{aligned} 0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \bar{Y}^d_{Tt+s} \\ &\times \left(g\left(\frac{\bar{P}_{Ht}(i)}{\mathcal{P}_{Tt+s}}\right) + \frac{\bar{P}_{Ht}(i)}{\mathcal{P}_{Tt+s}} g'\left(\frac{\bar{P}_{Ht}(i)}{\mathcal{P}_{Tt+s}}\right) - \frac{e^{v_{t+s}} MC_{Tt+s}}{\mathcal{P}_{Tt+s}} g'\left(\frac{\bar{P}_{Ht}(i)}{\mathcal{P}_{Tt+s}}\right)\right), \end{aligned}$$

or

$$\begin{aligned} 0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \left(\mathcal{P}^r_{Tt+s}\right)^{\theta_\tau-\epsilon} \left(\frac{P_{t+s}}{P_t}\right)^{\theta_\tau-\epsilon-1} \left\{ -\frac{\theta_\tau(1+\epsilon-\theta_\tau)}{\epsilon-\theta_\tau} \bar{P}^r_{Ht}(i) \right. \\ &\left. -\frac{\epsilon}{\theta_\tau-\epsilon}\left(\frac{P_t}{P_{t+s}}\right)^{\theta_\tau-\epsilon} \left(\mathcal{P}^r_{Tt+s}\right)^{-(\theta_\tau-\epsilon)} \bar{P}^r_{Ht}(i)^{1+\theta_\tau-\epsilon} + \theta_\tau \frac{P_{t+s}}{P_t} e^{v_{t+s}} MC^r_{t+s} \right\}, \end{aligned}$$

where $\mathcal{P}^r_{Tt+s} \equiv \mathcal{P}_{Tt+s}/P_{t+s}$, $\bar{P}^r_{Ht}(i) \equiv \bar{P}_{Ht}(i)/P_t$, and $MC^r_{Tt+s} = MC_{Tt+s}/P_{t+s}$. With index i omitted for notational simplicity, this equation can be written as

$$\frac{\theta_\tau(1+\epsilon-\theta_\tau)}{\epsilon-\theta_\tau} V_{h1,t}\bar{P}^r_{Ht} + \frac{\epsilon}{\theta_\tau-\epsilon} V_{h2,t}\left(\bar{P}^r_{Ht}\right)^{1+\theta_\tau-\epsilon} = \theta_\tau V_{h3,t}, \tag{C.37}$$

where

$$\begin{aligned} V_{h1,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \left(\mathcal{P}^r_{Tt+s}\right)^{\theta_\tau-\epsilon} \left(\frac{P_t}{P_{t+s}}\right)^{1+\epsilon-\theta_\tau}, \\ V_{h2,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \frac{P_t}{P_{t+s}}, \\ V_{h3,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \left(\mathcal{P}^r_{Tt+s}\right)^{\theta_\tau-\epsilon} \left(\frac{P_t}{P_{t+s}}\right)^{\epsilon-\theta_\tau} e^{v_{t+s}} MC^r_{Tt+s}. \end{aligned}$$

The auxiliary variables $V_{h1,t}$, $V_{h2,t}$, and $V_{h3,t}$ can be written recursively as

$$V_{h1,t} = \frac{\bar{Y}^d_{Tt}}{\bar{C}^{1/\sigma}_t} \left(\mathcal{P}^r_{Tt}\right)^{\theta_\tau-\epsilon} + \beta\xi_p E_t\left[\left(\frac{1}{\pi_{t+1}}\right)^{1+\epsilon-\theta_\tau} V_{h1,t+1}\right], \tag{C.38}$$

$$V_{h2,t} = \frac{\bar{Y}^d_{Tt}}{\bar{C}^{1/\sigma}_t} + \beta\xi_p E_t\left(\frac{1}{\pi_{t+1}} V_{h2,t+1}\right), \tag{C.39}$$

$$V_{h3,t} = \frac{\bar{Y}^d_{Tt}}{\bar{C}^{1/\sigma}_t} \left(\mathcal{P}^r_{Tt}\right)^{\theta_\tau-\epsilon} e^{v_t} MC^r_{Tt} + \beta\xi_p E_t\left[\left(\frac{1}{\pi_{t+1}}\right)^{\epsilon-\theta_\tau} V_{h3,t+1}\right]. \tag{C.40}$$

The value of the firm that chooses currency H as the invoice currency is given by

$$\begin{aligned}
\bar{\Pi}_{Ht}^{H} &\equiv \frac{\Pi_{Ht}^{H}}{P_t \bar{C}_t^{1/\sigma}} \frac{1}{1-\gamma_\tau} \\
&= \frac{\theta_\tau}{\theta_\tau - \epsilon} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \left(\mathcal{P}_{Tt+s}^r\right)^{(\theta_\tau - \epsilon)} \left(\frac{P_t}{P_{t+s}}\right)^{1+\epsilon-\theta_\tau} \left(\bar{P}_{Ht}^r\right)^{1+\epsilon-\theta_\tau} \\
&\quad - \frac{\theta_\tau}{\theta_\tau - \epsilon} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \left(\mathcal{P}_{Tt+s}^r\right)^{\theta_\tau - \epsilon} \left(\frac{P_t}{P_{t+s}}\right)^{\epsilon-\theta_\tau} e^{v_{t+s}} MC_{Tt+s}^r \left(\bar{P}_{Ht}^r\right)^{-(\theta_\tau - \epsilon)}, \\
&\quad - \frac{\epsilon}{\theta_\tau - \epsilon} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \frac{P_t}{P_{t+s}} \bar{P}_{Ht}^r + \frac{\epsilon}{\theta_\tau - \epsilon} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} e^{v_{t+s}} MC_{Tt+s}^r \\
&= \frac{\theta_\tau}{\theta_\tau - \epsilon} \left(V_{h1,t} \bar{P}_{Ht}^r - V_{h3,t}\right) \left(\bar{P}_{Ht}^r\right)^{\epsilon - \theta_\tau} - \frac{\epsilon}{\theta_\tau - \epsilon} \left(V_{h2,t} \bar{P}_{Ht}^r - V_{h4,t}\right), \qquad \text{(C.41)}
\end{aligned}$$

where

$$V_{h4,t} = E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} e^{v_{t+s}} MC_{Tt+s}^r.$$

The auxiliary variable $V_{h4,t}$ can be written recursively as

$$V_{h4,t} = \frac{\bar{Y}_{Tt}^d}{\bar{C}_t^{1/\sigma}} e^{v_t} MC_{Tt}^r + \beta\xi_p E_t V_{h4,t+1}. \qquad \text{(C.42)}$$

Tradable goods firms price setting (Goods, Market, Currency) = (H,H,F) Consider a price setting problem in which the Home firm i sets its price $P_{Ht}^F(i) \equiv P_{Ht}(i)/\mathcal{E}_t$ in the Home market using currency F:

$$\Pi_{Ht}^F = \max_{\left\{\bar{P}_{Ht}^F(i)\right\}} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\mathcal{E}_{t+s} \bar{P}_{Ht}^F(i) - e^{v_{t+s}} MC_{Tt+s}\right) Y_{H,t+s|t}(i),$$

subject to the demand curve

$$Y_{H,t+s|t}(i) = g\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ht}^F(i)}{\mathcal{P}_{Tt+s}}\right)(1-\gamma_\tau)\bar{Y}_{Tt+s}^d.$$

The first-order condition with respect to $\bar{P}_{Ht}^F(i)$ is

$$\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \bar{Y}_{Tt+s}^d \mathcal{E}_{t+s} \\
&\quad \times \left(g\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ht}^F(i)}{\mathcal{P}_{Tt+s}}\right) + \frac{\mathcal{E}_{t+s}\bar{P}_{Ht}^F(i)}{\mathcal{P}_{Tt+s}} g'\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ht}^F(i)}{\mathcal{P}_{Tt+s}}\right) - \frac{e^{v_{t+s}} MC_{Tt+s}}{\mathcal{P}_{Tt+s}} g'\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ht}^F(i)}{\mathcal{P}_{Tt+s}}\right)\right),
\end{aligned}$$

or

$$\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \mathcal{E}_{t+s}^r \frac{P_t^*}{P_{t+s}^*} \left\{ -\frac{\theta_\tau(1+\epsilon-\theta_p)}{\epsilon-\theta_\tau} \left(\frac{\mathcal{E}_{t+s}^r}{\mathcal{P}_{Tt+s}^r} \frac{P_t^*}{P_{t+s}^*}\right)^{-(\theta_\tau-\epsilon)} \bar{P}_{Ht}^{Fr}(i) \right. \\
&\quad \left. - \frac{\epsilon}{\theta_\tau - \epsilon} \left(\bar{P}_{Ht}^{Fr}(i)\right)^{1+\theta_\tau-\epsilon} + \theta_\tau \frac{e^{v_{t+s}} MC_{Tt+s}^r}{\mathcal{P}_{t+s}^r} \left(\frac{\mathcal{E}_{t+s}^r}{\mathcal{P}_{Tt+s}^r} \frac{P_t^*}{P_{t+s}^*}\right)^{-(1+\theta_\tau-\epsilon)} \right\}.
\end{aligned}$$

With index i omitted for notational simplicity, this equation can be written as

$$\frac{\theta_\tau (1+\epsilon-\theta_\tau)}{\epsilon-\theta_p} V_{h1,t}^F \bar{P}_{Ht}^{Fr} + \frac{\epsilon}{\theta_\tau-\epsilon} V_{h2,t}^F \left(\bar{P}_{Ht}^{Fr}\right)^{1+\theta_\tau-\epsilon} = \theta_\tau V_{h3,t}^F, \tag{C.43}$$

where

$$\begin{aligned}
V_{h1,t}^F &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \left(\mathcal{P}_{Tt+s}^r\right)^{\theta_\tau-\epsilon} \left(\mathcal{E}_{t+s}^r \frac{P_t^*}{P_{t+s}^*}\right)^{1+\epsilon-\theta_\tau}, \\
V_{h2,t}^F &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \mathcal{E}_{t+s}^r \frac{P_t^*}{P_{t+s}^*}, \\
V_{h3,t}^F &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \left(\mathcal{P}_{Tt+s}^r\right)^{\theta_\tau-\epsilon} \left(\mathcal{E}_{t+s}^r \frac{P_t^*}{P_{t+s}^*}\right)^{\epsilon-\theta_p} e^{v_{t+s}} MC_{Tt+s}^r.
\end{aligned}$$

The auxiliary variables $V_{h1,t}^F$, $V_{h2,t}^F$, and $V_{h3,t}^F$ can be written recursively as

$$V_{h1,t}^F = \frac{\bar{Y}_{Tt}^d}{\bar{C}_t^{1/\sigma}} \left(\mathcal{P}_{Tt}^r\right)^{\theta_\tau-\epsilon} \left(\mathcal{E}_t^r\right)^{1+\epsilon-\theta_\tau} + \beta\xi_p E_t \left[\left(\frac{1}{\pi_{t+1}^*}\right)^{1+\epsilon-\theta_\tau} V_{h1,t+1}^F\right], \tag{C.44}$$

$$V_{h2,t}^F = \frac{\bar{Y}_{Tt}^d}{\bar{C}_t^{1/\sigma}} \mathcal{E}_t^r + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}^*} V_{h2,t+1}^F\right), \tag{C.45}$$

$$V_{h3,t}^F = \frac{\bar{Y}_{Tt}^d}{\bar{C}_t^{1/\sigma}} \left(\mathcal{P}_{Tt}^r\right)^{\theta_\tau-\epsilon} \left(\mathcal{E}_t^r\right)^{\epsilon-\theta_\tau} e^{v_t} MC_{Tt}^r + \beta\xi_p E_t \left[\left(\frac{1}{\pi_{t+1}^*}\right)^{\epsilon-\theta_\tau} V_{h3,t+1}^F\right]. \tag{C.46}$$

The value of the firm that chooses currency F as the invoice currency is given by

$$\begin{aligned}
\bar{\Pi}_{Ht}^F &\equiv \frac{\Pi_{Ht}^F}{P_t \bar{C}_t^{1/\sigma}} \frac{1}{1-\gamma_\tau} \\
&= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \left(\mathcal{E}_{t+s}^r \bar{P}_{Ht}^{Fr} \frac{P_t^*}{P_{t+s}^*} - e^{v_{t+s}} MC_{Tt+s}^r\right) g\left(\frac{\mathcal{E}_{t+s}^r \bar{P}_{Ht}^{Fr}}{\mathcal{P}_{Tt+s}^r} \frac{P_t^*}{P_{t+s}^*}\right), \\
&= \frac{1}{\theta_\tau-\epsilon} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^d}{\bar{C}_{t+s}^{1/\sigma}} \left(\mathcal{E}_{t+s}^r \bar{P}_{Ht}^{Fr} \frac{P_t^*}{P_{t+s}^*} - e^{v_{t+s}} MC_{Tt+s}^r\right) \left[\theta_\tau \left(\frac{\mathcal{E}_{t+s}^r \bar{P}_{Ht}^{Fr}}{\mathcal{P}_{Tt+s}^r} \frac{P_t^*}{P_{t+s}^*}\right)^{-(\theta_\tau-\epsilon)} - \epsilon\right], \\
&= \frac{\theta_\tau}{\theta_\tau-\epsilon} \left(V_{h1,t}^F \bar{P}_{H,t}^{Fr} - V_{h3,t}^F\right) \left(\bar{P}_{Ht}^{Fr}\right)^{\epsilon-\theta_\tau} - \frac{\epsilon}{\theta_\tau-\epsilon} \left(V_{h2,t}^F \bar{P}_{Ht}^{Fr} - V_{h4,t}\right).
\end{aligned} \tag{C.47}$$

Tradable goods firms price setting (Goods, Market, Currency) = (F, H, H) Consider a price setting problem in which the Foreign tradable goods firm i sets its price in the Home market using currency H:

$$\Pi_{Ft}^H = \max_{\{\bar{P}_{Ft}(i)\}} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}^*}{\Lambda_t^*} \left(\frac{\bar{P}_{Ft}(i)}{\mathcal{E}_{t+s}} - e^{v_{t+s}^*} MC_{Tt+s}^*\right) Y_{F,t+s|t}(i),$$

subject to the demand curve

$$Y_{F,t+s|t}(i) = g\left(\frac{\bar{P}_{Ft}(i)}{\mathcal{P}_{Tt+s}}\right) \frac{\gamma_\tau n}{1-n} \bar{Y}_{Tt+s}^d.$$

The first-order condition with respect to $\bar{P}_{Ft}(i)$ is

$$\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda^*_{t+s}}{\Lambda^*_t} \frac{\bar{Y}^d_{Tt+s}}{\mathcal{E}_{t+s}} \\
&\times \left(g\left(\frac{\bar{P}_{Ft}(i)}{\mathcal{P}_{Tt+s}}\right) + \frac{\bar{P}_{Ft}(i)}{\mathcal{P}_{Tt+s}} g'\left(\frac{\bar{P}_{Ft}(i)}{\mathcal{P}_{Tt+s}}\right) - \frac{\mathcal{E}_{t+s} e^{v^*_{t+s}} MC^*_{Tt+s}}{\mathcal{P}_{Tt+s}} g'\left(\frac{\bar{P}_{Ft}(i)}{\mathcal{P}_{Tt+s}}\right)\right),
\end{aligned}$$

or

$$\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{1}{\bar{C}^{*1/\sigma}_{t+s}} \frac{\bar{Y}^d_{Tt+s}}{\mathcal{E}^r_{t+s}} \left[-\frac{\theta_\tau (1+\epsilon-\theta_\tau)}{\epsilon-\theta_p} \left(\mathcal{P}^r_{Tt+s}\right)^{\theta_\tau-\epsilon} \bar{P}^r_{Ft}(i) \left(\frac{P_{t+s}}{P_t}\right)^{\theta_\tau-\epsilon-1} \right. \\
&\left. -\frac{\epsilon}{\theta_\tau-\epsilon} \frac{P_t}{P_{t+s}} \bar{P}^r_{Ft}(i)^{1+\theta_\tau-\epsilon} + \theta_\tau \mathcal{E}^r_{t+s} e^{v^*_{t+s}} MC^{*r}_{Tt+s} \left(\mathcal{P}^r_{Tt+s}\right)^{\theta_\tau-\epsilon} \left(\frac{P_{t+s}}{P_t}\right)^{\theta_p-\epsilon} \right].
\end{aligned}$$

This equation can be written as

$$\frac{\theta_\tau (1+\epsilon-\theta_\tau)}{\epsilon-\theta_\tau} V_{f1,t} \bar{P}^r_{Ft} + \frac{\epsilon}{\theta_\tau-\epsilon} V_{f2,t} \left(\bar{P}^r_{Ft}\right)^{1+\theta_\tau-\epsilon} = \theta_\tau V_{f3,t} \tag{C.48}$$

where

$$\begin{aligned}
V_{f1,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{*1/\sigma}_{t+s}} \frac{1}{\mathcal{E}^r_{t+s}} \left(\mathcal{P}^r_{Tt+s}\right)^{\theta_\tau-\epsilon} \left(\frac{P_t}{P_{t+s}}\right)^{1+\epsilon-\theta_\tau}, \\
V_{f2,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{*1/\sigma}_{t+s}} \frac{1}{\mathcal{E}^r_{t+s}} \frac{P_t}{P_{t+s}}, \\
V_{f3,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{*1/\sigma}_{t+s}} e^{v^*_{t+s}} MC^{*r}_{Tt+s} \left(\mathcal{P}^r_{Tt+s}\right)^{\theta_\tau-\epsilon} \left(\frac{P_t}{P_{t+s}}\right)^{\epsilon-\theta_\tau}.
\end{aligned}$$

These auxiliary variables can be written recursively as

$$V_{f1,t} = \frac{\bar{Y}^d_{Tt}}{\bar{C}^{*1/\sigma}_t} \frac{1}{\mathcal{E}^r_t} \left(\mathcal{P}^r_{Tt}\right)^{\theta_\tau-\epsilon} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{1+\epsilon-\theta_\tau} V_{f1,t+1}, \tag{C.49}$$

$$V_{f2,t} = \frac{\bar{Y}^d_{Tt}}{\bar{C}^{*1/\sigma}_t} \frac{1}{\mathcal{E}^r_t} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right) V_{f2,t+1}, \tag{C.50}$$

$$V_{f3,t} = \frac{\bar{Y}^d_{Tt}}{\bar{C}^{*1/\sigma}_t} e^{v^*_t} MC^{*r}_{Tt} \left(\mathcal{P}^r_{Tt}\right)^{\theta_\tau-\epsilon} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\epsilon-\theta_\tau} V_{f3,t+1}. \tag{C.51}$$

The value of the firm that chooses currency H as the invoice currency is given by

$$\begin{aligned}
\bar{\Pi}^H_{F,t} &\equiv \frac{\Pi^H_{F,t}}{P^*_t \bar{C}^{*1/\sigma}_t} \frac{(1-n)}{n\gamma_\tau} \\
&= \frac{\theta_\tau}{\theta_\tau-\epsilon} \left(V_{f1,t} \bar{P}^r_{Ft} - V_{f3,t}\right) \left(\bar{P}^r_{Ft}\right)^{\epsilon-\theta_\tau} - \frac{\epsilon}{\theta_\tau-\epsilon} \left(V_{f2,t} \bar{P}^r_{Ft} - V_{f4,t}\right),
\end{aligned} \tag{C.52}$$

where

$$V_{f4,t} = E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{*1/\sigma}_{t+s}} e^{v^*_{t+s}} MC^{*r}_{Tt+s} = \frac{\bar{Y}^d_{Tt}}{\bar{C}^{*1/\sigma}_t} e^{v^*_t} MC^{*r}_{Tt} + \beta\xi_p E_t V_{f4,t+1}. \tag{C.53}$$

Tradable goods firms price setting (Goods, Market, Currency) = (F, H, F) Consider a price setting problem in which the Foreign tradable goods firm i sets its price in the Home market using currency F:

$$\Pi_{Ft}^{F}=\max_{\left\{\bar{P}_{Ft}^{F}(i)\right\}} E_t\sum_{s=0}^{\infty}(\beta\xi_p)^s\frac{\Lambda_{t+s}^{*}}{\Lambda_t^{*}}\left(\bar{P}_{Ft}^{F}(i)-e^{v_{t+s}^{*}}MC_{Tt+s}^{*}\right)g\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ft}^{F}(i)}{\mathcal{P}_{Tt+s}}\right)\frac{\gamma_\tau n}{1-n}\bar{Y}_{Tt+s}^{d}.$$

The first-order condition is

$$\begin{aligned}0 &= E_t\sum_{s=0}^{\infty}(\beta\xi_p)^s\frac{\Lambda_{t+s}^{*}}{\Lambda_t^{*}}\bar{Y}_{Tt+s}^{d}\left(g\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ft}^{F}(i)}{\mathcal{P}_{Tt+s}}\right)\right.\\&\left.+\frac{\mathcal{E}_{t+s}\bar{P}_{Ft}^{F}(i)}{\mathcal{P}_{Tt+s}}g'\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ft}^{F}(i)}{\mathcal{P}_{Tt+s}}\right)-\frac{\mathcal{E}_{t+s}e^{v_{t+s}^{*}}MC_{Tt+s}^{*}}{\mathcal{P}_{Tt+s}}g'\left(\frac{\mathcal{E}_{t+s}\bar{P}_{Ft}^{F}(i)}{\mathcal{P}_{Tt+s}}\right)\right),\end{aligned}$$

or

$$\begin{aligned}0 &= E_t\sum_{s=0}^{\infty}(\beta\xi_p)^s\frac{\bar{Y}_{Tt+s}^{d}}{\bar{C}_{t+s}^{*1/\sigma}}\left(-\frac{\theta_\tau(1-\theta_\tau+\epsilon)}{\epsilon-\theta_\tau}\left(\frac{\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{r}}\right)^{\epsilon-\theta_\tau}\left(\frac{P_t^{*}}{P_{t+s}^{*}}\right)^{1+\epsilon-\theta_\tau}\bar{P}_{Ft}^{Fr}(i)\right.\\&\left.-\frac{\epsilon}{\theta_\tau-\epsilon}\left(\frac{P_t^{*}}{P_{t+s}^{*}}\right)\bar{P}_{Ft}^{Fr}(i)^{1+\theta_\tau-\epsilon}+\theta_\tau e^{v_{t+s}^{*}}MC_{Tt+s}^{*r}\left(\frac{\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{r}}\right)^{\epsilon-\theta_\tau}\left(\frac{P_t^{*}}{P_{t+s}^{*}}\right)^{\epsilon-\theta_\tau}\right).\end{aligned}$$

This equation can be written as

$$\frac{\theta_\tau(1-\theta_\tau+\epsilon)}{\epsilon-\theta_p}V_{f1,t}^{F}\bar{P}_{Ft}^{Fr}+\frac{\epsilon}{\theta_\tau-\epsilon}V_{f2,t}^{F}\left(\bar{P}_{Ft}^{Fr}\right)^{1+\theta_\tau-\epsilon}=\theta_\tau V_{f3,t}^{F} \tag{C.54}$$

where

$$\begin{aligned}V_{f1,t}^{F} &= E_t\sum_{s=0}^{\infty}(\beta\xi_p)^s\frac{\bar{Y}_{Tt+s}^{d}}{\bar{C}_{t+s}^{*1/\sigma}}\left(\frac{\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{r}}\right)^{\epsilon-\theta_\tau}\left(\frac{P_t^{*}}{P_{t+s}^{*}}\right)^{1+\epsilon-\theta_\tau},\\V_{f2,t}^{F} &= E_t\sum_{s=0}^{\infty}(\beta\xi_p)^s\frac{\bar{Y}_{Tt+s}^{d}}{\bar{C}_{t+s}^{*1/\sigma}}\left(\frac{P_t^{*}}{P_{t+s}^{*}}\right),\\V_{f3,t}^{F} &= E_t\sum_{s=0}^{\infty}(\beta\xi_p)^s\frac{\bar{Y}_{Tt+s}^{d}}{\bar{C}_{t+s}^{*1/\sigma}}e^{v_{t+s}^{*}}MC_{Tt+s}^{*r}\left(\frac{\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{r}}\right)^{\epsilon-\theta_\tau}\left(\frac{P_t^{*}}{P_{t+s}^{*}}\right)^{\epsilon-\theta_\tau}.\end{aligned}$$

These auxiliary variables can be written recursively as

$$V_{f1,t}^{F} = \frac{\bar{Y}_{Tt}^{d}}{\bar{C}_t^{*1/\sigma}}\left(\frac{\mathcal{E}_t^{r}}{\mathcal{P}_{Tt}^{r}}\right)^{\epsilon-\theta_\tau}+\beta\xi_pE_t\left(\frac{1}{\pi_{t+1}^{*}}\right)^{1+\epsilon-\theta_\tau}V_{f1,t+1}^{F}, \tag{C.55}$$

$$V_{f2,t}^{F} = \frac{\bar{Y}_{Tt}^{d}}{\bar{C}_t^{*1/\sigma}}+\beta\xi_pE_t\left(\frac{1}{\pi_{t+1}^{*}}\right)V_{f2,t+1}^{F}, \tag{C.56}$$

$$V_{f3,t}^{F} = \frac{\bar{Y}_{Tt}^{d}}{\bar{C}_t^{*1/\sigma}}e^{v_t^{*}}MC_{Tt}^{*r}\left(\frac{\mathcal{E}_t^{r}}{\mathcal{P}_{Tt}^{r}}\right)^{\epsilon-\theta_\tau}+\beta\xi_pE_t\left(\frac{1}{\pi_{t+1}^{*}}\right)^{\epsilon-\theta_\tau}V_{f3,t+1}^{F}. \tag{C.57}$$

The value of the firm that chooses currency F as the invoice currency is given by

$$
\begin{aligned}
\bar{\Pi}^F_{Ft} &\equiv \frac{\Pi^F_{Ft}}{P^*_t \bar{C}^{*1/\sigma}_t} \frac{(1-n)}{n\gamma_\tau} = \frac{1}{\theta_\tau - \epsilon} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^d_{Tt+s}}{\bar{C}^{*1/\sigma}_{t+s}} \left(\frac{P^*_t}{P^*_{t+s}} \bar{P}^{Fr}_{Ft} - e^{v^*_{t+s}} MC^{*r}_{Tt+s} \right) \\
&\quad \times \left(\theta_\tau \left(\frac{\mathcal{E}^r_{t+s} \bar{P}^{Fr}_{Ft}}{\mathcal{P}^r_{Tt+s}} \frac{P^*_t}{P^*_{t+s}} \right)^{\epsilon-\theta_\tau} - \epsilon \right) \\
&= \frac{\theta_\tau}{\theta_\tau - \epsilon} \left(V^F_{f1,t} \bar{P}^{Fr}_{Ft} - V^F_{f3,t} \right) \left(\bar{P}^{Fr}_{Ft} \right)^{\epsilon-\theta_\tau} - \frac{\epsilon}{\theta_\tau - \epsilon} \left(V^F_{f2,t} \bar{P}^{Fr}_{Ft} - V_{f4,t} \right). \qquad \text{(C.58)}
\end{aligned}
$$

Tradable goods firms price setting (Goods, Market, Currency) = (H, F, F) Consider a price setting problem in which the Home tradable goods firm i sets its price in the Foreign market using currency F:

$$
\Pi^{*F}_{Ht} = \max_{\{\bar{P}^*_{Ht}(i)\}} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\mathcal{E}_{t+s} \bar{P}^*_{Ht}(i) - e^{v_{t+s}} MC_{Tt+s} \right) Y^*_{H,t+s|t}(i),
$$

subject to the demand curve

$$
Y^*_{H,t+s|t}(i) = g\left(\frac{\bar{P}^*_{Ht}(i)}{\mathcal{P}^*_{Tt+s}} \right) \frac{\gamma^*_\tau (1-n)}{n} \bar{Y}^{d*}_{Tt+s},
$$

where $\bar{Y}^{d*}_{Tt+s} \equiv \bar{C}^*_{Tt+s} + \bar{X}^*_{t+s} + \bar{G}^*_{Tt+s}$. The first-order condition is

$$
\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^{d*}_{Tt+s}}{\bar{C}^\sigma_{t+s}} \left\{ \frac{P^*_t}{P^*_{t+s}} \mathcal{E}^r_{t+s} g\left(\frac{\bar{P}^{*r}_{Ht}(i)}{\mathcal{P}^{*r}_{Tt+s}} \frac{P^*_t}{P^*_{t+s}} \right) \right. \\
&\quad \left. + \frac{P^*_t}{P^*_{t+s}} \frac{\mathcal{E}^r_{t+s} \bar{P}^{*r}_{Ht}(i)}{\mathcal{P}^{*r}_{Tt+s}} g'\left(\frac{\bar{P}^{*r}_{Ht}(i)}{\mathcal{P}^{*r}_{Tt+s}} \frac{P^*_t}{P^*_{t+s}} \right) - \frac{e^{v_{t+s}} MC^r_{Tt+s}}{\mathcal{P}^{*r}_{Tt}} g'\left(\frac{\bar{P}^{*r}_{Ht}(i)}{\mathcal{P}^{*r}_{Tt+s}} \frac{P^*_t}{P^*_{t+s}} \right) \right\}
\end{aligned}
$$

or

$$
\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^{d*}_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \left\{ -\frac{\theta_\tau (1-\theta_\tau+\epsilon)}{\epsilon-\theta_\tau} \left(\frac{P^*_t}{P^*_{t+s}} \right)^{1+\epsilon-\theta_\tau} \frac{\mathcal{E}^r_{t+s}}{\left(\mathcal{P}^{*r}_{Tt+s}\right)^{\epsilon-\theta_\tau}} \bar{P}^{*r}_{Ht}(i) \right. \\
&\quad \left. - \frac{\epsilon}{\theta_\tau - \epsilon} \frac{P^*_t}{P^*_{t+s}} \mathcal{E}^r_{t+s} \bar{P}^{*r}_{Ht}(i)^{\theta_\tau+1-\epsilon} + \theta_\tau \frac{e^{v_{t+s}} MC^r_{Tt+s}}{\left(\mathcal{P}^{*r}_{Tt+s}\right)^{\epsilon-\theta_\tau}} \left(\frac{P^*_t}{P^*_{t+s}} \right)^{\epsilon-\theta_\tau} \right\}.
\end{aligned}
$$

This equation can be written as

$$
\frac{\theta_\tau (1-\theta_\tau+\epsilon)}{\epsilon-\theta_p} V^*_{h1,t} \bar{P}^{*r}_{Ht} + \frac{\epsilon}{\theta_\tau-\epsilon} V^*_{h2,t} \left(\bar{P}^{*r}_{Ht} \right)^{\theta_\tau+1-\epsilon} = \theta_\tau V^*_{h3,t}, \qquad \text{(C.59)}
$$

where

$$
\begin{aligned}
V^*_{h1,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^{d*}_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \frac{\mathcal{E}^r_{t+s}}{\left(\mathcal{P}^{*r}_{Tt+s}\right)^{\epsilon-\theta_\tau}} \left(\frac{P^*_t}{P^*_{t+s}} \right)^{1+\epsilon-\theta_\tau}, \\
V^*_{h2,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^{d*}_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \mathcal{E}^r_{t+s} \frac{P^*_t}{P^*_{t+s}}, \\
V^*_{h3,t} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}^{d*}_{Tt+s}}{\bar{C}^{1/\sigma}_{t+s}} \frac{e^{v_{t+s}} MC^r_{Tt+s}}{\left(\mathcal{P}^{*r}_{Tt+s}\right)^{\epsilon-\theta_\tau}} \left(\frac{P^*_t}{P^*_{t+s}} \right)^{\epsilon-\theta_\tau}.
\end{aligned}
$$

These auxiliary variables can be written recursively as

$$V_{h1,t}^{*} = \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{1/\sigma}}\frac{\mathcal{E}_{t}^{r}}{\left(\mathcal{P}_{Tt}^{*r}\right)^{\epsilon-\theta_{\tau}}} + \beta\xi_{p}E_{t}\left(\frac{1}{\pi_{t+1}^{*}}\right)^{1+\epsilon-\theta_{\tau}} V_{h1,t+1}^{*}, \tag{C.60}$$

$$V_{h2,t}^{*} = \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{1/\sigma}}\mathcal{E}_{t}^{r} + \beta\xi_{p}E_{t}\left(\frac{1}{\pi_{t+1}^{*}}\right)V_{h2,t+1}^{*}, \tag{C.61}$$

$$V_{h3,t}^{*} = \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{1/\sigma}}\frac{e^{v_{t}}MC_{Tt}^{r}}{\left(\mathcal{P}_{Tt}^{*r}\right)^{\epsilon-\theta_{\tau}}} + \beta\xi_{p}E_{t}\left(\frac{1}{\pi_{t+1}^{*}}\right)^{\epsilon-\theta_{\tau}} V_{h3,t+1}^{*}. \tag{C.62}$$

The value of the Home tradable goods firm i that chooses currency F in country F is given by

$$\begin{aligned}
\bar{\Pi}_{Ht}^{*F} &\equiv \frac{\Pi_{Ht}^{*F}}{P_{t}\bar{C}_{t}^{1/\sigma}}\frac{n}{(1-n)\gamma_{\tau}^{*}} \\
&= \frac{1}{\theta_{\tau}-\epsilon}E_{t}\sum_{s=0}^{\infty}(\beta\xi_{p})^{s}\frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+s}^{1/\sigma}}\left(\mathcal{E}_{t+s}^{r}\bar{P}_{Ht}^{*r}\frac{P_{t}^{*}}{P_{t+s}^{*}} - e^{v_{t+s}}MC_{Tt+s}^{r}\right)\left(\theta_{\tau}\left(\frac{\bar{P}_{Ht}^{*r}}{\mathcal{P}_{Tt+s}^{*r}}\frac{P_{t}^{*}}{P_{t+s}^{*}}\right)^{\epsilon-\theta_{\tau}} - \epsilon\right) \\
&= \frac{\theta_{\tau}}{\theta_{\tau}-\epsilon}\left(V_{h1,t}^{*}\bar{P}_{Ht}^{*r} - V_{h3,t}^{*}\right)\left(\bar{P}_{Ht}^{*r}\right)^{\epsilon-\theta_{\tau}} - \frac{\epsilon}{\theta_{\tau}-\epsilon}\left(V_{h2,t}^{*}\bar{P}_{Ht}^{*r} - V_{h4,t}^{*}\right),
\end{aligned} \tag{C.63}$$

where

$$V_{h4,t}^{*} = E_{t}\sum_{s=0}^{\infty}(\beta\xi_{p})^{s}\frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+1}^{1/\sigma}}e^{v_{t+s}}MC_{Tt+s}^{r} = \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{1/\sigma}}e^{v_{t}}MC_{Tt}^{r} + \beta\xi_{p}E_{t}V_{h4,t+1}^{*}. \tag{C.64}$$

Tradable goods firms price setting (Goods, Market, Currency) = (H, F, H) Consider a price setting problem in which the Home tradable goods firm i sets its price in country F using currency H:

$$\Pi_{Ht}^{*H} = \max_{\left\{\bar{P}_{Ht}^{*H}(i)\right\}} E_{t}\sum_{s=0}^{\infty}(\beta\xi_{p})^{s}\frac{\Lambda_{t+s}}{\Lambda_{t}}\left(\bar{P}_{Ht}^{*H}(i) - e^{v_{t+s}}MC_{Tt+s}\right)g\left(\frac{\bar{P}_{Ht}^{*H}(i)/\mathcal{E}_{t+s}}{\mathcal{P}_{Tt+s}^{*}}\right)\frac{\gamma_{\tau}^{*}(1-n)}{n}\bar{Y}_{Tt+s}^{d*}.$$

The first-order condition is

$$\begin{aligned}
0 = {} & E_{t}\sum_{s=0}^{\infty}(\beta\xi_{p})^{s}\frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+s}^{1/\sigma}}\frac{P_{t}}{P_{t+s}}\left\{g\left(\frac{\bar{P}_{Ht}^{*Hr}(i)/\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{*r}}\frac{P_{t}}{P_{t+s}}\right)\right. \\
& \left.+\frac{\bar{P}_{Ht}^{*Hr}(i)/\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{*r}}\frac{P_{t}}{P_{t+s}}g'\left(\frac{\bar{P}_{Ht}^{*Hr}(i)/\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{*r}}\frac{P_{t}}{P_{t+s}}\right) - \frac{e^{v_{t+s}}MC_{Tt+s}^{r}/\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{*r}}g'\left(\frac{\bar{P}_{Ht}^{*Hr}(i)/\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{*r}}\frac{P_{t}}{P_{t+s}}\right)\right\}
\end{aligned}$$

or

$$\begin{aligned}
0 = {} & E_{t}\sum_{s=0}^{\infty}(\beta\xi_{p})^{s}\frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+s}^{1/\sigma}}\frac{P_{t}}{P_{t+s}}\left\{-\frac{\theta_{\tau}(1+\epsilon-\theta_{\tau})}{\epsilon-\theta_{p}}\left(\frac{1}{\mathcal{P}_{Tt+s}^{*r}\mathcal{E}_{t+s}^{r}}\frac{P_{t}}{P_{t+s}}\right)^{\epsilon-\theta_{\tau}}\bar{P}_{Ht}^{*Hr}(i)\right. \\
& \left.-\frac{\epsilon}{\theta_{\tau}-\epsilon}\bar{P}_{Ht}^{*Hr}(i)^{1+\theta_{\tau}-\epsilon} + \theta_{\tau}\frac{e^{v_{t+s}}MC_{Tt+s}^{r}/\left(\mathcal{E}_{t+s}^{r}\right)^{\epsilon-\theta_{\tau}}}{\left(\mathcal{P}_{Tt+s}^{*r}\right)^{\epsilon-\theta_{\tau}}}\left(\frac{P_{t}}{P_{t+s}}\right)^{\epsilon-\theta_{\tau}-1}\right\}.
\end{aligned}$$

This equation can be written as

$$\frac{\theta_{\tau}(1+\epsilon-\theta_{\tau})}{\epsilon-\theta_{p}}V_{h1,t}^{*H}\bar{P}_{Ht}^{*Hr} + \frac{\epsilon}{\theta_{\tau}-\epsilon}V_{h2,t}^{*H}\left(\bar{P}_{Ht}^{*Hr}\right)^{1+\theta_{\tau}-\epsilon} = \theta_{\tau}V_{h3,t}^{*H} \tag{C.65}$$

where

$$
\begin{aligned}
V_{h1,t}^{*H} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+s}^{1/\sigma}} \frac{1}{\left(\mathcal{P}_{Tt+s}^{*r} \mathcal{E}_{t+s}^{r}\right)^{\epsilon-\theta_\tau}} \left(\frac{P_t}{P_{t+s}}\right)^{1+\epsilon-\theta_\tau}, \\
V_{h2,t}^{*H} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+s}^{1/\sigma}} \frac{P_t}{P_{t+s}}, \\
V_{h3,t}^{*H} &= E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+s}^{1/\sigma}} \frac{e^{v_{t+s}} MC_{Tt+s}^{r}}{\left(\mathcal{P}_{Tt+s}^{*r} \mathcal{E}_{t+s}^{r}\right)^{\epsilon-\theta_\tau}} \left(\frac{P_t}{P_{t+s}}\right)^{\epsilon-\theta_\tau}.
\end{aligned}
$$

These auxiliary variables can be written recursively as

$$
\begin{aligned}
V_{h1,t}^{*H} &= \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{1/\sigma}} \frac{1}{\left(\mathcal{P}_{Tt}^{*r} \mathcal{E}_{t}^{r}\right)^{\epsilon-\theta_\tau}} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{1+\epsilon-\theta_\tau} V_{h1,t+1}^{*H}, && \text{(C.66)} \\
V_{h2,t}^{*H} &= \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{1/\sigma}} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right) V_{h2,t+1}^{*H}, && \text{(C.67)} \\
V_{h3,t}^{*H} &= \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{1/\sigma}} \frac{e^{v_t} MC_{Tt}^{r}}{\left(\mathcal{P}_{Tt}^{*r} \mathcal{E}_{t}^{r}\right)^{\epsilon-\theta_\tau}} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}}\right)^{\epsilon-\theta_\tau} V_{h3,t+1}^{*H}. && \text{(C.68)}
\end{aligned}
$$

The value of the Home tradable goods firm i that sets the price in country F using currency H is given by

$$
\begin{aligned}
\bar{\Pi}_{Ht}^{*H} &\equiv \frac{\Pi_{Ht}^{*H}}{P_t C_t^{1/\sigma}} \frac{n}{(1-n)\gamma_\tau^*} \\
&= \frac{1}{\theta_\tau - \epsilon} E_t \sum_{s=0}^{\infty} (\beta\xi_p)^s \frac{\bar{Y}_{Tt+s}^{d*}}{\bar{C}_{t+1}^{1/\sigma}} \left(\bar{P}_{Ht}^{*Hr} \frac{P_t}{P_{t+1}} - e^{v_{t+s}} MC_{Tt+s}^{r}\right) \left(\theta_\tau \left(\frac{\bar{P}_{Ht}^{*Hr}/\mathcal{E}_{t+s}^{r}}{\mathcal{P}_{Tt+s}^{*r}} \frac{P_t}{P_{t+s}}\right)^{-(\theta_\tau-\epsilon)} - \epsilon\right) \\
&= \frac{\theta_\tau}{\theta_\tau - \epsilon} \left(V_{h1,t}^{*H} \bar{P}_{Ht}^{*Hr} - V_{h3,t}^{*H}\right) \left(\bar{P}_{Ht}^{*Hr}\right)^{\epsilon-\theta_\tau} - \frac{\epsilon}{\theta_\tau - \epsilon} \left(V_{h2,t}^{*H} \bar{P}_{Ht}^{*Hr} - V_{h4,t}^{*}\right). && \text{(C.69)}
\end{aligned}
$$

Tradable goods firms price setting (Goods, Market, Currency) = (F, F, F) Consider a price setting problem in which the Foreign tradable goods firm i sets its price in country F using currency F. This problem is symmetric to that for the Home firm i, so that the solution is characterized by similar conditions, given by

$$
\frac{\theta_\tau (1+\epsilon-\theta_\tau)}{\epsilon - \theta_p} V_{f1,t}^{*} \bar{P}_{Ft}^{*r} + \frac{\epsilon}{\theta_\tau - \epsilon} V_{f2,t}^{*} \left(\bar{P}_{Ft}^{*r}\right)^{1+\theta_\tau-\epsilon} = \theta_\tau V_{f3,t}^{*}, \tag{C.70}
$$

where

$$
\begin{aligned}
V_{f1,t}^{*} &= \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{*1/\sigma}} \left(\mathcal{P}_{Tt}^{*r}\right)^{\theta_\tau-\epsilon} + \beta\xi_p E_t \left[\left(\frac{1}{\pi_{t+1}^{*}}\right)^{1+\epsilon-\theta_\tau} V_{f1,t+1}^{*}\right], && \text{(C.71)} \\
V_{f2,t}^{*} &= \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{*1/\sigma}} + \beta\xi_p E_t \left(\frac{1}{\pi_{t+1}^{*}} V_{f2,t+1}^{*}\right), && \text{(C.72)} \\
V_{f3,t}^{*} &= \frac{\bar{Y}_{Tt}^{d*}}{\bar{C}_{t}^{*1/\sigma}} \left(\mathcal{P}_{Tt}^{*r}\right)^{\theta_\tau-\epsilon} e^{v_t^*} MC_{Tt}^{*r} + \beta\xi_p E_t \left[\left(\frac{1}{\pi_{t+1}^{*}}\right)^{\epsilon-\theta_\tau} V_{f3,t+1}^{*}\right]. && \text{(C.73)}
\end{aligned}
$$

Since we assume that the Foreign firms set their prices using their own currency in country F, i.e. $\lambda_F^* = 0$, neither the firm's problem with H-currency pricing nor the invoice-currency choice problem is not considered.

Price indices of non-tradable goods The price index of the non-tradable goods is given by (A.12) and it can be decomposed as

$$\left(P_{Nt}^{r}\right)^{1-\theta_n}=\lambda_N\left(P_{Nt}^{Hr}\right)^{1-\theta_n}+\left(1-\lambda_N\right)\left(\mathcal{E}_t^r P_{Nt}^{Fr}\right)^{1-\theta_n}, \tag{C.74}$$

where P_{Nt}^{Hr} and P_{Nt}^{Fr} are price indices of the non-tradable goods in real terms in the cases of H-currency pricing and F-currency pricing, respectively. These price indices can be written recursively as

$$\left(P_{Nt}^{Hr}\right)^{1-\theta_n}=\left(1-\xi_p\right)\left(\bar{P}_{Nt}^{r}\right)^{1-\theta_n}+\xi_p\left(\frac{P_{Nt-1}^{Hr}}{\pi_t}\right)^{1-\theta_n}, \tag{C.75}$$

$$\left(P_{Nt}^{Fr}\right)^{1-\theta_n}=\left(1-\xi_p\right)\left(\bar{P}_{Nt}^{Fr}\right)^{1-\theta_n}+\xi_p\left(\frac{P_{Nt-1}^{Fr}}{\pi_t^*}\right)^{1-\theta_n}, \tag{C.76}$$

For country F, because an invoice currency is assumed to be currency F only, the price index is given by

$$\left(P_{Nt}^{*r}\right)^{1-\theta_n}=\left(1-\xi_p\right)\left(\bar{P}_{Nt}^{*r}\right)^{1-\theta_n}+\xi_p\left(\frac{P_{Nt-1}^{*r}}{\pi_t^*}\right)^{1-\theta_n}. \tag{C.77}$$

Price indices of tradable goods The price indices for the tradable goods, P_{Tt} and $\mathcal{P}_{Tt}$, are given by equations (A.16) and (A.17), which are reproduced here for convenience:

$$\begin{aligned}
1&=\frac{1-\gamma_\tau}{n}\int_0^n G\left(g\left(\frac{P_{Ht}^r(i)}{\mathcal{P}_{Tt}^r}\right)\right)di+\frac{\gamma_\tau}{1-n}\int_0^{1-n} G\left(g\left(\frac{P_{Ft}^r(i)}{\mathcal{P}_{Tt}^r}\right)\right)di,\\
P_{Tt}^r&=\frac{1-\gamma_\tau}{n}\int_0^n P_{Ht}^r(i)\,g\left(\frac{P_{Ht}^r(i)}{\mathcal{P}_{Tt}^r}\right)di+\frac{\gamma_\tau}{1-n}\int_0^{1-n} P_{Ft}^r(i)\,g\left(\frac{P_{Ft}^r(i)}{\mathcal{P}_{Tt}^r}\right)di,
\end{aligned}$$

where $P_{Ht}^r(i)=P_{Ht}(i)/P_t$, $P_{Ft}^r(i)=P_{Ft}(i)/P_t$, $\mathcal{P}_{Tt}^r=\mathcal{P}_{Tt}/P_t$, and $P_{Tt}^r=P_{Tt}/P_t$. These two equations can be expanded as

$$\begin{aligned}
\left(\mathcal{P}_{Tt}^r\right)^{\epsilon+1-\theta_\tau}&=\frac{1-\gamma_\tau}{n}\int_0^n P_{Ht}^r(i)^{\epsilon+1-\theta_\tau}di+\frac{\gamma_\tau}{1-n}\int_0^{1-n} P_{Ft}^r(i)^{\epsilon+1-\theta_\tau}di,\\
P_{Tt}^r&=\frac{\theta_\tau}{\theta_\tau-\epsilon}\mathcal{P}_{Tt}^r-\frac{\epsilon}{\theta_\tau-\epsilon}\left[\frac{1-\gamma_\tau}{n}\int_0^n P_{Ht}^r(i)\,di+\frac{\gamma_\tau}{1-n}\int_0^{1-n} P_{Ft}^r(i)\,di\right].
\end{aligned}$$

First consider the initial equation of the above two equations. It can be decomposed as

$$\left(\mathcal{P}_{Tt}^r\right)^{\epsilon+1-\theta_\tau}=\left(P_{Tt}^{Hr}\right)^{\epsilon+1-\theta_\tau}+\left(\mathcal{E}_t^r P_{Tt}^{Fr}\right)^{\epsilon+1-\theta_\tau}, \tag{C.78}$$

where P_{Tt}^{Hr} and P_{Tt}^{Fr} denote the price indices of the tradable goods in real terms in the cases of H-currency pricing and F-currency pricing, respectively. These indices can be written recursively as

$$\left(P_{Tt}^{Hr}\right)^{\epsilon+1-\theta_\tau} = (1-\xi_p)\left[(1-\gamma_\tau)\lambda_H\left(\bar{P}_{Ht}^{r}\right)^{\epsilon+1-\theta_\tau} + \gamma_\tau\lambda_F\left(\bar{P}_{Ft}^{r}\right)^{\epsilon+1-\theta_\tau}\right] + \xi_p\left(\frac{P_{Tt-1}^{Hr}}{\pi_t}\right)^{\epsilon+1-\theta_\tau}, \tag{C.79}$$

$$\left(P_{Tt}^{Fr}\right)^{\epsilon+1-\theta_\tau} = (1-\xi_p)\left[(1-\gamma_\tau)(1-\lambda_H)\left(\bar{P}_{Ht}^{Fr}\right)^{\epsilon+1-\theta_\tau} + \gamma_\tau(1-\lambda_F)\left(\bar{P}_{Ft}^{Fr}\right)^{\epsilon+1-\theta_\tau}\right] + \xi_p\left(\frac{P_{Tt-1}^{Fr}}{\pi_t^*}\right)^{\epsilon+1-\theta_\tau}, \tag{C.80}$$

Next consider the second equationso the above two equations, which can be written as

$$P_{Tt}^{r} = \frac{\theta_\tau}{\theta_\tau-\epsilon}\mathcal{P}_{Tt}^{r} - \frac{\epsilon}{\theta_\tau-\epsilon}\left(\Delta_{2t}^{H} + \mathcal{E}_t^{r}\Delta_{2t}^{F}\right), \tag{C.81}$$

where

$$\Delta_{2t}^{H} = (1-\xi_p)\left[(1-\gamma_\tau)\lambda_H\bar{P}_{Ht}^{r} + \gamma_\tau\lambda_F\bar{P}_{Ft}^{r}\right] + \xi_p\left(\frac{\Delta_{2t-1}^{H}}{\pi_t}\right), \tag{C.82}$$

$$\Delta_{2t}^{F} = (1-\xi_p)\left[(1-\gamma_\tau)(1-\lambda_H)\bar{P}_{Ht}^{Fr} + \gamma_\tau(1-\lambda_F)\bar{P}_{Ft}^{Fr}\right] + \xi_p\left(\frac{\Delta_{2t-1}^{F}}{\pi_t^*}\right), \tag{C.83}$$

The corresponding price indices for country F are symmetric to those of country H except that the F-currency is used exclusively in country F:

$$\left(\mathcal{P}_{Tt}^{*r}\right)^{\epsilon+1-\theta_\tau} = (1-\xi_p)(1-\gamma_\tau^*)\left(\bar{P}_{Ft}^{*r}\right)^{\epsilon+1-\theta_\tau} + (1-\xi_p)\gamma_\tau^*\left(\bar{P}_{Ht}^{*r}\right)^{\epsilon+1-\theta_\tau} + \xi_p\left(\frac{\mathcal{P}_{Tt-1}^{*r}}{\pi_t^*}\right)^{\epsilon+1-\theta_\tau}, \tag{C.84}$$

$$P_{Tt}^{*r} = \frac{\theta_\tau}{\theta_\tau-\epsilon}\mathcal{P}_{Tt}^{*r} - \frac{\epsilon}{\theta_\tau-\epsilon}\Delta_{2t}^{*}, \tag{C.85}$$

where

$$\Delta_{2t}^{*} = (1-\xi_p)(1-\gamma_\tau^*)\bar{P}_{Ft}^{*r} + (1-\xi_p)\gamma_\tau^*\bar{P}_{Ht}^{*r} + \xi_p\frac{\Delta_{2t-1}}{\pi_t^*}. \tag{C.86}$$

Price indices of the total consumption bundle The price index of the total consumption bundle in country H is given by equation (A.16). In real terms, it is written as

$$1 = \left(P_{Nt}^{r}\right)^{\gamma_n}\left(P_{Tt}^{r}\right)^{1-\gamma_n}. \tag{C.87}$$

For country F, an equation for the price index is similary given by

$$1 = \left(P_{Nt}^{*r}\right)^{\gamma_n}\left(P_{Tt}^{*r}\right)^{1-\gamma_n}. \tag{C.88}$$

Market clearing for non-tradable goods The supply of the non-tradable goods bundle in per capita terms is given by

$$\bar{Y}_{Nt} = A_t\left(\frac{\bar{L}_{Nt}}{1-\phi_n}\right)^{1-\phi_n}\left(\frac{\bar{X}_{Nt}}{\phi_n}\right)^{\phi_n}. \tag{C.89}$$

Market clearing requires that the supply is equal to demand, which is given by equation (A.57). Then, the market clearing condition can be written in per capita terms as

$$\bar{Y}_{Nt} = \Delta_{Nt}^{-\theta_n} \left(P_{Nt}^r\right)^{\theta_n} \bar{Y}_{Nt}^d, \tag{C.90}$$

where

$$\bar{Y}_{Nt}^d = \left(P_{Nt}^r\right)^{-1} \gamma_n \left(\bar{C}_t + \bar{G}_t\right),$$

Here, $\bar{G}_t$ is the exogenous government spending in terms of the consumption bundle. In the main text, the govenment spending is omited so that $\bar{G}_t = 0$. Also, the price dispersion Δ_{Nt} in equation (C.90) is given by $\Delta_{Nt} = \left[(1/n)\int_0^n P_{Nt}^r(i)^{-\theta_n} di\right]^{-\frac{1}{\theta_n}}$, which can be decomposed as

$$\Delta_{Nt}^{-\theta_n} = \lambda_N \left(\Delta_{Nt}^H\right)^{-\theta_n} + (1-\lambda_N)\left(\mathcal{E}_t^r \Delta_{Nt}^F\right)^{-\theta_n}, \tag{C.91}$$

where Δ_{Nt}^H and Δ_{Nt}^F are given by

$$\left(\Delta_{Nt}^H\right)^{-\theta_n} =(1-\xi_p)\left(\bar{P}_{Nt}^r\right)^{-\theta_n} + \xi_p \left(\frac{\Delta_{Nt-1}^H}{\pi_t}\right)^{-\theta n}, \tag{C.92}$$

$$\left(\Delta_{Nt}^F\right)^{-\theta_n} =(1-\xi_p)\left(\bar{P}_{Nt}^{Fr}\right)^{-\theta_n} + \xi_p \left(\frac{\Delta_{Nt-1}^{Fr}}{\pi_t^*}\right)^{-\theta_n}, \tag{C.93}$$

In country F, it is assumed that currency F is exclusively used. Equations pertaining to market clearing for the non-tradable goods in country F are then given by

$$\bar{Y}_{Nt}^* = A_t^* \left(\frac{\bar{L}_{Nt}^*}{1-\phi_n}\right)^{1-\phi_n} \left(\frac{\bar{X}_{Nt}^*}{\phi_n}\right)^{\phi_n}, \tag{C.94}$$

$$\bar{Y}_{Nt}^* = \left(\Delta_{Nt}^*\right)^{-\theta_n} \left(P_{Nt}^{*r}\right)^{\theta_n} \bar{Y}_{Nt}^{d*}, \tag{C.95}$$

$$\bar{Y}_{Nt}^{d*} = \left(P_{Nt}^{*r}\right)^{-1} \gamma_n \left(\bar{C}_t^* + \bar{G}_t^*\right),$$

$$\left(\Delta_{Nt}^*\right)^{-\theta_n} = (1-\xi_p)\left(\bar{P}_{Nt}^{*r}\right)^{-\theta_n} + \xi_p \left(\frac{\Delta_{Nt-1}^*}{\pi_t^*}\right)^{-\theta_n}. \tag{C.96}$$

Market clearing for tradable goods The supply of the tradable goods bundle in per capita terms is given by

$$\bar{Y}_{Tt} = A_t \left(\frac{\bar{L}_{Tt}}{1-\phi_\tau}\right)^{1-\phi_\tau} \left(\frac{\bar{X}_{Tt}}{\phi_\tau}\right)^{\phi_\tau}. \tag{C.97}$$

A market clearing condition for tradable goods is given by equation (A.55), which can be written in per capita terms as

$$\bar{Y}_{Tt} = (1-\gamma_\tau)\bar{Y}_{Tt}^d \frac{1}{n}\int_0^n g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right) di + \frac{\gamma_\tau^*(1-n)}{n}\bar{Y}_{Tt}^{d*}\frac{1}{n}\int_0^n g\left(\frac{P_{Ht}^*(i)}{\mathcal{P}_{Tt}^*}\right) di, \tag{C.98}$$

where $\bar{Y}_{Tt}^{d}$ and $\bar{Y}_{Tt}^{d*}$, are given, respectively, by

$$
\begin{aligned}
\bar{Y}_{Tt}^{d} &= \left(P_{Tt}^{r}\right)^{-1}(1-\gamma_n)\left(\bar{C}_t+\bar{G}_t\right)+\bar{X}_t, \\
\bar{Y}_{Tt}^{d*} &= \left(P_{Tt}^{*r}\right)^{-1}(1-\gamma_n)\left(\bar{C}_t^{*}+\bar{G}_t^{*}\right)+\bar{X}_t^{*}.
\end{aligned}
$$

where $\bar{X}_t=\bar{X}_{Tt}+\bar{X}_{Nt}$ and $\bar{X}_t^{*}=\bar{X}_{Tt}^{*}+\bar{X}_{Nt}^{*}$ The two integral terms in equation (C.98) can be explicitly written as

$$
\begin{aligned}
\frac{1}{n}\int_0^n g\left(\frac{P_{Ht}(i)}{\mathcal{P}_{Tt}}\right)di &= \left(\frac{\epsilon-\theta_\tau\left(\Delta_{1H,t}/\mathcal{P}_t^{r}\right)^{\epsilon-\theta_\tau}}{\epsilon-\theta_\tau}\right), \\
\frac{1}{n}\int_0^n g\left(\frac{P_{Ht}^{*}(i)}{\mathcal{P}_{Tt}^{*}}\right)di &= \left(\frac{\epsilon-\theta_\tau\left(\Delta_{1H,t}^{*}/\mathcal{P}_t^{*r}\right)^{\epsilon-\theta_\tau}}{\epsilon-\theta_\tau}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\left(\Delta_{1H,t}\right)^{\epsilon-\theta_\tau} &= \frac{1}{n}\int_0^n\left(P_{Ht}^{r}(i)\right)^{\epsilon-\theta_\tau}di=\lambda_H\left(\Delta_{1H,t}^{H}\right)^{\epsilon-\theta_\tau}+(1-\lambda_H)\left(\mathcal{E}_t^{r}\Delta_{1H,t}^{F}\right)^{\epsilon-\theta_\tau}, \\
\left(\Delta_{1H,t}^{*}\right)^{\epsilon-\theta_\tau} &= \frac{1}{n}\int_0^n\left(P_{Ht}^{*r}(i)\right)^{\epsilon-\theta_\tau}di.
\end{aligned}
$$

Keeping in mind that F-currency pricing is exclusively used in the Foreign country, the price dispersion terms can be written recursively as

$$
\left(\Delta_{1H,t}^{H}\right)^{\epsilon-\theta_\tau} = (1-\xi_p)\left(\bar{P}_{Ht}^{r}\right)^{\epsilon-\theta_\tau}+\xi_p\left(\frac{\Delta_{1H,t-1}^{H}}{\pi_t}\right)^{\epsilon-\theta_\tau}, \tag{C.99}
$$

$$
\left(\Delta_{1H,t}^{F}\right)^{\epsilon-\theta_\tau} = (1-\xi_p)\left(\bar{P}_{Ht}^{Fr}\right)^{\epsilon-\theta_\tau}+\xi_p\left(\frac{\Delta_{1H,t-1}^{F}}{\pi_t^{*}}\right)^{\epsilon-\theta_\tau}, \tag{C.100}
$$

$$
\left(\Delta_{1H,t}^{*}\right)^{\epsilon-\theta_\tau} = (1-\xi_p)\left(\bar{P}_{Ht}^{*r}\right)^{\epsilon-\theta_\tau}+\xi_p\left(\frac{\Delta_{1H,t-1}^{*}}{\pi_t^{*}}\right)^{\epsilon-\theta_\tau}. \tag{C.101}
$$

For country F, a market clearing condition regarding the tradable goods is symmetric, given by

$$
\bar{Y}_{Tt}^{*}=\left(\frac{\epsilon-\theta_\tau\left(\Delta_{1F,t}^{*}/\mathcal{P}_t^{*r}\right)^{\epsilon-\theta_\tau}}{\epsilon-\theta_p}\right)(1-\gamma^{*})\bar{Y}_{Tt}^{d*}+\left(\frac{\epsilon-\theta_\tau\left(\Delta_{1F,t}/\mathcal{P}_t^{r}\right)^{\epsilon-\theta_\tau}}{\epsilon-\theta_\tau}\right)\gamma\frac{n}{1-n}\bar{Y}_{Tt}^{d}, \tag{C.102}
$$

where

$$\bar{Y}^*_{Tt} = A^*_t \left(\frac{\bar{L}^*_{Tt}}{1-\phi_\tau}\right)^{1-\phi_\tau} \left(\frac{\bar{X}^*_{Tt}}{\phi_\tau}\right)^{\phi_\tau}, \tag{C.103}$$

$$\left(\Delta^*_{1F,t}\right)^{\epsilon-\theta_\tau} = (1-\xi_p)\left(\bar{P}^{*r}_{Ft}\right)^{\epsilon-\theta_\tau} + \xi_p \left(\frac{\Delta^*_{1F,t-1}}{\pi^*_t}\right)^{\epsilon-\theta_\tau}, \tag{C.104}$$

$$\left(\Delta_{1F,t}\right)^{\epsilon-\theta_\tau} = (1-\lambda_F)\left(\mathcal{E}^r_t \Delta^F_{1F,t}\right)^{\epsilon-\theta_\tau} + \left(\Delta^H_{1F,t}\right)^{\epsilon-\theta_\tau},$$

$$\left(\Delta^F_{1F,t}\right)^{\epsilon-\theta_\tau} = (1-\xi_p)\left(\bar{P}^{Fr}_{Ft}\right)^{\epsilon-\theta_\tau} + \xi_p \left(\frac{\Delta^F_{1F,t-1}}{\pi^*_t}\right)^{\epsilon-\theta_\tau}, \tag{C.105}$$

$$\left(\Delta^H_{1F,t}\right)^{\epsilon-\theta_\tau} = (1-\xi_p)\left(\bar{P}^{Hr}_{Ft}\right)^{\epsilon-\theta_\tau} + \xi_p \left(\frac{\Delta^H_{1F,t-1}}{\pi_t}\right)^{\epsilon-\theta_\tau}. \tag{C.106}$$

Complete asset markets Under the assumption of complete asset markets, price equalization in the state-contingent claims leads to the risk-sharing condition (A.6), which can be written as

$$\frac{1}{\mathcal{E}^r_t}\left(\frac{\bar{C}_t}{\bar{C}^*_t}\right)^{\frac{1}{\sigma}} = \frac{1}{\mathcal{E}^r_{t+1}}\left(\frac{\bar{C}_{t+1}}{\bar{C}^*_{t+1}}\right)^{\frac{1}{\sigma}}.$$

Combined with the assumption of initially zero net foreign assets and with the assumption of an exogenous shock to the exchange rate z_{et}, the above equation is reduced to

$$1 = \frac{1}{\mathcal{E}^r_t}\left(\frac{\bar{C}_t}{\bar{C}^*_t}\right)^{\frac{1}{\sigma}} e^{z_{et}}. \tag{C.107}$$

Equilibrium conditions To summarize, the system of equations for this economy consists of:

- *Households problem*: 2 equations (C.1) and (C.2) with $\bar{C}_t$ and $\bar{C}^*_t$.
- *Wage setting problem*: 15 equations (C.3)-(C.17) with $\bar{W}^r_t$, $V_{w1,t}$, $V_{w2,t}$, Π^H_{wt}, $\bar{W}^{*r}_t$, $V^*_{w1,t}$, $V^*_{w2,t}$, $\bar{W}^{Fr}_t$, $V^F_{w1,t}$, $V^F_{w2,t}$, Π^F_{wt}, W^r_t, W^{Hr}_t, W^{Fr}_t, and W^{*r}_t.
- *Firms' cost minimization problem*: 8 equations (C.18)-(C.25) with MC^r_{Nt}, MC^r_{Tt}, $\bar{L}_{Nt}$, $\bar{L}_{Tt}$, MC^{*r}_{Nt}, MC^{*r}_{Tt}, $\bar{L}^*_{Nt}$, and $\bar{L}^*_{Tt}$.
- *Non-tradable goods firms price setting*: 11 equations (C.26)-(C.36) with $\bar{P}^r_{Nt}$, $V_{n1,t}$, $V_{n2,t}$, $\bar{\Pi}^H_{Nt}$, $\bar{P}^{Fr}_{Nt}$, $V^F_{n1,t}$, $V^F_{n2,t}$, $\bar{\Pi}^F_{Nt}$, $\bar{P}^*_{Nt}$, $V^*_{n1,t}$, and $V^*_{n2,t}$.
- *Tradable goods firms price setting: (Goods, Market, Currency)*
  - *(H, H, H)*: 6 equations (C.37)-(C.42) with $\bar{P}^r_{Ht}$,$V_{h1,t}$, $V_{h2,t}$, $V_{h3,t}$, $V_{h4,t}$, and $\bar{\Pi}^H_{Ht}$.
  - *(H, H, F)*: 5 equations (C.43)-(C.47) with $\bar{P}^{Fr}_{Ht}$, $V^F_{h1,t}$, $V^F_{h2,t}$, $V^F_{h3,t}$, and $\bar{\Pi}^F_{Ht}$.
  - *(F, H, H)*: 6 equations (C.48)-(C.53) with $\bar{P}^r_{Ft}$, $V_{f1,t}$, $V_{f2,t}$, $V_{f3,t}$, $V_{f4,t}$, and $\bar{\Pi}^H_{Ft}$.
  - *(F, H, F)*: 5 equations (C.54)-(C.58) with $\bar{P}^{Fr}_{Ft}$, $V^F_{f1,t}$, $V^F_{f2,t}$, $V^F_{f3,t}$, and $\bar{\Pi}^F_{Ft}$.
  - *(H, F, F)*: 6 equations (C.59)-(C.64) with $\bar{P}^{*r}_{Ht}$, $V^*_{h1,t}$, $V^*_{h2,t}$, $V^*_{h3,t}$, $V^*_{h4,t}$, and $\bar{\Pi}^{*F}_{Ht}$.
  - *(H, F, H)*: 5 equations (C.65)-(C.69) with $\bar{P}^{*Hr}_{Ht}$, $V^{*H}_{h1,t}$, $V^{*H}_{h2,t}$, $V^{*H}_{h3,t}$, and $\bar{\Pi}^{*H}_{Ht}$.

- *(F, F, F)*: 4 equations (C.70)-(C.73) with $\bar{P}^{*r}_{Ft}$, $V^*_{f1,t}$, $V^*_{f2,t}$, and $V^*_{f3,t}$.

- *Price indices for non-tradable goods*: 4 equations (C.74) and (C.77) with P^r_{Nt}, P^{Hr}_{Nt}, P^{Fr}_{Nt}, and P^{*r}_{Nt}.
- *Price indices of tradable goods*: 9 equations (C.78)-(C.86) with $\mathcal{P}^r_{Tt}$, P^{Hr}_{Tt}, P^{Fr}_{Tt}, π_t, Δ^H_{2t}, Δ^F_{2t}, $\mathcal{P}^{*r}_{Tt}$, π^*_t, and Δ^*_{2t}.
- *Price indices of consumption bundles*: 2 equations (C.87) and (C.88) with P^r_{Tt} and P^{*r}_{Tr}.
- *Market clearing for non-tradable goods*: 8 equations (C.89)-(C.96) with $\bar{Y}_{Nt}$, Δ_{Nt}, Δ^H_{Nt}, Δ^F_{Nt}, $\bar{X}_{Nt}$, $\bar{Y}^*_{Nt}$, Δ^*_{Nt}, and $\bar{X}^*_{Nt}$.
- *Market clearing for tradable goods*: 10 equations (C.97)-(C.106) with $\bar{Y}_{Tt}$, $\bar{X}_{Tt}$, $\Delta^H_{1H,t}$, $\Delta^F_{1H,t}$, $\Delta^*_{1H,t}$, $\bar{Y}^*_{Tt}$, $\bar{X}^*_{Tt}$, $\Delta^*_{1F,t}$, $\Delta^F_{1F,t}$, and $\Delta^H_{1F,t}$.
- *Exchange rate*: 1 equation (C.107) with $\mathcal{E}^r_t$.
- *Monetary policy rules*: two equations (31) and its counterpart for country F with R_t and R^*_t
- *Shocks*: the stochastic processes for the neutral technology a_t, the monetary policy shock $\epsilon_{r,t}$, the exchange rate shock z_{et}, and corresponding stochastic processes for country F. Additional shocks include the government spending shock $\eta_{g,t}$ and the cost-push shock v_t and corresponding shocks for country F.

Welfare measure The welfare measure for country H is given by the average expected household utility, given by

$$\begin{aligned} SW_0 =& \frac{1}{n}\int_0^n E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\frac{1}{\sigma}} C_t(j)^{1-\frac{1}{\sigma}} - \frac{\psi}{1+\frac{1}{\nu}} L_t(j)^{1+\frac{1}{\nu}} \right) dj, \\ =& \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\frac{1}{\sigma}} C_t(j)^{1-\frac{1}{\sigma}} - \frac{\psi}{1+\frac{1}{\nu}} \left(\frac{1}{n}\int_0^n L_t(j)^{1+\frac{1}{\nu}} dj \right) \right). \end{aligned}$$

The consumption is the same across households, $C_t(j) = \bar{C}_t$, thanks to the state-contingent securities on wage change opportunities, but the labor supply is heterogeneous. The average of the labor disutility can be written as

$$\begin{aligned} \frac{1}{n}\int_0^n L_t(j)^{1+\frac{1}{\nu}} dj =& \left[\frac{1}{n}\int_0^n \left(\frac{W_t(j)}{W_t} \right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} dj \right] \bar{L}_t^{1+\frac{1}{\nu}} \\ =& W_t^{\theta_w\left(1+\frac{1}{\nu}\right)} \left[\lambda_w \left(W^H_{\nu,t} \right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} + (1-\lambda_w) \left(\mathcal{E}_t W^F_{\nu,t} \right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} \right] \bar{L}_t^{1+\frac{1}{\nu}}, \\ =& \left[\lambda_w \left(\frac{W^{Hr}_{\nu,t}}{W^r_t} \right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} + (1-\lambda_w) \left(\frac{\mathcal{E}^r_t W^{Fr}_{\nu,t}}{W^r_t} \right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} \right] \bar{L}_t^{1+\frac{1}{\nu}}, \end{aligned}$$

where $W^H_{\nu,t}$ and $W^F_{\nu,t}$ are auxiliary wage indices for the H-currency and F-currency wages, respectively, and $W^{Hr}_{\nu,t} = W^H_{\nu,t}/P_t$ and $W^{Fr}_{\nu,t} = W^F_{\nu,t}/P^*_t$. The social welfare and wage dispersions can be written recursively as

$$SW_t = \frac{1}{1-\frac{1}{\sigma}}\bar{C}_t^{1-\frac{1}{\sigma}} - \frac{\psi}{1+\frac{1}{\nu}}\left[\lambda_w\left(\frac{W_{\nu,t}^{Hr}}{W_t^r}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} + (1-\lambda_w)\left(\frac{\mathcal{E}_t^r W_{\nu,t}^{Fr}}{W_t^r}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)}\right]\bar{L}_t^{1+\frac{1}{\nu}} + \beta E_t SW_{t+1},$$

$$\left(W_{\nu,t}^{Hr}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} = (1-\xi_w)\left(\bar{W}_t^r\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} + \xi_w\left(\frac{W_{\nu,t-1}^{Hr}}{\pi_t}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)},$$

$$\left(W_{\nu,t}^{Fr}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} = (1-\xi_w)\left(\bar{W}_t^{Fr}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} + \xi_w\left(\frac{W_{\nu,t-1}^{Fr}}{\pi_t^*}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)}.$$

Similarly, the social welfare for country F can be written as

$$SW_t^* = \frac{1}{1-\frac{1}{\sigma}}\left(\bar{C}_t^*\right)^{1-\frac{1}{\sigma}} - \frac{\psi}{1+\frac{1}{\nu}}\left(\frac{W_{\nu,t}^{Fr*}}{W_t^{r*}}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)}\left(\bar{L}_t^*\right)^{1+\frac{1}{\nu}} + \beta E_t SW_{t+1}^*,$$

$$\left(W_{\nu,t}^{Fr*}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} = (1-\xi_w)\left(\bar{W}_t^{r*}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)} + \xi_w\left(\frac{W_{\nu,t-1}^{Fr*}}{\pi_t}\right)^{-\theta_w\left(1+\frac{1}{\nu}\right)}.$$

Steady state In steady state, the inflation rates are $\pi = \pi^* = 1$. The nominal interest rates are $R = R^* = 1/\beta$. The real exchange rate is also unity: $\mathcal{E}^r = 1$. As shown below, under the assumption of symmetry between countries H and F, per capita variables in country H – $\bar{C}$, $\bar{L}$, $\bar{L}_N$, $\bar{L}_T$, $\bar{Y}_N$, $\bar{Y}_T$, $\bar{X}_N$, $\bar{X}_T$ – become the same as those in country F.

In steady state, $P_N^r = \bar{P}_N^r = \bar{P}_N^{Fr} = \Delta_N$ and $P_T^r = \bar{P}_H^r = \bar{P}_H^{Fr} = \bar{P}_F^r = \bar{P}_F^{Fr} = \mathcal{P}_T^r = \Delta_2 = \Delta_{1H} = \Delta_{1F}$. From the corresponding price setting equations, the prices P_N^r and P_T^r are given by

$$P_N^r = \frac{\theta_n}{\theta_n - 1}MC_N^r = \frac{\theta_n}{\theta_n - 1}\left(W^r\right)^{1-\phi_n}\left(P_T^r\right)^{\phi_n},$$
$$P_T^r = \frac{\theta_\tau}{\theta_\tau - 1}MC_T^r = \frac{\theta_\tau}{\theta_\tau - 1}\left(W^r\right)^{1-\phi_\tau}\left(P_T^r\right)^{\phi_\tau}.$$

From equation (C.87), the price P_N^r is given as $P_N^r = \left(P_T^r\right)^{\frac{\gamma_n - 1}{\gamma_n}}$. By using this, the above two equations can be written as

$$P_T^r = \left[\frac{\theta_n}{\theta_n - 1}\left(W^r\right)^{1-\phi_n}\right]^{\frac{\gamma_n}{\gamma_n(1-\phi_n)-1}},$$
$$P_T^r = \left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1}{1-\phi_\tau}} W^r.$$

Solving for W^r and P_T^r yields

$$W^r = \left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{\gamma_n(1-\phi_n)-1}{1-\phi_\tau}}\left(\frac{\theta_n - 1}{\theta_n}\right)^{\gamma_n},$$
$$P_T^r = \left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1-\phi_n}{1-\phi_\tau}\gamma_n}\left(\frac{\theta_n - 1}{\theta_n}\right)^{\gamma_n}.$$

If $\theta_\tau = \theta_n = \theta$, the relative price P_T^r is given as $P_T^r = [\theta/(\theta-1)]^{(\phi_\tau - \phi_n)\gamma_n/(1-\phi_\tau)}$. If the share of intermediate input is greater for the tradable sector than the non-tradable sector (i.e. if $\phi_\tau > \phi_n$) as well, then $P_T^r > 1$ and $P_N^r < 1$. From equations (C.20) and (C.21), the ratios of labor to intermediate input are given by

$$\frac{\bar{L}_N}{\bar{X}_N} = \frac{1-\phi_n}{\phi_n}\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1}{1-\phi_\tau}},$$
$$\frac{\bar{L}_T}{\bar{X}_T} = \frac{1-\phi_\tau}{\phi_\tau}\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1}{1-\phi_\tau}}.$$

From equations (C.89) and (C.97), the supplies of the non-tradable goods bundle and the tradable goods bundle in per capita terms are given, respectively, by

$$\bar{Y}_N = A\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1-\phi_n}{1-\phi_\tau}}\left(\frac{1}{\phi_n}\right)\bar{X}_N,$$
$$\bar{Y}_T = A\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)\left(\frac{1}{\phi_\tau}\right)\bar{X}_T.$$

From the market clearing conditions (C.90) and (C.98):

$$A\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1-\phi_n}{1-\phi_\tau}}\left(\frac{1}{\phi_n}\right)\bar{X}_N = \bar{Y}_N = \frac{\gamma_n(1+\eta_g)}{P_N^r}\bar{C}, \tag{C.108}$$
$$A\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)\left(\frac{1}{\phi_\tau}\right)\bar{X}_T = \bar{Y}_T = \frac{(1-\gamma_n)(1+\eta_g)}{P_T^r}\bar{C} + \left(\bar{X}_N + \bar{X}_T\right), \tag{C.109}$$

where $\bar{G} = \eta_g \bar{C}$ and a symmetry between countries H and F is used in the second equation. In the main text, the government spending is assumed to be zero so that $\eta_g = 0$. These two equations have three unknowns: $\bar{X}_N$, $\bar{X}_T$, and $\bar{C}$. One more equation is needed to pin down these variables. From equations (C.3)-(C.17) about the labor supply:

$$\begin{aligned}
V_{w1} &= V_{w1}^F = V_{w1}^* = \frac{1}{1-\beta\xi_w}\bar{L}^{1+\frac{1}{\nu}},\\
V_{w2} &= V_{w2}^F = V_{w2}^* = \frac{1}{1-\beta\xi_w}\frac{W^r\bar{L}}{C^{\frac{1}{\sigma}}},\\
W^r &= \bar{W}^r = \bar{W}^{Fr} = \bar{W}^{*r} = W^{*r} = \frac{\theta_w\psi}{\theta_w - 1}\bar{L}^{\frac{1}{\nu}}\bar{C}^{\frac{1}{\sigma}}.
\end{aligned}$$

From the final equation above:

$$\left(\frac{\theta_w - 1}{\theta_w\psi}\right)^{\nu}(W^r)^{\nu}\bar{C}^{-\frac{\nu}{\sigma}} = \bar{L} = \bar{L}_N + \bar{L}_T = \left(\frac{\bar{L}_N}{\bar{X}_N}\right)\bar{X}_N + \left(\frac{\bar{L}_T}{\bar{X}_T}\right)\bar{X}_T, \tag{C.110}$$

where the ratios of labor to intermediate input are already given and constant. Substituting equation (C.108) into equations (C.109) and (C.110) yields

$$\left[A\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)\left(\frac{1}{\phi_\tau}\right) - 1\right]\bar{X}_T = \left[\frac{P_N^r}{P_T^r}\frac{(1-\gamma_n)}{\gamma_n}A\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1-\phi_n}{1-\phi_\tau}}\left(\frac{1}{\phi_n}\right) + 1\right]\bar{X}_N,$$
$$\left(\frac{\theta_w - 1}{\theta_w\psi}\right)^{\nu}(W^r)^{\nu}\left[A\left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1-\phi_n}{1-\phi_\tau}}\left(\frac{1}{\phi_n}\right)\frac{P_N^r}{\gamma_n(1+\eta_g)}\right]^{-\frac{\nu}{\sigma}}\bar{X}_N^{-\frac{\nu}{\sigma}} = \left(\frac{\bar{L}_N}{\bar{X}_N}\right)\bar{X}_N + \left(\frac{\bar{L}_T}{\bar{X}_T}\right)\bar{X}_T.$$

These two equations can be solved for $\bar{X}_N$ and $\bar{X}_T$. With $\bar{X}_N$ and $\bar{X}_T$ in hand, the following variables can be derived: $\bar{X}$, $\bar{L}$, $\bar{L}_N$, $\bar{L}_T$, and $\bar{C}$.

Instead of computing $\bar{L}$ for a given ψ, it is convenient to calibrate ψ so that $\bar{L} = 1$ for normalization. Then, from equation (C.110), $\bar{X}_N = (\bar{L}_N/\bar{X}_N)^{-1} - (\bar{L}_N/\bar{X}_N)^{-1}(\bar{L}_T/\bar{X}_T)\bar{X}_T$. The per capita labor in the tradable sector can be pinned down as

$$\bar{X}_T = \frac{\frac{P_N^r}{P_T^r}\frac{(1-\gamma_n)}{\gamma_n} A \left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1-\phi_n}{1-\phi_\tau}} \left(\frac{1}{\phi_n}\right) + 1}{\left[A\left(\frac{\theta_\tau}{\theta_\tau-1}\right)\left(\frac{1}{\phi_\tau}\right) - 1\right]\frac{\bar{L}_N}{\bar{X}_N} + \left[\frac{P_N^r}{P_T^r}\frac{(1-\gamma_n)}{\gamma_n} A \left(\frac{\theta_\tau}{\theta_\tau - 1}\right)^{\frac{1-\phi_n}{1-\phi_\tau}} \left(\frac{1}{\phi_n}\right) + 1\right]\frac{\bar{L}_T}{\bar{X}_T}}.$$

With $\bar{X}_T$ in hand, $\bar{X}_N$, $\bar{X}$, $\bar{C}$, $\bar{L}_T$, and $\bar{L}_N$ can be derived. Finally, ψ is pinned down as

$$\psi = \frac{\theta_w - 1}{\theta_w} W^r \bar{C}^{-\frac{1}{\sigma}}.$$

The other variables in steady state are derived as follows. From equations (C.37)-(C.73),

$$\begin{aligned}
V_{h1} &= V_{h1}^F = V_{f1} = V_{f1}^F = V_{h1}^* = V_{h1}^{*H} = V_{f1}^* = \frac{(P_T^r)^{\theta_\tau - \epsilon}}{1-\beta\xi_p}\frac{\bar{Y}_T^d}{\bar{C}^{1/\sigma}}, \\
V_{h2} &= V_{h2}^F = V_{f2} = V_{f2}^F = V_{h2}^* = V_{h2}^{*H} = V_{f2}^* = \frac{1}{1-\beta\xi_p}\frac{\bar{Y}_T^d}{\bar{C}^{1/\sigma}}, \\
V_{h3} &= V_{h3}^F = V_{f3} = V_{f3}^F = V_{h3}^* = V_{h3}^{*H} = V_{f3}^* = \frac{(P_T^r)^{\theta_\tau - \epsilon}}{1-\beta\xi_p}\frac{\bar{Y}_T^d}{\bar{C}^{1/\sigma}} MC_T^r, \\
V_{h4} &= V_{f4} = V_{h4}^* = \frac{1}{1-\beta\xi_p}\frac{\bar{Y}_T^d}{\bar{C}^{1/\sigma}} MC_T^r \\
\bar{\Pi}_H^H &= \bar{\Pi}_H^F = \bar{\Pi}_F^H = \bar{\Pi}_F^F = \bar{\Pi}_H^{*F} = \bar{\Pi}_H^{*H} = \frac{\theta_\tau}{\theta_\tau - \epsilon}\left(V_{h1} - V_{h3}\right)\left(\bar{P}_H^r\right)^{-(\theta_\tau - \epsilon)} - \frac{\epsilon}{\theta_\tau - \epsilon}\left(V_{h2} - V_{h4}\right), \\
V_{n1} &= V_{n1}^F = V_{n1}^* = \frac{(P_N^r)^{\theta_n}}{1-\beta\xi_p}\frac{\bar{Y}_N^d}{\bar{C}^{1/\sigma_p}} MC_N^r, \\
V_{n2} &= V_{n2}^F = V_{n2}^* = \frac{(P_N^r)^{\theta_n}}{1-\beta\xi_p}\frac{\bar{Y}_N^d}{\bar{C}^{1/\sigma_p}},
\end{aligned}$$

where $\bar{Y}_T^d = (P_T^r)^{-1}(1-\gamma_n)(1+\eta_g)\bar{C} + \bar{X}_T + \bar{X}_N$ and $\bar{Y}_N^d = (P_N^r)^{-1}\gamma_n(1+\eta_g)\bar{C}$.