The Rise of Corporate Social Responsibility: When Firms Set Personalized Prices for Network Goods

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Abstract

We propose a theory of corporate social responsibility by linking it to the firm's product markets. The firm's product exhibits network effects in the sense that the product's value to each consumer increases with the number of consumers. Moreover, with the technology development, the firm can adopt personalized pricing for each consumer. We show that such a firm could use social responsibility as a commitment device for low product prices, which helps overcome the coordination problem among consumers. In this way, the firm's material payoff increases, supporting the notion of "doing well by doing good."

Keywords: Corporate social responsibility, network effect, personalized pricing, coordination

JEL Classification: D11, G32, L20, M14

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1 Introduction

Many of today's most popular firms are selling products that exhibit network effects; that is, the value created by the product increases with the number of users adopting the same product. Examples include social media platforms (e.g., Twitter and LinkedIn), information technology providers (e.g., Apple and Huawei), and video game companies. Firms selling these network products can use new tools, such as machine learning, to predict individual consumers' willingness to pay and then use those data to adopt personalized pricing strategies. For instance, firms can deliver a customized price in the form of a discount to a universally posted price via a mobile application or other channel.

Meanwhile, a growing number of firms are making corporate social responsibility (CSR) an integral part of their business strategies. Among the dimensions of CSR, social responsibility toward consumers is seen as particularly important. For example, Huawei, the leading provider of information and communication technology infrastructure, has actively launched a technology (RuralStar Pro) to provide mobile connections to previously uncovered rural areas at the optimal cost and fastest speed.[1] In 2018, the world's fifth largest smartphone maker Xiaomi announced that it would forever limit its net profit margin of hardware sales to a maximum of 5%.[2] The firm will distribute the extra profits to users by "reasonable" means if the margin exceeds the ceiling, as it restated its philosophy of making innovation affordable for everyone. Some gaming companies are active new players in corporate social responsibility as well. For instance, Riot Games, the company behind the massively popular multiplayer game League of Legends has matched millions in donations made by their global eSports community.

In this paper, we provide a unified conceptual framework to understand these seemingly unrelated trends. We aim to understand various CSR practices across different firms/industries and the normative implications of these CSR practices. In our model, a monopolistic firm sells a product to two consumers to maximize its profit. To study the firm's possible social responsible

[1] See https://www.huawei.com/en/news/2021/2/rural-ruralstar-iab-relay-ruralstar-pro.

[2] See https://asia.nikkei.com/Business/Companies/China-s-Xiaomi-caps-hardware-profit-margin-at-5-indefinitely.

preference, we assume that before selling its product, the firm can commit to an objective function that places a positive weight on consumer surplus in addition to its profit from selling the product. In other words, the stated objective function is credible in that the firm's subsequent pricing strategy is implemented such that the stated objective function is maximized. However, the firm ultimately pursues material payoffs so that the weight on consumer surplus in the stated objective function will be optimally chosen to maximize its profit.

Two ingredients in our model are crucial to give rise to CSR, that is, the de facto profit-maximizing firm would commit to caring about consumer surplus. First, the product is a network good in that the product's value to each consumer increases with the number of users. Examples of network goods include communication devices/services, video games, and digital platforms such as Amazon and LinkedIn. By consuming the product, a consumer derives utility consisting of two components: The intrinsic value which represents the consumer's individual basic willingness to pay for the product, and the network value which is achieved if and only if the other consumer also purchases. Second, we assume that the firm adopts personalized pricing, which is facilitated by the collection of consumer data and the development of technology. Specifically, the firm simultaneously offers potentially different prices to the two different consumers, and each consumer decides whether or not to purchase after observing her individualized price. Importantly, under personalized pricing, a consumer only observes her own price and does not know anything about the price of the other consumer.

We show that under network effects and personalized pricing, it is optimal for the profit-maximizing firm to commit to being socially responsible for consumer surplus. In equilibrium, the firm chooses a positive weight on consumer surplus in its stated objective function. Therefore, CSR endogenously arises as part of the profit-maximizing strategy for the firm, which lends support to the notion of "doing well by doing good."

To understand the commitment role of corporate social responsibility, let us first consider the effect of personalized pricing on the firm profit. Under personalized pricing, the firm can charge different prices to the consumers based on their different willingness to pay, and this price discrimination benefits the firm. However, personalized pricing also implies that a consumer cannot observe the other consumer's price when making purchase decisions. This price non-transparency can cause a coordination problem among consumers in the presence of the network effect. Noting that a consumer can derive the network value only when the other consumer also buys the product. Therefore, when making the purchase decision, a consumer is concerned that if the firm charges a high price to the other consumer, the product's customer base would be so limited that she might be unable to derive the network value. This concern can become valid in equilibrium.

Specifically, if prices are transparent, when the firm raises the price to one consumer, not only the focal consumer but also the other consumer will be less likely to buy the product. It is intuitive that the price increase directly discourages the focal consumer from buying the product. Then, the other consumer expects a smaller customer base for the product and, thus a lower likelihood of deriving the network value, thereby refraining from the purchase. By contrast, under personalized pricing, each consumer's price is unobservable to the other. When offering prices, the firm will not internalize the negative effect of increasing a consumer's price on the other, which results in inefficiently high prices and fulfills consumers' beliefs.

Social responsibility can kick in as a way to improve the firm profit. By being socially responsible for consumer surplus, the firm credibly commits to low prices offered to both consumers. This alleviates each consumer's concern about the high price charged to the other consumer, allowing them to coordinate their purchase decisions and encouraging them to buy the product. When the extent of social responsibility is mild, this gain from higher consumer demand overshadows the loss due to a lower profit margin, leading to the firm's material payoff increasing in the social responsibility. However, the relationship can be reversed if the firm commits too much to social responsibility. This trade-off thus determines the optimal level of social responsibility committed by the firm.

In practice, consumers may engage in sequential purchases rather than make simultaneous purchases. Moreover, two consumers in our baseline model are clearly a simplified setting and there exist many more consumers in the economy. We show that our key insight that a profit-maximizing firm would find optimal to commit to social responsibility toward consumers continues to hold in these extensions. Finally, the firm may not be purely profit oriented, but rather cares about consumer surplus to some extent. We find that under such a more general firm objective function, the firm still finds it optimal to commit to a greater level of social responsibility, which helps the firm overcome the price unobservability problem and better achieve its purpose.

Our framework provides a novel perspective of corporate social responsibility and offers new empirical implications. First, one salient feature of our theory is to link firms' social responsibility toward consumers to their product properties, thereby generating testable cross-sectional variation in social responsibility practices across firms and industries. Specifically, for firms or industries whose products are featured with high network value, our model predicts that they would devote considerable resources to being socially responsible toward consumers. This prediction is aligned with the casual observations in the cases of Huawei and Xiaomi and the gaming industry. In addition, our channel crucially hinges on the consumers being well aware of the CSR practices adopted by the firm. Thus, the same types of firms or industries with high-network-value products should be more active in the related CSR disclosure. The KPMG (2020) survey consistently shows that the Technology, Media & Telecommunications (TMT) sector, whose products are typically featured with the network value, has the highest sustainability reporting rate in sample firms.

Second, CSR in our model is closely related to the technology development that enables firms to adopt personalized pricing for consumers. This is broadly consistent with the recent trend of the widely used data and artificial intelligence algorithms in business and firms' increasing attention to social responsibility.

Our paper relates to the growing literature on corporate social responsibility (e.g., Friedman, 1970; Hart and Zingales, 2017; Bénabou and Tirole, 2010). Some theories also rationalize firm sustainability from the profit perspective. For instance, Baron (2001) argues that CSR can be used as a strategic deterrence of regulation. Albuquerque and Cabral (2021) consider strategic interaction among firms and show that firms may choose to deviate from profit maximization and yet yield higher profits. We add to this literature by showing that CSR can help coordinate consumers' purchase decisions in the presence of network effects, which generates testable implications about CSR activities across firms and industries. As argued by Liang and Renneboog (2017, p. 854), the existing "doing good – doing well" statements hardly explain the cross-firm variation in CSR; that is, if on average CSR enhances firm value, why do some companies adopt a CSR-oriented strategy whereas others do so to a lesser extent. Our theory thus provides a framework to understand the cross-firm/industry variation.

Our work also relates to the literature that examines the use of delegation as a means of strategic commitment (e.g., Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987; Aggarwal and Samwick, 1999; Bova and Yang, 2017). A principal's use of an agent to play a game can cause the principal to choose an agent with unaligned interests or to set a strategic compensation scheme that makes the agent take a different action in subsequent games. In our setting, one way for the firm to commit to social responsibility can be to hire an agent that has more social responsible awareness or to design a compensation scheme that induces this awareness.

The potential for individualized, personalized pricing is recognized by Shapiro et al. (1999) and is reviewed in surveys by Fudenberg and Villas-Boas (2012) and Seele et al. (2021). Since personalized pricing becomes more feasible when firms have access to vast quantities of personal data, our paper is also related to the burgeoning literature on big data and its price implications. For instance, when consumers have privacy choices, Ichihashi (2020) shows that consumers benefit from revealing information to enjoy personalized recommendations, but the seller can use the same information to price discriminate. With more personal data available, more consumers might be reached, and there might be less exclusion (Bergemann and Bonatti, 2019). We demonstrate interesting interactions between using personal data to target pricing decisions and the adoption of corporate social responsibility. The uniform pricing benchmark also connects our paper to the related literature (e.g., Cavallo et al., 2014; Cavallo, 2018; DellaVigna and Gentzkow, 2019). Hajihashemi et al. (2022) examine the effect of personalized pricing on prices and consumer and producer surplus in the presence of network effects. We differentiate by focusing on the non-transparency issue rooted in personalized pricing and explore how the firm can use corporate social responsibility to mitigate it.

Our paper is also related to the literature on network effects (e.g., Farrell and Saloner, 1985, 1986; Katz and Shapiro, 1985, 1994; Hagiu and Wright, 2020). We highlight the coordination problem created by the network effect and the role of corporate social responsibility in overcoming such payoff externality among consumers (Myatt and Wallace, 2012).

2 Model Setup

Consider an economy with one monopolistic firm and two consumers. There are two dates $\{0, 1\}$. On date 0, the firm announces its objective function to maximize its material payoff. On date 1, the firm offers each consumer a price to maximize the stated objective function announced on the previous date. After observing her own price, each consumer simultaneously decides whether or not to buy the good. At the end of date 1, the firm's profit and consumers' utility are realized.

2.1 Network Good

The firm sells a product to two consumers $\{H, L\}$, and the product has a network property; we simply call it a network good. Specifically, by consuming the product, consumer i derives utility as follows:

$$\mathcal{U}_i = v_i + \lambda \cdot \mathbb{1}(\text{consumer } j \text{ makes a purchase}), \tag{1}$$

where $i, j \in \{H, L\}$ and $i \neq j$. As in Katz and Shapiro (1985), there are two components in consumer i's utility. The first component v_i is consumer i's basic willingness to pay for the product. It represents the product's intrinsic value or stand-alone value to consumers, which is a random variable drawn from a distribution and can be different across the two consumers. For simplicity, we assume that the basic willingness to pay follows a uniform distribution. We consider heterogeneous consumers in that their v_i follows heterogeneous uniform distributions. Without loss of generality, we normalize consumer L's support as $[0,1]$, that is, $v_L \sim U[0,1]$ and $v_H \sim U[0,a]$, where $a \geq 1$. The realization of each v_i remains the private information of consumer i until the end of date 1.

The second component in the utility function (1) captures the network effect. The parameter $\lambda \in (0,1)$ represents the magnitude of the network value inherent in the good and is common knowledge. We assume that $\lambda < 1$ so the network value is not too large compared with the product's stand-alone value. The network value is achieved if and only if both consumers purchase the good. Many products are characterized by such a network effect, under which the value of the product to each user increases with the number of users (Katz and Shapiro, 1985, 1994; Farrell and Saloner, 1985; Liebowitz, Margolis, and Hirshleifer, 1999; Shapiro, Varian, Carl, et al., 1999). Examples of markets with a network effect include communication devices (e.g., fax machines and modems), communication services (e.g., telephone, e-mail, and Internet online services), complementary products (e.g., video games), and platforms (e.g., LinkedIn).

2.2 Personalized Pricing

At the beginning of date 1, the firm simultaneously offers prices to the two consumers. The price is personalized; we use $p_i \geq 0$ to denote the price provided to consumer i, where $i \in \{H, L\}$. Personalized pricing has been greatly facilitated in the era of big data and advanced artificial intelligence algorithms. Big data has lowered the costs of collecting customer-level information, making it easier for sellers to identify new customer segments to target those populations with customized marketing and pricing plans. For instance, it is now possible to track users' location via various apps, their browser and search history, and whom and what they like on social networks such as Twitter and LinkedIn. In addition, the growing industry of data brokers and information intermediaries that buy and sell customer lists enables sellers to assemble a digital profile of individual consumers. This massive volume of data, combined with the power of machine learning algorithms, has given rise to a wide and varied range of personalized services (including news content, advertising, etc), as well as personalized pricing.

Personalized pricing also implies that each price is only observable to its target consumer; that is, consumer j cannot directly observe consumer i's price p_i. In the big-data era, this price non-transparency is realistic; for instance, a customer's personalized pricing through individualized coupons is hardly observable by another consumer (e.g., Allender, Liaukonyte, Nasser, and Richards, 2021). In addition, consumer privacy or fairness concerns may incentivize the firm not to announce personalized prices, which limits price transparency.

2.3 Corporate Social Responsibility toward Consumers

On date 0, the firm can credibly announce to become socially responsible and to what extent it cares about consumers.[3] Specifically, suppose that the firm's stated objective function takes the following form:

$$\Pi = \pi + \gamma \cdot CS, \tag{2}$$

where π is the firm's expected material payoff from selling the good and CS is consumer surplus. We denote $\alpha_i \in [0, 1]$ as the probability of consumer i purchasing the product. We normalize the firm's marginal cost to zero so that the expected material payoff is

$$\pi = \alpha_L \, p_L + \alpha_H \, p_H. \tag{3}$$

The consumer surplus CS is computed as the expected unconditional surplus of both consumers before their purchase decisions and the realization of the product's intrinsic value to each consumer.

Furthermore, the choice variable $\gamma \geq 0$ in (2) captures the firm's concern for consumer surplus: By selecting a higher value of γ, the firm commits to being more socially responsible to-

[3]Since there are only the firm and its two consumers in our economy, their combined surplus essentially represents social welfare.

ward consumers; that is, the consumer surplus weighs more in the firm's stated objective function. Such responsibility toward consumers includes, for instance, charging low prices so that the existing consumers derive higher utility and more consumers can afford the product, as in the examples of Huawei and Xiaomi. In the extreme case of $\gamma = 0$, the firm does not commit to any corporate social responsibility.

The stated objective function is credible because the firm's subsequent pricing strategy is set to maximize this objective. In reality, this commitment is feasible through firms' public announcement about their sustainability projects and the reputation cost upon breach, e.g., Huawei's official launch of RuralStar Pro solution and Xiaomi announcing its profit cap, as mentioned in the introduction. In addition, as Ceccarelli, Glossner, Homanen, and Schmidt (2021) suggest, having investors that participate in collaborative engagements organized by the Principles for Responsible Investing (PRI) also demonstrates the firm's commitment to sustainability.

Despite the stated objective function and the credible commitment to implement the said principle, the firm ultimately cares about the material payoff π. Therefore, the choice of the stated objective function, characterized by γ, is endogenously chosen at the beginning of date 0 so that the material payoff π is maximized.

3 Equilibrium Characterization

In this section we characterize the equilibrium by first solving the firm's optimal pricing strategy and consumers' optimal purchase strategy given the social responsibility γ in Section 3.1 and then the optimal social responsibility in Section 3.2. We show that a profix-maximizing firm would commit to being social responsible in equilibrium. Throughout, we consider only pure strategy, perfect Bayesian-Nash equilbria.

3.1 Optimal Firm Pricing and Consumer Purchase

Personalized pricing means not only that the two consumers may receive different prices, but also that they are unable to observe each other's prices when making purchase decisions. Since the utility a consumer derives from consumption depends on whether the other consumer also joins the network associated with that product, the consumer will base her purchase decision on the expected purchase behavior of the other consumer. Suppose that consumer L believes that consumer H purchases the product with probability $\hat{\alpha}_H$; similarly, consumer H's belief about the probability of consumer L purchasing the product is $\hat{\alpha}_L$. In computing the equilibrium, we need to specify how a consumer forms beliefs about the other consumer's purchase probability upon observing her own price, particularly when receiving an unexpected (off-equilibrium) price. We consider passive beliefs as commonly used in the literature (e.g., Hart and Tirole, 1990; O'Brien and Shaffer, 1992; McAfee and Schwartz, 1994).[4]

Under passive beliefs, when a consumer receives an offer different from what she expects in the candidate equilibrium, she does not revise her beliefs about the price offered to the other consumer. Passive beliefs can be justified based on the notion that consumers view unexpected offers as trembles by the firm. Then, together with the fact that in equilibrium consumers must hold consistent beliefs, a consumer's belief is a constant fixed at its equilibrium level. That is, upon receiving an off-equilibrium price $p_i \neq p_i^*$, consumer i does not update $\hat{p}_j$, her belief about p_j. In other words, $\hat{p}_j$ is a constant. Because $\hat{p}_j$ is a constant, $\hat{\alpha}_j$, consumer i's belief about j's purchase probability, is also a constant.

For a given level of social responsibility γ, the equilibrium characterization in this subgame depends on whether consumers' purchase probability takes interior value or not; that is, whether $\alpha_i \in (0,1)$ or not, where $i \in \{H, L\}$. In the main text, for illustration purpose we only focus on the case in which both consumers' purchase probabilities take interior values, i.e., $\alpha_H, \alpha_L \in (0,1)$. The discussion of other cases are delegated to the appendix.

We start with consumers' purchase decisions. Consumer L is willing to purchase if and only if

[4] The equilibrium concept is in the same spirit of sequential equilibrium (Kreps and Wilson, 1982).

the expected consumption utility outweighs the price, i.e., $v_L + \lambda\hat{\alpha}_H \geq p_L$. Or, equivalently, $v_L \geq p_L - \lambda\hat{\alpha}_H$. Note that under passive beliefs, consumer L's belief $\hat{\alpha}_H$ about H's purchase probability is not affected by her price p_L. Similarly, consumer H purchases if and only if $v_H \geq p_H - \lambda\hat{\alpha}_L$. Since the two consumers' basic willingness to pay is assumed to be uniformed distributed, we can compute their respective purchase probability as follows:

$$\alpha_L = \Pr(v_L \geq p_L - \lambda\hat{\alpha}_H | p_L, \hat{\alpha}_H) = 1 - (p_L - \lambda\hat{\alpha}_H), \tag{4}$$

$$\alpha_H = \Pr(v_H \geq p_H - \lambda\hat{\alpha}_L | p_H, \hat{\alpha}_L) = 1 - \frac{1}{a}(p_H - \lambda\hat{\alpha}_L). \tag{5}$$

At the beginning of date 1, understanding how consumers make their purchase decisions (see equations (4) and (5)), the firm determines prices p_L and p_H to maximize the stated objective function: $\Pi = \pi + \gamma \cdot CS$, as given by equation (2). The firm's expected material payoff is the summation of the profit made from the two consumers as given by equation (3). The expected consumer surplus is the summation of the two consumers' ex ante utility from consumption:

$$CS = \underbrace{\int_{p_L - \lambda\hat{\alpha}_H}^{1} (v - p_L)dv + \alpha_L\alpha_H\lambda}_{\text{consumer 1's utility}} + \underbrace{\int_{p_H - \lambda\hat{\alpha}_L}^{a} \frac{1}{a}(v - p_H)dv + \alpha_L\alpha_H\lambda}_{\text{consumer 2's utility}}. \tag{6}$$

Maximizing the stated objective function Π yields the firm's optimal pricing strategy as follows:

$$p_L = \frac{a\left(\gamma\left(-\left(\hat{\alpha}_H + 2\right)\lambda + \gamma - 3\right) + 2\hat{\alpha}_H\lambda + 2\right) - 2\gamma\lambda^2\left(2\left(\hat{\alpha}_H\gamma\lambda + \gamma\right) + \hat{\alpha}_L(1-\gamma)\right)}{a(2-\gamma)^2 - 4\gamma^2\lambda^2}, \tag{7}$$

$$p_H = \frac{a^2(2-\gamma)(1-\gamma) + a\lambda\left(2\gamma\left(\hat{\alpha}_H(\gamma-1)\lambda - 2\gamma\lambda - 1\right) - \hat{\alpha}_L(\gamma-2)\right) - 4\hat{\alpha}_L\gamma^2\lambda^3}{a(2-\gamma)^2 - 4\gamma^2\lambda^2}, \tag{8}$$

which are functions of the beliefs about the two consumers' purchase probability. Then, applying the fact that consumers must hold consistent beliefs in equilibrium (i.e., $\hat{\alpha}_L = \alpha_L$ and $\hat{\alpha}_H = \alpha_H$) to equations (4), (5), (7) and (8), we obtain that in equilibrium the two consumers purchase the product with the following respective probability[5]

$$\alpha_L^* = \frac{a(2+\lambda-\gamma(1-\lambda))}{a(2-\gamma)^2-\lambda^2(1+\gamma)^2}, \tag{9}$$

$$\alpha_H^* = \frac{a(2-\gamma)+\lambda(1+\gamma)}{a(2-\gamma)^2-\lambda^2(1+\gamma)^2}. \tag{10}$$

Simple calculation of equations (9) and (10) shows that consumer L is more likely to buy the product than consumer H, i.e., $\alpha_L^* > \alpha_H^*$. While consumer L has a lower average willingness to pay, the firm charges a lower price to her, giving her more purchase incentives. Therefore, to verify that both consumers' purchase probability lies within 0 and 1, i.e., $\alpha_L^*, \alpha_H^* \in (0,1)$, we only need to make sure that $\alpha_L^* < 1$. We can show that this is indeed the case when $\gamma < \bar{\gamma}_L$, where $\bar{\gamma}_L$ is given by (A1) in the appendix. Intuitively, when the firm does not care much about consumer surplus, it charges high product prices, only catering to consumers with high willingness to pay.

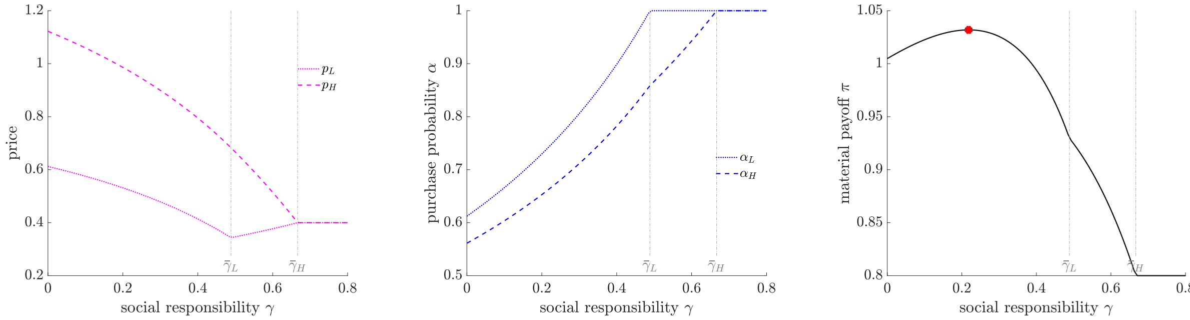
And replacing $\hat{\alpha}_L = \alpha_L^*$ and $\hat{\alpha}_H = \alpha_H^*$ in equations (7) and (8) and inserting equations (9) and (10) yields the equilibrium prices as follows:

$$p_L^* = \frac{a\left(2+\gamma^2+\lambda-\gamma(3+2\lambda)\right)-\gamma(1+\gamma)\lambda^2}{a(2-\gamma)^2-\lambda^2(1+\gamma)^2}, \tag{11}$$

$$p_H^* = a\frac{a(\gamma^2-3\gamma+2)-\lambda\left(\gamma^2\lambda+\gamma(2+\lambda)-1\right)}{a(2-\gamma)^2-\lambda^2(1+\gamma)^2}. \tag{12}$$

Equations (9) through (12) clearly show that the two consumers' optimal purchase strategy and the firm's optimal pricing strategy are all functions of γ, the firm's choice of social responsibility. In Figure 1, we plot how the firm's social responsibility affects these variables. When the firm starts to commit to social responsibility (i.e., $\gamma < \bar{\gamma}_L$), it charges lower prices for both consumers and in turn consumers are more likely to buy the product. This is intuitive since if consumer surplus weighs more in a generic firm's objective function, the firm naturally lowers its product price to transfer more rent to the consumers. And consumers respond by making the purchase more likely.

[5] One necessary condition for $\alpha_i^* \geq 0$ is $a(2-\gamma)^2 - \lambda^2(1+\gamma)^2 > 0$, which is equivalent to $\gamma < \frac{2\sqrt{a}-\lambda}{\lambda+\sqrt{a}}$.



(a) Price (b) Purchasing probability (c) material payoff

This figure plots the effect of social responsibility γ on the optimal product prices set by the firm (Panel a), consumers' purchase probability (Panel b), and the firm's material payoff (Panel c). The red dot in Panel (c) indicates the maximum material payoff. The parameters are $a = 2$ and $\lambda = 0.4$.

Figure 1: Main model: Personalized pricing

As the social responsibility γ continues to increase, consumer L certainly buys the product. While the price p_H charged for consumer H still decreases in γ, the price p_L charged for consumer 1 becomes an increasing function of it. As shown in equation (A2), in this case the price charged for consumer L is simply the expected network value the consumer can derive. Since a higher γ implies lower p_H and thus higher purchase probability α_H of consumer H, consumer L finds it more plausible to derive the network value and thus willing to accept higher price. When the increasing γ exceeds $\bar{\gamma}_H$, both consumers certainly buy the product and the firm charges a constant price at the level of the network value.

3.2 Optimal Corporate Social Responsibility

We emphasize that all variables α_L^*, α_H^*, p_L^*, and p_H^* are functions of the social responsibility γ in equations (9) through (12). Inserting these equations into the firm's material payoff function (3) we can express the firm's profit as follows:

$$\pi(\gamma) = \frac{a\left(\begin{array}{c} a\left(\gamma^3\left(\lambda^2+2\lambda-1\right)+\gamma^2\left(5-3\lambda^2\right)-\gamma\left(3\lambda^2+12\lambda+8\right)+\lambda^2+8\lambda+4\right) \\ -a^2(\gamma-1)(\gamma-2)^2-(\gamma+1)\lambda^2\left(\gamma^2(2\lambda-1)+2\gamma(\lambda+2)-1\right) \end{array}\right)}{\left(a(2-\gamma)^2-\lambda^2(\gamma+1)^2\right)^2}. \quad (13)$$

As discussed in Section 3.1, the firm's material payoff takes the form (13) when $\gamma < \bar{\gamma}_L$. We further find that at the point $\gamma = 0$, if the firm commits to a marginally higher level of social responsibility, its material payoff will increase. Mathematically, taking the derivative of $\pi(\gamma)$ in equation (13) with respect to γ and setting $\gamma = 0$ yields

$$\frac{d\pi(\gamma)}{d\gamma} \mid_{\gamma=0} > 0. \quad (14)$$

Therefore, even if the firm essentially aims to maximize material payoff, it finds it optimal to commit to being socially responsible toward consumers. The following proposition summarizes this key insight of our paper.

Proposition 1 (Endogenous rise of CSR)**.** *Under personalized pricing, there exists a unique equilibrium in which the firm commits to social responsibility; that is,* $\gamma^* > 0$*.*

Panel (c) of Figure 1 numerically confirms this finding. It shows that when the firm starts to commit to social responsibility, its material payoff will improve. Only when the level of social responsibility becomes very high will this social responsibility erodes the firm's profit. The red dot indicates the maximum material payoff that can be achieved by the firm, which clearly occurs when the profit-maximizing firm commits to a positive level of social responsibility.

In the next section, we discuss in detail how personalized pricing and the network effect are the two crucial driving forces for this key insight.

4 Driving Forces

In this section, we illustrate how personalized pricing and network effects can induce the firm to strategically commit to being social responsible even though it only cares about its material payoff.

4.1 The Role of Price Discrimination and Price Transaprency

On one hand, personalized pricing suggests potential price discrimination against consumers. After learning about the distribution of each consumer's basic willingness to pay, the firm can charge different prices to different consumers for the same product based on their different willingness to pay. On the other hand, personalized pricing also implies price unobservability. That is, each consumer can only observe her own price offer, but not the one received by the other consumer. In reality, many factors can give price to this price non-transparency. For instance, while it is feasible for the firm to inform the two consumers about the prices in our simplified model, this task becomes very cumbersome when there are many consumers as in real life. The consumers may also find it time-consuming to collect and analyze all the personalized prices. In addition, fairness and privacy concerns among consumers may also prohibit price transparency.

To transparently show the role of personalized pricing in driving our key result, we first present two benchmarks, namely uniform pricing and transparent pricing, and then contrast the main personalized pricing model to these two benchmarks.

4.1.1 Uniform pricing.

Under uniform pricing, the two consumers are charged the same prices. Hence, when observing her price, one consumer also effectively observes the other consumer's price. The other model setup is the same as in the main model and the equilibrium concept is subgame perfect equilibrium (SPE).

On date 1, consumer L observes the product price p and calculates consumer H's actual purchase probability α_H. Then, consumer L purchases the good if and only if the expected utility exceeds the purchase cost: $v_L + \lambda\alpha_H \geq p$, or equivalently, the basic willingness to pay exceeds a certain threshold, i.e., $v_L \geq p - \lambda\alpha_H$. Since v_L is assumed to follow a standard uniform distribution, consumer L's purchase probability is characterized by $\alpha_L = 1 - (p - \lambda\alpha_H)$. Similarly, consumer H's purchase probability is $\alpha_H = 1 - \frac{1}{a}(p - \lambda\alpha_L)$. Solving the two equations yields the two consumers' equilibrium purchase probabilities as follows:[6]

$$\alpha_L = \frac{a + a\lambda - p(a+\lambda)}{a - \lambda^2} \text{ and } \alpha_H = \frac{a + \lambda - p(1+\lambda)}{a - \lambda^2}. \tag{15}$$

Clearly, when the firm charges higher prices, both consumers are less willing to buy the product.

Next, as in Section 3.1, the firm computes its stated objective function $\Pi = \pi + \gamma \cdot CS$ under the pricing policy p, where the material payoff is $\pi = p(\alpha_L + \alpha_H)$ and the expected consumer surplus is $CS = \int_{p-\lambda\alpha_H}^{1}(v-p)dv + \int_{p-\lambda\alpha_L}^{a}\frac{1}{a}(v-p)dv + 2\alpha_L\alpha_H\lambda$. As a credible commitment, the firm chooses the pricing strategy p to maximize the stated objective function Π, which yields the optimal uniform price as follows:

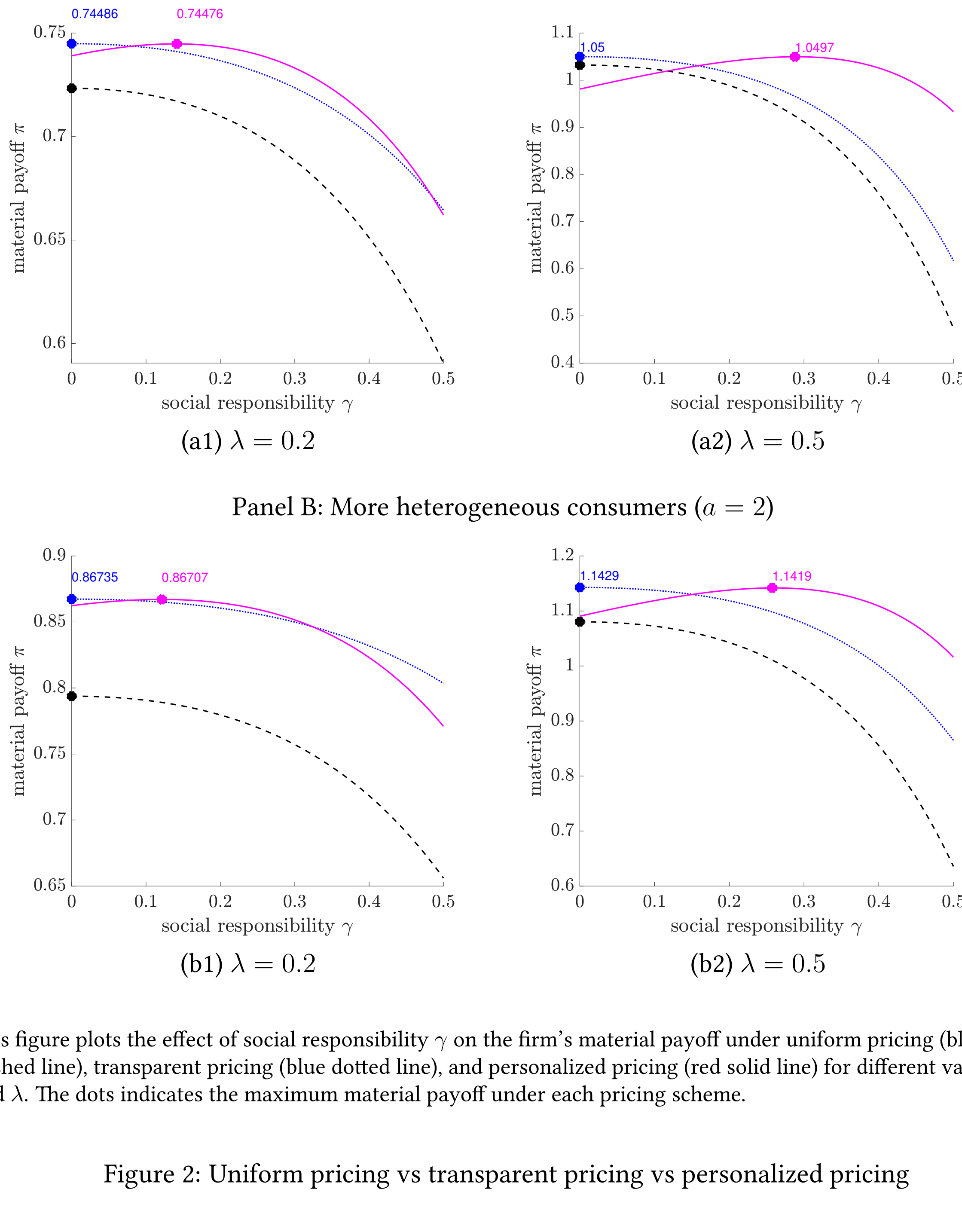
$$p = \frac{2a(\lambda+1)(\lambda+a)\gamma + \lambda^3 + a\lambda(\lambda+1)^2 - \lambda(a^2+2a) - 2a^2}{((1+a)\lambda^2 + 4a\lambda + a(1+a))\gamma + 4\lambda^3 + 2(1+a)\lambda^2 - 4a\lambda - 2a(1+a)}. \tag{16}$$

Finally, using equations (15) and (16) we can express the firm's material payoff $\pi = p\,(\alpha_H + \alpha_L)$. Further analysis shows that π monotonically decreases in γ (i.e., $\frac{\partial\pi}{\partial\gamma} < 0$), and hence the optimal social responsibility chosen by the firm is zero. That is, the profit-maximizing firm does not commit to any corporate social responsibility under uniform pricing. The following lemma summarizes the equilibrium in this benchmark, where the superscript "U" indicates the equilibrium under uniform pricing.

Lemma 1 (Uniform pricing). *Under uniform pricing, there exists a unique equilibrium in which (1)*

[6] As in Section 3.1, we focus on the interior case where the consumers' purchase probability $\alpha_i < 1$, where $i \in \{H, L\}$, which occurs when $\gamma < \bar{\gamma}^0$, where $\bar{\gamma}^0$ is given by (A11). This greatly simplifies exposition without affecting the equilibrium characterization. We leave the discussion of the full cases to the appendix; see the end of proof of Lemma 1.

the firm does not commit to any social responsibility, i.e., $\gamma^U = 0$, *(2) the price offered by the firm to the two consumers is* $p^U = \frac{1}{2}\frac{2a+\lambda+a\lambda}{1+a+2\lambda}$, *and (3) consumer* 1 *purchases if and only if* $v_L \geq \frac{(p^U-\lambda)(a+\lambda)}{a-\lambda^2}$, *whereas consumer* 1 *purchases if and only if* $v_H \geq \frac{a(p^U-\lambda)(1+\lambda)}{a-\lambda^2}$.



Panel A: Less heterogeneous consumers ($a = 1.5$)

(a1) $\lambda = 0.2$ (a2) $\lambda = 0.5$

Panel B: More heterogeneous consumers ($a = 2$)

(b1) $\lambda = 0.2$ (b2) $\lambda = 0.5$

This figure plots the effect of social responsibility γ on the firm's material payoff under uniform pricing (black dashed line), transparent pricing (blue dotted line), and personalized pricing (red solid line) for different values of a and λ. The dots indicates the maximum material payoff under each pricing scheme.

Figure 2: Uniform pricing vs transparent pricing vs personalized pricing

The black dashed line in Figure 2 plots how the firm's social responsibility affects its material payoff under uniform pricing, with the black dot indicates the maximum material payoff that can be achieved by the firm. Consistent with Lemma 1, the profit-maximizing firm optimally chooses not to commit to any social responsibility.

4.1.2 Transparent pricing.

The second benchmark features transparent pricing. That is, while the firm may offer different prices to different consumers, consumers can observe each other's price. While as argued in the beginning of Section 4.1 transparent pricing may not be feasible in practice, it is still a relevant benchmark for us to investigate the intuition.

On date 1, consumer L observes the price offered to consumer H and calculates consumer H's actual purchase probability α_H. Then, consumer L purchases the good if and only if the derived utility exceeds the purchase cost: $v_L + \lambda\alpha_H \geq p_L$, or equivalently, $v_L \geq p_L - \lambda\alpha_H$. As derived above, consumer L's purchase probability is characterized by $\alpha_L = 1 - (p_L - \lambda\alpha_H)$. Similarly, consumer H's purchase probability is $\alpha_H = 1 - \frac{1}{a}(p_H - \lambda\alpha_L)$. Solving the two equations yields the two consumers' purchase strategies as follows:

$$\alpha_L = \frac{a(1-p_L) + \lambda(a-p_H)}{a-\lambda^2}, \tag{17}$$

$$\alpha_H = \frac{a - p_H + \lambda(1-p_L)}{a-\lambda^2}. \tag{18}$$

Different from equation (4), equation (17) shows that under transparent pricing, consumer H's price p_H directly affects consumer L's purchase decision. In fact, a higher price p_H will lower not only H's purchase probability, but also L's, i.e., $\frac{\partial\alpha_H}{\partial p_H} < 0$ and $\frac{\partial\alpha_L}{\partial p_H} < 0$ as shown in equations (17) and (18). When p_H increases, H becomes less willing to buy the product; observing this higher price, consumer L knows that the chance for her to derive the network value is lower and thus she responds by being less likely to buy the product as well.

On date 0, using equations (17) and (18), the firm computes its stated objective function $\Pi = \pi + \gamma \cdot CS$, where the material payoff is $\pi = \alpha_L p_L + \alpha_H p_H$ and the expected consumer surplus is $CS = \int_{p_L - \lambda\alpha_H}^{1} (v - p_L)dv + \int_{p_H - \lambda\alpha_L}^{a} \frac{1}{a}(v - p_H)dv + 2\alpha_L\alpha_H\lambda$. As a credible commitment, the firm chooses the pricing strategies p_i and p_j to maximize the stated objective function Π, which yields

$$p_L = \frac{a(2 + \gamma^2 - \gamma(3 + \lambda) - 2\lambda^2)}{a(2 - \gamma)^2 - 4\lambda^2}, \tag{19}$$

$$p_H = \frac{a\left(a(2 - 3\gamma + \gamma^2) - \lambda(\gamma + 2\lambda)\right)}{a(2 - \gamma)^2 - 4\lambda^2}. \tag{20}$$

Finally, inserting equations (17)–(20) into the firm's material payoff $\pi = p_L\alpha_L + p_H\alpha_H$ and maximizing it yields the optimal social responsibility. We find that in equilibrium the profit-maximizing firm does not commit to any corporate social responsibility. The following proposition summarizes the equilibrium under transparent pricing, where the superscript "T" indicates the case with transparent prices.

Lemma 2 (Transparent pricing)**.** *Under transparent pricing, there exists a unique equilibrium in which (1) the firm does not commit to any social responsibility, i.e., $\gamma^T = 0$, (2) the prices offered by the firm to the two consumers are $p_L^T = \frac{1}{2}$ and $p_H^T = \frac{a}{2}$, and (3) consumer L makes the purchase if and only if $v_L \geq \frac{a(p_L^T - \lambda) + \lambda(p_H^T - \lambda)}{a - \lambda^2}$, and consumer H makes the purchase if and only if $v_H \geq \frac{a(p_H^T - (1 + \lambda - p_L^T)\lambda)}{a - \lambda^2}$.*

The blue dotted line in Figure 2 plots how the firm's social responsibility affects its material payoff under transparent pricing, with the blue dot indicates the firm's maximum material payoff. As in the uniform-pricing case and consistent with Lemma 2, under transparent pricing the profit-maximizing firm optimally chooses not to commit to any social responsibility.

4.1.3 Social responsibility under personalized pricing.

Compared with uniform pricing, personalized pricing enables the firm to charge different prices based on each consumer's willingness to pay, thereby improving the firm's profit. However, personalized pricing may hurt the firm profit since the price unobservability causes a coordination problem among consumers, which can be a serious concern in the presence of the network effect.

Recall that the network value can only be derived when both consumers make the purchase. Under personalized pricing, when a consumer cannot observe the price received by the other consumer, she is concerned that once the firm charges a high price to the other consumer and thus deters the purchase, she wouldn't be able to enjoy the network benefit. Indeed, this concern can become valid in equilibrium; that is, the firm indeed charges high prices to consumers when their prices are not observable to each other. To understand it, we compare personalized pricing with transparent pricing which, as an ideal case for the profit-maximizing firm, preserves the benefit of price discrimination but overcomes the price unobservability issue, by decomposing the effect of one consumer's price on the firm's material payoff. Applying the chain rule to the firm's profit function $\pi = \alpha_1 p_1 + \alpha_2 p_2$ with respect to the price p_i yields the following:

$$\frac{d\pi}{dp_i} = \underbrace{\alpha_i}_{\text{Price margin effect } (>0)} + \underbrace{\frac{\partial \alpha_i}{\partial p_i} p_i}_{\text{Direct demand effect } (<0)} + \underbrace{p_j \frac{\partial \alpha_j}{\partial p_i}}_{\substack{\text{Indirect demand effect} \\ -\text{ transparent pricing: } (<0) \\ -\text{ personalized pricing: } (=0)}}. \tag{21}$$

where $i, j \in \{L, H\}$ and $i \neq j$.

When prices are transparent, if the firm raises the price p_i, there are three effects. First, as p_i increases, the firm enjoys a higher profit margin from selling to consumer i. This is captured by the first term in equation (21) and we term it as the "price margin effect." Second, an increase in p_i will directly decrease consumer i's purchase probability, which is the "direct demand effect" in equation (21). The third effect, termed as the "indirect demand effect" in equation (21), concerns the externality between the two consumers; specifically, consumer j will respond to the increasing price offered to consumer i by reducing the likelihood of purchase. Intuitively, if consumer i becomes less likely to buy the product, consumer j derives less utility from the product due to the loss of the network value. While the first effect works to the benefit of the firm's profit, the latter two hurt it.

Under personalized pricing, consumer i's price p_i becomes unobservable to consumer j. Thus, while the first two effects remain, the third indirect demand effect vanishes, i.e., the third term in equation (21) becomes zero. That is, once consumer j cannot observe consumer i's price, she will not respond to the change in consumer i's price. Instead, consumer j forms a belief about consumer i's purchase probability when making purchase decisions. Since the third effect hurts the firm's profit and hence only forces the firm to lower product prices, its absence means that the firm tends to set a higher product price to consumer i, compared with the level under transparent pricing. In the absence of the unobservability friction, the firm's prices under transparent pricing are optimal. Thus, this higher price must be inefficiently high for the profit-maximizing firm. Indeed, since consumer j's belief must be consistent in equilibrium, she well anticipates the high price charged by the firm to consumer i and thus is concerned about the product's consumer base when deciding whether to buy the product, which ultimately limits the product's customer base.

Overall, this coordination problem induced by price unobservability harms the firm. Whether the firm makes a higher profit under personalized pricing relative to that under transparent pricing depends on the trade-off between the price-discrimination benefit and the coordination cost.

We use Figure 2 to further illustrate the trade-off. The pink solid line in Figure 2 plots the effect of social responsibility on the firm's material payoff under personalized pricing. For now we fix $\gamma = 0$. The firm's profit under personalized pricing can be higher or lower than that under uniform pricing, depending on the trade-off between benefit and cost. The former price-discrimination benefit is governed by the consumer heterogeneity parameter a: the benefit of price discrimination is more salient when consumers become more heterogeneous. The latter coordination cost is governed by the network value λ: the consumers are more concerned about coordination when the network value is higher. As Figure 2 consistently shows, when a higher a favors the firm under transparent pricing whereas a higher λ disfavors it.

Having introduced the coordination problem under personalized pricing, we now explain the role of corporate social responsibility in mitigating this issue. We argue that via corporate social responsibility, the firm can commit to low product prices, thereby boosting the consumer demand, sustaining the network value, and improving the firm profit. Specifically, a marginal increase in γ commits the firm to offer lower price p_i to consumer i, which alleviates consumer j's concern about the product's consumer base and encourages j to make purchases. While low product prices imply low profit margin for the firm, it gets compensated by the stronger consumer demand. When γ is low, the profit-margin loss is dominated by gain from higher demand, causing the firm's material payoff increase in γ. By contrast, when γ is high, the profit-margin loss prevails, yielding a decreasing material payoff with respect to γ. The firm thus trades off the two opposing effects when setting the optimal level of social responsibility. Taken together, as the pink line in Figure 2 shows, the firm's material payoff exhibits a hump-shaped pattern with respect to γ, and the equilibrium social responsibility features a positive level. That is, a de facto profit-maximizing firm would commit to corporate social responsibility in equilibrium.

One natural question arises: Can the firm use social responsibility as a commitment device to achieve the first-best profit under transparent pricing? We investigate this question using Figure 2. First, the figure confirms that a personalized-pricing firm, by committing to an optimal level of social responsibility, can always achieve higher profit than a uniform-pricing firm; that is, the red dot always features higher firm profit than the blue dot. Second, while the social responsibility commitment helps the firm mitigate the price unobservability friction while maintaining the benefit of price discrimination, its maximum profit is still lower than what a transparent-pricing firm can achieve.

4.2 The Role of Network Effects

Network effects represent explicit benefits that are generated or gained when an individual aligns the individual's behavior with the behavior of others (Easley and Kleinberg, 2010). The products of many popular firms in the digital age such as Twitter and LinkedIn feature the network effect.

In this section, we examine the role of network effect in generating the firm's social responsibility by conducting comparative statics with respect to the network value parameter λ. When the product features no network value ($\lambda = 0$), we find that under personalized pricing, the firm optimally commits to no social responsibility, i.e., $\gamma^* = 0$. Recall the three effects of varying the price to consumer i as mentioned in equation (21). In the absence of the network value, the third effect concerning the externality between the two consumers vanishes. Now, consumer j does not even care about consumer i's purchase decisions or her price p_i; that is, whether or not consumer i purchases the product does not directly affect consumer j's utility. As such, even if consumer j cannot observe i's price, her demand for the product will not be affected by the unobservability issue and the price set by the profit-maximizing firm is optimal in achieving its highest expected profit. In this case, any commitment to corporate social responsibility only deviates from the profit-maximizing objective, thereby hurting the firm's profit.

Moreover, we find that a larger network value induces a higher level of social responsibility in equilibrium. Again, a larger network value implies a more serious coordination problem among consumers: consumer j is more concerned about the product's consumer base when making purchase decisions, so the high price charged by a profit-maximizing firm will further lower consumer demand and harm its profit. As a result, the firm has to commit more to social responsibility toward consumers and sustain the optimal level of demand via low product price.

Finally, we also examine the normative implications of the network value. As the network effect λ strengthens, not only the firm makes a higher material payoff, but also consumers enjoy higher surplus, and thus the social surplus increases. This Pareto improvement result contrasts our finding with that in the literature (e.g., Baron, 2001; Albuquerque and Cabral, 2021).

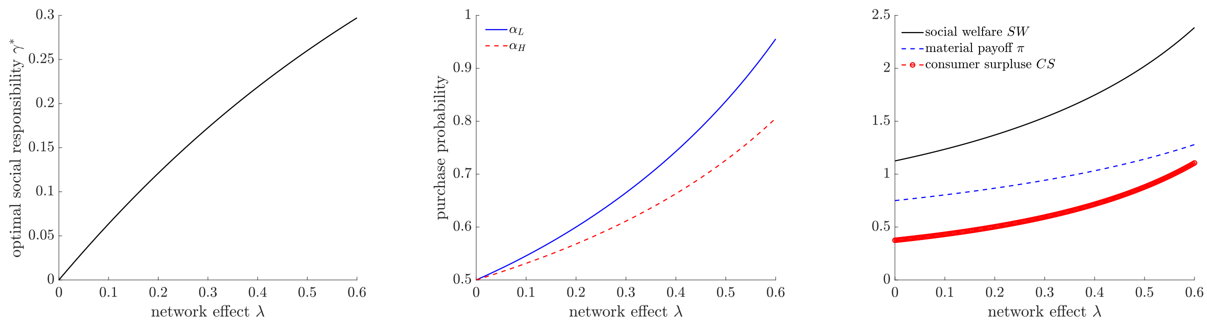
We analytically show these results when the network value is sufficiently small, as summarized in Proposition 2. Figure 1 numerically confirms the intuition for a wider range of parameters.

Proposition 2 (Network Value)**.** (1) *When the product features no network value, the firm commits to no social responsibility in equilibrium. That is, when $\lambda = 0$, we have $\gamma^* = 0$.*

(2) *For sufficiently low network value, the following holds:*

(2.1) *The firm's optimal level of social responsibility monotonically increases in its product's network value. That is, γ^* increases in λ.*

(2.2) *The firm's material payoff, consumer surplus, and social welfare all monotonically increase* *in the network value of the firm's product.*



(a) Optimal CSR (b) Purchase probability (c) Welfare

This figure plots the effect of network value on the firm's optimal choice of social responsibility in Panel (a) and on the firm's material payoff, consumer surplus, and social welfare in Panel (b). The parameter is $a = 2$.

Figure 3: The effect of network value

5 Extensions and Variations

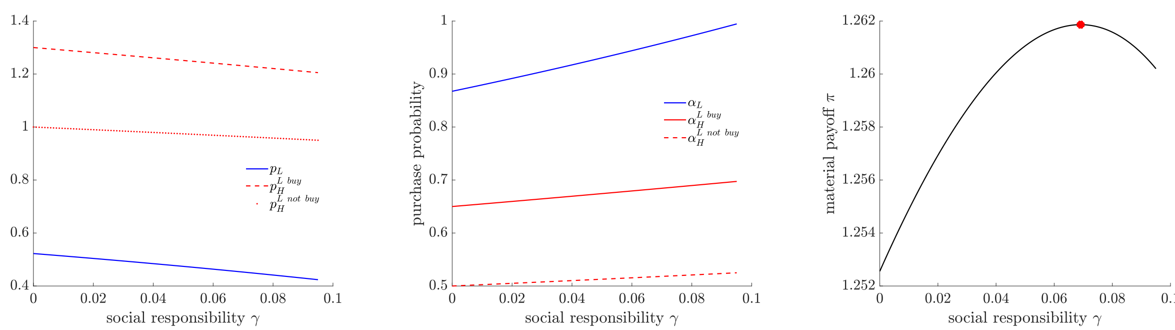
In this section, we consider several variations and extensions of our baseline model to demonstrate the robustness of the key insight.

5.1 Sequential Purchase by Consumers

In the baseline model, we assume that the two consumers simultaneously make the purchase decisions and they cannot observe each other's prices. In this section, we relax this assumption. Compared with the baseline model, the setup difference occurs only on date 1. Without loss of generality, suppose that on date 1, consumer L moves first to decide whether or not to buy the product; after observing consumer L's purchase decisions, consumer H makes her own purchase decision.

We solve the game using backward induction by first characterizing the firm's equilibrium pricing strategy and consumers' equilibrium purchase strategies on date 1 given the firm's social responsibility preference γ. First, on date 1, conditional on consumer L's purchase decisions, the firm optimally sets the price for consumer H to maximize its stated objective function. Upon observing consumer L's purchase decisions and her own product price p_H, consumer H decides to buy the product only when the derived utility exceeds the price p_H. Second, anticipating how her own purchase decision affects the subsequent firm pricing p_H and consumer H's purchase decision, consumer L buys the product if and only if her expected utility exceeds the purchase cost p_L. Understanding how consumer L makes her purchase decision and consumer H's response, the firm in turn sets the price p_L such that the stated objective function is maximized. The following proposition summarizes the equilibrium social responsibility chosen by the firm.

Proposition 3 (Sequential purchase)**.** *Suppose that consumer L moves first and consumer H follows. There exists a unique equilibrium in which the firm commits to social responsibility, i.e., $\gamma^* > 0$.*



(a) Pricing strategies (b) Purchase probability (c) material payoff

This figure plots the effect of social responsibility γ on the firm's pricing strategies (Panel a), consumers' purchase probability (Panel b), and the firm's material payoff (Panel c). The red dot in Panel (c) indicates the maximum material payoff. The parameters are $\lambda = 0.6$ and $a = 1.5$.

Figure 4: Extension: Sequential purchases by consumers

Proposition 3 shows that when consumers make sequential purchase, the firm optimally commits to being socially responsible. Figure 4 graphically illustrates this result: When the firm becomes more socially responsible (i.e., caring about consumer surplus), it lowers its product prices and transfers more rent to consumers. The consumers respond by becoming more likely to buy the product. The resultant material payoff is hump-shaped with respect to the social responsibility. Thus, the de facto profit-maximizing firm commits to social responsibility toward consumers in equilibrium; that is, $\gamma^* > 0$.

The intuition for these results is as follows. The sequential purchase creates asymmetry between the two consumers. Again, we start with a firm that does not commit to any social responsibility. First, consumer L's purchase decision precedes that of consumer H, and thus when consumer H makes her purchase decision the consumer base (and hence the network value) does not concern consumer H at all. That is, if consumer L purchases the product, consumer H knows that she could derive the network value if she also buys the product; otherwise, if consumer L does not purchase, she would not enjoy the network value anyway. Therefore, the firm could charge a higher price p_H for consumer H in the former case than in the latter, and the price is optimal in achieving the maximum material payoff for the firm.

Second, for consumer L, whether consumer H buys the product or not remains unknown when making the purchase decision. Thus, consumer L is concerned about the firm offering a high price to consumer H so that H becomes less likely to purchase the product, thereby affecting the product's customer base and lowering her utility. Such a concern is valid in equilibrium because even if the firm charges a high price to the following consumer H, consumer L won't respond to it by scaling back her purchase (recall the third effect in equation (21)). Without consideration of this negative effect, the firm indeed charges a high price to the following consumer, which limits the customer base and thus lowers consumer L's utility.

Now, as in the main model, the firm's responsibility toward consumers can serve as a commitment device for low product prices and help the firm overcome the price non-transparency problem. To be specific, by committing to caring about consumer surplus, the firm will set low product prices for both consumers, in particular for consumer H. This helps alleviate consumer L's concern regarding the subsequent high price p_H and the resulting small customer base, which encourages consumer L to buy the product in the first place. Therefore, even if the firm ultimately aims at maximizing material payoff, in equilibrium it optimally commits to being socially responsible for consumer surplus.

5.2 Multiple Consumers

There are only two consumers in the baseline model, which is clearly a simplified setting to highlight the insight. In this section, we introduce multiple types of consumers and demonstrate the robustness of the insight. Suppose there is a continuum type of consumers, where the type $t \sim U[0, a]$ and the basic willingness to pay of the type-t consumers satisfies $v_t \sim U[0, 1 + t]$. Under personalized pricing, the firm can charge different prices to different types of consumers and each consumer cannot observe the others' prices. The following proposition summarizes the results in this extended economy.

Proposition 4 (Multiple consumers)**.** *Suppose that there is a continuum of consumer types. When the network value is positive but sufficiently small, in equilibrium the firm commits to being socially responsible, i.e.,* $\gamma^* > 0$*.*

Proposition 4 confirms our key insight in the extended economy with multiple consumers. In fact, the presence of more consumers does not alter the coordination problem among consumers. That is, when making purchase decisions, a consumer is still concerned that the high price charged to the other consumers will deter their purchase and thus impairs her network value. Therefore, the firm still finds it optimal to use social responsibility as a commitment device to lower product prices, maintain the consumer base, and thus improving the material payoff.

5.3 A More General Firm Objective Function

In the baseline model, we assume that the firm aims to maximize the material payoff only. In practice, the firm may truly care about consumer surplus to some extent. We thus relax this assumption by considering a more general firm's objective function. Denote the firm's true objective function as Z. Suppose that the firm loads a weight w of consumer surplus in its true objective function, that is,

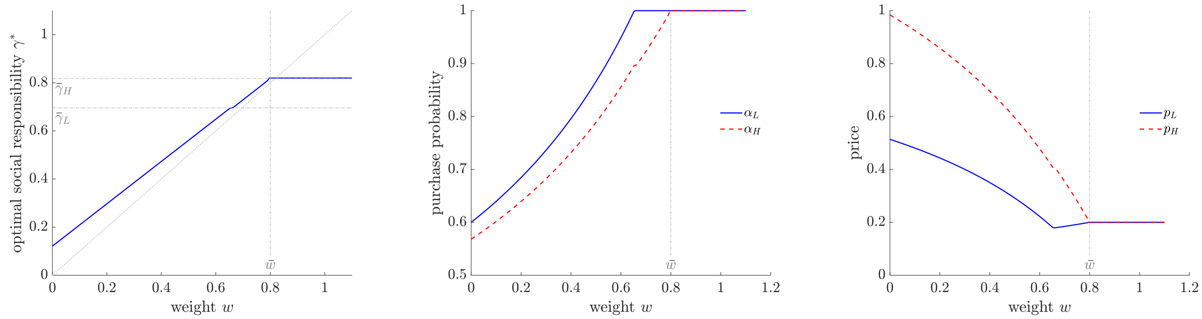
$$Z = \pi + w \cdot CS. \tag{22}$$

The baseline model is nested here by setting $w = 0$. When $w = 1$, the firm becomes a social planner in this economy. In this extension, the firm chooses γ to maximize its true objective function Z, leading to the optimal γ^* as a function of w. The following proposition summarizes the results in this extended economy and Figure 5 plots the effect of w on the equilibrium choice of γ, consumers' purchase probability, and the firm's pricing.

Proposition 5 (More general firm objective function)**.** *Suppose that the firm truly places the weight w on the consumer surplus. When $w < \bar{w} \equiv 1 - \frac{2\lambda}{a}$, the optimal choice of γ exceeds w, i.e., $\gamma^* > w$. When $w > \bar{w}$, the optimal choice of γ is independent of w.*

Proposition 5 shows that when the true weight w on consumer surplus is low, the firm tends to commit to a greater level of social responsibility than that is loaded in its true objective function. The baseline model has demonstrated that when the firm only aims to maximize the material payoff, it still finds it optimal to commit to being socially responsible, that is, $\gamma^* > w = 0$. The relationship $\gamma^* > w$ holds in more general cases because the firm tends to set inefficiently high product prices in the presence of the price unobservability, which deters the consumers' purchase. Therefore, committing to being more socially responsible helps the firm avoid the opportunistic pricing behavior and restore the efficient pricing to the best possible.

When the true weight w on consumer surplus is large (i.e., $w > \bar{w}$), any value of γ exceeding $\bar{\gamma}_H$ will achieve the same level of optimal utility for the firm. In this case, even if the firm cares much about its consumers, the best it can do is to lower the prices enough so that both consumers buy the product with certainty and derive both the basic benefit and the network value. As shown in panels b and c, both consumers buy the product for sure and both prices are constant at the level of the network value.



(a) Optimal γ (b) Purchase probability (c) Prices

This figure plots the effect of the weight parameter w on the firm's optimal choice of γ (Panel a), consumers' purchase probability (Panel b), and the firm's prices (Panel c). The parameters are $\lambda = 0.2$ and $a = 2$. In Panel a, when $w > \bar{w}$, any $\gamma \geq \bar{\gamma}_H$ can be the optimal γ^*, and we plot $\gamma = \bar{\gamma}_H$ for simplicity.

Figure 5: Extension: More general firm objective function

6 Conclusion

Corporate social responsibility has gained increasing attention worldwide. We propose a parsimonious framework that rationalizes why a profit-maximizing firm would commit to being socially responsible for consumers. We show that if the firm's product is featured with network value, under personalized pricing corporate social responsibility helps the firm commit to low prices offered to consumers, overcome the coordination problem faced by them, and thus improve firm profits. Therefore, our model provides one justification for the notion of "doing well by doing good." In other words, a firm's profit maximization and its socially responsible awareness can be well aligned.

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Appendix: Proofs

Full Equilibrium Characterization in Section 3.1

To begin with, we argue that it is never optimal for the firm to not sell the product or to sell only to one consumer, that is, $\alpha_L = 0$ or $\alpha_H = 0$. We prove it by contradiction. Suppose not; that is, $\alpha_i = 0$, where $i = H$ or L. Then, the firm can always lower the price p_i to induce the consumer to make purchase with positive probability. On the one hand, the firm can make positive (instead of zero) expected profit from the consumer i. On the other hand, due to the network effect, the other consumer will be more likely to buy the product and the firm can potentially make higher expected profit as well. Therefore, $\alpha_i = 0$ cannot be sustained in equilibrium and we only need to consider the case in which both consumers buy the product with positive probability, that is, $\alpha_L > 0$ and $\alpha_H > 0$.

Case 1: $\alpha_L, \alpha_H < 1$**.** In the main text, we have discussed the case in which $\alpha_L, \alpha_H \in (0,1)$. We now solve conditions that sustain the interior purchase probability. Using equations (9) and (10), we know that

$$\alpha_L^* - \alpha_H^* = \frac{(a-1)(1+\gamma)\lambda}{a(2-\gamma)^2 - \lambda^2(1+\gamma)^2} > 0.$$

That is, consumer L is more likely to make the purchase than consumer H. Furthermore, equations (9) and (10) imply that to ensure $\alpha_L^*, \alpha_H^* > 0$ we need $\gamma < \frac{2\sqrt{a}-\lambda}{\sqrt{a}+\lambda}$. Taking the derivative of α_L in equation (9) with respect to γ yields

$$\begin{aligned}
\frac{\partial \alpha_L}{\partial \gamma} &= \frac{a\left((\gamma+1)\lambda^2(\lambda+5-\gamma(1-\lambda)) + a(2-\gamma)(-\gamma(1-\lambda)+4\lambda+2)\right)}{\left(a(2-\gamma)^2 - (\gamma+1)^2\lambda^2\right)^2} \\
&> \frac{a\left((\gamma+1)\lambda^2(\lambda+5-\frac{2+\lambda}{1-\lambda}(1-\lambda)) + a(2-\gamma)(-\frac{2+\lambda}{1-\lambda}(1-\lambda)+4\lambda+2)\right)}{\left(a(2-\gamma)^2 - (\gamma+1)^2\lambda^2\right)^2} \\
&= \frac{3a\lambda(a(2-\gamma)+\lambda(1+\gamma))}{\left(a(2-\gamma)^2 - (\gamma+1)^2\lambda^2\right)^2} > 0,
\end{aligned}$$

where the first inequality follows because $\gamma < \frac{2+\lambda}{1-\lambda}$ (noting that $\gamma < \frac{2\sqrt{a}-\lambda}{\sqrt{a}+\lambda} < \frac{2+\lambda}{1-\lambda}$) and the second inequality follows because $\gamma < 2$ (noting that $\gamma < \frac{2\sqrt{a}-\lambda}{\sqrt{a}+\lambda} < 2$). Therefore, α_L is an increasing function of γ. Solving $\alpha_L < 1$ yields

$$\gamma < \bar{\gamma}_L \equiv \frac{-\sqrt{a}\sqrt{a\lambda^2 + 10a\lambda + a + 24\lambda^2} + a\lambda + 3a + 2\lambda^2}{2(a-\lambda^2)}. \tag{A1}$$

Therefore, when $\gamma < \bar{\gamma}_L$, we have $\alpha_L^*, \alpha_H^* < 1$.

Case 2: $\alpha_L = 1$ and $\alpha_H < 1$. As discussed in Case 1, consumer L is more likely to buy the product than consumer H. Therefore, when γ is high, it is possible that consumer L certainly buys the product whereas consumer H still purchases with probability strictly less than 1, i.e., $\alpha_L = 1$ and $\alpha_H < 1$.

Since consumer L buys the product with certainty, consumer H must derive the network value and thus will buy the product if and only if $v_H \geq p_H - \lambda$, which occurs with probability $\alpha_H = 1 - \frac{1}{a}(p_H - \lambda)$. $\alpha_L = 1$ also implies that even the consumer L with the lowest willingness to pay ($v_L = 0$) wants to buy the product, and thus the price charged to consumer L cannot exceed the expected network value: $p_L \leq \lambda\alpha_H$.

The firm thus chooses p_L and p_H to maximize $\Pi = \pi + CS$, where $\pi = p_L + \alpha_H p_H$ and $CS = \int_0^1 (v - p_L)dv + \int_{p_H-\lambda}^{a} \frac{1}{a}(v - p_H)dv + 2\alpha_H\lambda$. Taking the first-order derivative of Π with respect to p_H and setting it to zero yields the optimal price:

$$p_H^* = \frac{a(1-\gamma) + \lambda(1-2\gamma)}{2-\gamma}. \tag{A2}$$

The resulting consumer H's purchase probability is thus

$$\alpha_H^* = \frac{a + \lambda + \gamma\lambda}{a(2-\gamma)}. \tag{A3}$$

Thus, to ensure that $\alpha_H^* < 1$ we need

$$\gamma < \bar{\gamma}_H \equiv \frac{a-\lambda}{a+\lambda}. \tag{A4}$$

Simple comparison yields $\bar{\gamma}_L < \bar{\gamma}_H$, where $\bar{\gamma}_L$ and $\bar{\gamma}_H$ are given by equations (A1) and (A4) respectively. Taking the first-order derivative of Π with respect to p_L yields $\frac{\partial \Pi}{\partial p_L} = 1-\gamma$. Since $\bar{\gamma}_L < \bar{\gamma}_H < 1$, $\frac{\partial \Pi}{\partial p_L} > 0$. So the firm optimally sets the highest possible price for consumer L, namely, $p_L^* = \lambda \alpha_H^*$, where α_H^* is given by equation (A3).

Inserting into p_L^*, α_H^*, and p_H^* into the material payoff yields

$$\pi(\gamma) = -\frac{(\gamma-1)(a+3\lambda)(a+\gamma\lambda+\lambda)}{a(\gamma-2)^2}. \tag{A5}$$

Case 3: $\alpha_L = \alpha_H = 1$. We finally discuss the case in which both consumers certainly buy the product: $\alpha_L = \alpha_H = 1$, which can only occur when $\gamma > \bar{\gamma}_H$. When both consumers always buy the product, we must have $p_L \leq \lambda$ and $p_H \leq \lambda$. The firm thus chooses p_L and p_H to maximize $\Pi = \pi + \gamma \cdot CS$, where $\pi = p_L + p_H$ and $CS = \int_0^1 (v-p)dv + \int_0^a \frac{1}{a}(v-p)dv + 2\lambda$. Taking the derivative of Π with respect to p_L and p_H yields $\frac{\partial \Pi}{\partial p_L} = \frac{\partial \Pi}{\partial p_H} = 1-\gamma$. We thus discuss the following two subcases.

(1) If $\bar{\gamma}_H < \gamma < 1$ so that $\frac{\partial \Pi}{\partial p_L} = \frac{\partial \Pi}{\partial p_H} > 0$, the optimal prices are $p_L = p_H = \lambda$ and the firm makes optimal profit $\pi = 2\lambda$.

(2) If $\gamma > 1$ so that $\frac{\partial \Pi}{\partial p_L} = \frac{\partial \Pi}{\partial p_H} < 0$, the firm freely provides the good to the two consumers, i.e., $p_L = p_H = 0$, and makes zero material payoff. Clearly, $\gamma > 1$ is always suboptimal for the profit-maximizing firm.

Proof of Proposition 1

We have characterized the equilibrium firm prices and consumer purchase for a given γ in Section 3.1. Now we first prove that the optimal social responsibility γ^* must be positive. Inserting the equilibrium consumer purchase probability (9)-(10) and prices (11)-(12) into the material payoff function $\pi = p_L\alpha_L + p_H\alpha_H$, we obtain the profit as given by (13). Taking the first-order derivative of π with respect to γ yields

$$\frac{d\pi}{d\gamma} = \frac{a\begin{pmatrix} a^2(2-\gamma)\begin{pmatrix}\gamma^3\left(2\lambda^2+2\lambda-1\right)+\gamma^2\left(-10\lambda^2+12\lambda+4\right)\\ -4\gamma\left(5\lambda^2+9\lambda+1\right)+2\lambda(5\lambda+4)\end{pmatrix}\\ +a\lambda^2\begin{pmatrix}\gamma^4\left(\lambda^2+4\lambda-2\right)+\gamma^3\left(-8\lambda^2+12\lambda+14\right)\\ -12\gamma^2\lambda(\lambda+5)+\gamma\left(4\lambda^2-32\lambda-50\right)+7\lambda^2+36\lambda+20\end{pmatrix}\\ -a^3(2-\gamma)^3\gamma-(\gamma+1)^2\lambda^4\left(\gamma^2(2\lambda-1)+10\gamma-2\lambda-7\right)\end{pmatrix}}{\left(a(2-\gamma)^2-\lambda^2(1+\gamma)^2\right)^3}. \tag{A6}$$

Setting $\gamma \to 0$ yields

$$\frac{d\pi}{d\gamma}\mid_{\gamma\to 0} = \frac{a\lambda\left(4a^2(5\lambda+4)+a\left(7\lambda^2+36\lambda+20\right)\lambda+(2\lambda+7)\lambda^3\right)}{\left(4a-\lambda^2\right)^3} > 0.$$

Therefore, the optimal social responsibility must be positive, i.e., $\gamma^* > 0$.

We next show that the equilibrium social responsibility is unique. When $\gamma < \bar{\gamma}_L$, denote the optimal choice of γ as γ''. When $\alpha_L = 1$ and $\alpha_H < 1$, as discussed in the full equilibrium characterization of Section 3.1 in the appendix, the firm's material payoff is $\pi = p_L + \alpha_H p_H = \alpha_H(\lambda + p_H)$, which can be simplified to the following:

$$\pi(\gamma) = \frac{(1-\gamma)(a+3\lambda)(a+\gamma\lambda+\lambda)}{a(2-\gamma)^2}. \tag{A7}$$

We can show that in this case the profit function is hump-shaped and peaks at $\gamma = \gamma' \equiv \frac{2\lambda}{a+4\lambda}$. Therefore, the firm's optimal choice must be from γ', $\bar{\gamma}_L$, and $\bar{\gamma}_H$.

Finally, when $\alpha_L = \alpha_H = 1$, then the firm's material payoff is independent of γ. That is, when $\bar{\gamma}_H < \gamma < 1$, $\pi = 2\lambda$, and when $\gamma > 1$, $\pi = 0$.

We have shown that a profit-maximizing firm will not choose $\gamma > 1$. Taking into account of the convex constraint $0 \leq \gamma \leq 1$, the firm's optimal choice of social responsibility must be within $\{\bar{\gamma}_L, \bar{\gamma}_H, \bar{\gamma}', \bar{\gamma}''\}$.

Proof of Lemma 1

First note that the firm's material payoff does not directly depend on its social responsibility γ. We thus express the the material payoff as a function of the product price p charged by the firm. Inserting consumers' optimal purchase strategies (15) into the material payoff $\pi = p(\alpha_L + \alpha_H)$, we obtain

$$\pi(p) = \frac{p\big(\lambda + a(2+\lambda) - (1+a+2\lambda)p\big)}{a-\lambda^2}. \tag{A8}$$

Taking the first-order derivative of $\pi(p)$ with respect to p and setting it to zero yields

$$p^U = \frac{1}{2}\frac{2a+\lambda+a\lambda}{1+a+2\lambda}. \tag{A9}$$

Since $\pi(p)$ is a concave function, i.e., $\frac{\partial^2\pi}{\partial p^2} = -\frac{2(1+a+2\lambda)}{1-\lambda^2} < 0$, p^U is the unique price that maximizes the firm's material payoff. The optimal material payoff is

$$\pi(p^U) = \frac{(\lambda + a(2+\lambda))^2}{4(1+a+2\lambda)(a-\lambda^2)}. \tag{A10}$$

Next, denoting the optimal price function in equation (16) as $p = P(\gamma)$. Taking the first-order derivative with respect to γ yields

$$\begin{aligned}\frac{\partial P}{\partial \gamma} &= -\frac{(a-\lambda)^2\big(a^2\left(-\lambda^3+6\lambda^2+6\lambda+2\right)+a^3(3\lambda+2)+3a\lambda\left(2\lambda^2+2\lambda+1\right)-\lambda^3\big)}{\big(((1+a)\lambda^2+4a\lambda+a(1+a))\gamma+4\lambda^3+2(1+a)\lambda^2-4a\lambda-2a(1+a)\big)^2}\\ &< -\frac{4(a-\lambda)^2(1+\lambda)^3}{\big(((1+a)\lambda^2+4a\lambda+a(1+a))\gamma+4\lambda^3+2(1+a)\lambda^2-4a\lambda-2a(1+a)\big)^2} < 0,\end{aligned}$$

where the first inequality follows $a > 1$. Therefore, p is a monotonically decreasing function of γ. So, we solve for the optimal social responsibility via the inverse function: $\gamma^U = P^{-1}(p^U) = 0$; that is, the firm won't commit to any corporate social responsibility. Finally, using consumers' optimal purchase probability (15) we obtain that consumer L makes the purchase if and only if $v_L \geq \frac{(p^U-\lambda)(a+\lambda)}{a-\lambda^2}$ and consumer H makes the purchase if and only if $v_H \geq \frac{a(p^U-\lambda)(1+\lambda)}{a-\lambda^2}$.

Finally, as mentioned in Footnote 6, we only focus on the case in which γ is not too large so that $\alpha_L, \alpha_H < 1$ in the main text. Under uniform pricing, this occurs when

$$\gamma < \bar{\gamma}^U \equiv \frac{a(2-\lambda)-\lambda(1+4\lambda)}{\lambda+a(2+\lambda)}. \tag{A11}$$

The threshold $\bar{\gamma}^U$ is obtained by inserting (16) into equations (15) and solving $\alpha_L < 1$ and $\alpha_H < 1$. Noting that under uniform pricing $\alpha_L < 1$ and $\alpha_H < 1$ feature the same upper bound for γ. Therefore, the remaining case to be discussed is that $\alpha_L = \alpha_H = 1$, which occurs when $\gamma > \bar{\gamma}^U$.

When $\gamma > \bar{\gamma}^U$, both consumers buy the product for sure, which implies $p \leq \lambda$, i.e., the consumer with the lowest basic willingness to pay would like to buy the product. The firm's stated objective function is $\Pi = 2p + \int_0^1 (v-p)dv + \int_0^a \frac{1}{a}(v-p)dv + 2\lambda$, which can be simplified to

$$\Pi = 2p + \frac{\gamma}{2}(1+a-4p+4\lambda).$$

Taking the first-order derivative of Π with respect to p yields $\frac{\partial \Pi}{\partial p} = 2(1-\gamma)$. Noting that $\bar{\gamma}^U < 1$. We then discuss the following two subcases:

(1) If $\bar{\gamma}^U < \gamma < 1$, $\frac{\partial \Pi}{\partial p} > 0$. So the firm will choose the highest possible price: $p = \lambda$, and the resultant material payoff is $\pi = 2\lambda$.

(2) If $\gamma > 1$, $\frac{\partial \Pi}{\partial p} < 0$. So the optimal price is $p = 0$; that is, the firm freely provides the product to consumers. Thus, the resultant material payoff is 0.

Since the firm ultimately cares about the material payoff, Case (2) is obviously suboptimal to the firm compared with Case (1). Because the optimal material payoff under $\gamma < \bar{\gamma}^U$ is given by $\pi(p^U)$ in equation (A10) and

$$\pi(p^U) - 2\lambda = \frac{(a(2-\lambda)-\lambda(1+4\lambda))^2}{4(1+a+2\lambda)(a-\lambda^2)} > 0,$$

to choose $\gamma \in (\bar{\gamma}^U, 1)$ is suboptimal for the profit-maximizing firm. Overall, as stated in Footnote 6, it suffices to focus on the region in which γ is not too large, i.e., $\gamma < \bar{\gamma}^U$, when characterizing the firm's optimal choice of social responsibility under uniform pricing.

Proof of Lemma 2

The derivation is similar to that in the proof of Lemma 1. Noting that the firm's material payoff π does not directly depend on γ, we can insert equations (17) and (18) into $\pi = p_L\alpha_L + p_H\alpha_H$ and express π as a function of p_L and p_H as following:

$$\pi(p_L, p_H) = \frac{p_H\left(a + \lambda - 2\lambda p_L\right) + ap_L\left(\lambda - p_L + 1\right) - p_H^2}{a - \lambda^2}.$$

Taking the derivative of $\pi(p_L, p_H)$ with respect to p_L and setting it to zero yields $p_L(p_H) = \frac{a(1+\lambda)-2\lambda p_H}{2a}$. The second-order condition is $\frac{\partial\pi(p_L,p_H)}{\partial p_L^2} = -\frac{2a}{a-\lambda^2} < 0$. Similarly, maximizing $\pi(p_L, p_H)$ with respect to p_H generates $p_H(p_L) = \frac{a+\lambda-2\lambda p_L}{2}$. The second-order condition is $\frac{\partial\pi(p_L,p_H)}{\partial p_H^2} = -\frac{2}{a-\lambda^2} < 0$. The interaction of the two best response functions yields

$$p_L^T = \frac{1}{2} \text{ and } p_H^T = \frac{a}{2}, \tag{A12}$$

which are the prices that maximize the firm's material payoff. Denote equations (19) and (20) as $p_L = P_L(\gamma)$ and $p_H = P_H(\gamma)$. Setting $\gamma = 0$ in equations (19) and (20) reproduce the optimal prices (A12) for the firm. We next will show that this is the unique γ that generates these optimal prices.

Taking the first-order derivative of $P_L(\gamma)$ in equation (19) with respect to γ generates

$$\frac{\partial P_L(\gamma)}{\partial\gamma} = a\frac{-a\gamma^2(1-\lambda) + 4\gamma\left(a - \lambda^2\right) - 4(\lambda+1)\left(a - \lambda^2\right)}{\left(a(2-\gamma)^2 - 4\lambda^2\right)^2} < 0.$$

Similarly, taking the first-order derivative of $P_H(\gamma)$ in equation (20) with respect to γ generates

$$\frac{\partial P_H(\gamma)}{\partial \gamma} = a\frac{-a\gamma^2(a-\lambda) - 4a\gamma\left(a-\lambda^2\right) - 4\left(a^2 + a(1-\lambda)\lambda - \lambda^3\right)}{\left(a(2-\gamma)^2 - 4\lambda^2\right)^2} < 0.$$

Therefore, p_L and p_H are both monotonically decreasing functions of γ. And we confirm that $P_L^{-1}(p_L^T) = 0$ and $P_H^{-1}(p_H^T) = 0$.

Finally, inserting equations (19) and (20) into (17) and setting $\alpha_L < 1$ yields $\gamma < \frac{3}{2} - \frac{\sqrt{a+8a\lambda+16\lambda^2}}{2\sqrt{a}}$. Similarly, inserting equations (19) and (20) into (18) and setting $\alpha_H < 1$ yields $\gamma < \bar{\gamma}^T \equiv \frac{3}{2} - \frac{\sqrt{a+8\lambda(1+2\lambda)}}{2\sqrt{a}} < \frac{3}{2} - \frac{\sqrt{a+8a\lambda+16\lambda^2}}{2\sqrt{a}}$. Thus, when $\gamma < \bar{\gamma}^T$ we indeed have $\alpha_L, \alpha_H < 1$.

Proof of Proposition 2

When $\lambda = 0$, the firm's material payoff function (13) is simplified to $\pi(\gamma) = \frac{(1+a)(1-\gamma)}{(2-\gamma)^2}$. Note that to sustain positive profit, we need $\gamma < 1$. Taking the first-order derivative with respect to γ yields $\frac{d\pi}{d\gamma} = -\frac{(1+a)\gamma}{(2-\gamma)^3} < 0$. Therefore, the optimal choice of social responsibility is $\gamma^* = 0$. This proves Part (1) of the proposition.

Denote the right-hand-side of equation (A6) as $G(\gamma, \lambda)$. Thus, the optimal social responsibility γ^* is determined by the equation $G(\gamma, \lambda) = 0$. Via implicit function theorem, we can derive

$\frac{\partial \gamma}{\partial \lambda} = -\frac{\partial G/\partial \lambda}{\partial G/\partial \gamma}$, which is simplified as follows:

$$\frac{\partial \gamma}{\partial \lambda} = \frac{\begin{pmatrix} -a^2(\gamma-2)\lambda \begin{pmatrix} \gamma^5\left(2\lambda^2-\lambda-1\right)+2\gamma^4\left(4\lambda^2+17\lambda-6\right) \\ +\gamma^3\left(-84\lambda^2+101\lambda+37\right)-2\gamma^2\left(68\lambda^2+131\lambda-19\right) \\ +2\gamma\left(\lambda^2-100\lambda-66\right)+8\left(6\lambda^2+16\lambda+5\right) \end{pmatrix} \\ +a^3(\gamma-2)^3\left(\gamma^3(\lambda-1)+2\gamma^2(8\lambda-3)+\gamma(23\lambda+18)-2(5\lambda+2)\right) \\ +a(\gamma+1)^2\lambda^3 \begin{pmatrix} \gamma^4\left(\lambda^2+\lambda-2\right)-3\gamma^2\left(4\lambda^2+35\lambda-34\right) \\ +2\gamma^3(19-4\lambda)\lambda+4\gamma\left(\lambda^2-17\lambda-59\right) \\ +7\lambda^2+74\lambda+96 \end{pmatrix} \\ -(\gamma+1)^4\lambda^5\left(\gamma^2(\lambda-1)+10\gamma-\lambda-7\right) \end{pmatrix}}{\begin{pmatrix} 3a^2\lambda^2 \begin{pmatrix} \gamma^5\left(\lambda^2+2\lambda-1\right)+\gamma^4\left(-11\lambda^2+12\lambda+9\right) \\ -2\gamma^3\left(6\lambda^2+46\lambda+1\right)+\gamma^2\left(76\lambda^2+32\lambda-70\right) \\ +\gamma\left(31\lambda^2+216\lambda+96\right)-45\lambda^2-80\lambda-8 \end{pmatrix} \\ -a^3(\gamma-2)^2 \begin{pmatrix} \gamma^3\left(3\lambda^2+2\lambda-1\right)+\gamma^2\left(-15\lambda^2+24\lambda+3\right) \\ -6\gamma\lambda(11\lambda+8)+6\lambda^2-16\lambda-4 \end{pmatrix} \\ -a(\gamma+1)\lambda^4 \begin{pmatrix} \gamma^4\left(\lambda^2+6\lambda-3\right)+\gamma^3\left(-14\lambda^2+18\lambda+36\right) \\ -6\gamma^2\lambda(2\lambda+27)+\gamma\left(22\lambda^2-30\lambda-228\right) \\ +19\lambda^2+144\lambda+135 \end{pmatrix} \\ +a^4(\gamma-2)^4(\gamma+1)+(\gamma+1)^3\lambda^6\left(\gamma^2(2\lambda-1)-2\gamma(\lambda-8)-4\lambda-19\right) \end{pmatrix}}.$$

Setting $\lambda = 0$ we obtain that $\frac{\partial \gamma}{\partial \lambda} \mid_{\lambda=0} = \frac{2-8\gamma-\gamma^2}{(1+a)(1+\gamma)} = \frac{2}{1+a} > 0$, where the second equality follows $\gamma^* = 0$. That is, when $\lambda = 0$, a marginal increase in λ induces the firm to choose a higher level of social responsibility in equilibrium. This proves Part (2) of the proposition.

Finally, using the envelope theorem we can compute the derivative of the optimal profit function as:

$$\frac{\partial \pi(\gamma;\lambda)}{\partial \lambda}\mid_{\gamma=\gamma^*}= -\frac{2a\begin{pmatrix} (\gamma^*)^5(\lambda-1)\left(a^2-a(\lambda+1)\lambda+\lambda^3\right) \\ +(\gamma^*)^4\left(a^2(\lambda+4)+a\left(\lambda^2-12\lambda+1\right)\lambda+(4\lambda+1)\lambda^3\right) \\ +(\gamma^*)^3\left(a^2(2-15\lambda)+a\left(8\lambda^2+6\lambda-15\right)\lambda+2(3\lambda+4)\lambda^3\right) \\ +(\gamma^*)^2\left(a^2(13\lambda-28)+a\left(8\lambda^2+36\lambda+13\right)\lambda+4(\lambda+2)\lambda^3\right) \\ +\gamma^*\left(8a^2(2\lambda+5)+a\left(\lambda^2+6\lambda+16\right)\lambda+(\lambda+1)\lambda^3\right) \\ -4a^2(3\lambda+4)-a\lambda\left(\lambda^2+12\lambda+12\right)-\lambda^3 \end{pmatrix}}{\left((\gamma^*)^2\left(a-\lambda^2\right)-2\gamma^*\left(2a+\lambda^2\right)+4a-\lambda^2\right)^3}.$$

Setting $\lambda=0$ and $\gamma^*=0$ yields $\frac{\partial \pi(\gamma;\lambda)}{\partial \lambda}\mid_{\gamma=\gamma^*}=\frac{1}{2}>0$. Therefore, when λ is sufficiently small, a marginal increase in λ increases the firm's material payoff.

Furthermore, inserting equations (9) through (12) into equation (6), we obtain the equilibrium consumer surplus as

$$CS=\frac{a\left(a^2(\gamma-2)^2+a\left(\gamma^2\left(\lambda^2-4\lambda+1\right)+2\gamma\left(\lambda^2+2\lambda-2\right)+\lambda^2+8\lambda+4\right)+(\gamma+1)^2\lambda^2\right)}{2\left(a(\gamma-2)^2-(\gamma+1)^2\lambda^2\right)^2}. \tag{A13}$$

Taking the total derivative of CS with respect to λ and setting $\lambda=0$ and $\gamma=0$ yields $\frac{\partial CS}{\partial \lambda}\mid_{\lambda=0}=\frac{1}{4}>0$. Therefore, when λ is sufficiently small, a marginal increase in λ increases consumer surplus. Taken together, when λ is sufficiently small, a marginal increase in λ must increase social welfare as well. This completes the proof of Part (3) of the proposition.

Proof of Proposition 3

We solve the game using backward induction.

Case 1: $\alpha_L<1$. If consumer L makes the purchase, then consumer H purchases as well when $v_H\geq p_H-\lambda$, which occurs with probability $\alpha_H^{\text{L buy}}=1-\frac{1}{a}\left(p_H-\lambda\right)$ since v_H is assumed to follow uniform distribution $U[0,a]$. Accordingly, the firm will set p_H to maximize its stated objective function: $\left(1-\frac{1}{a}\left(p_H-\lambda\right)\right)p_H+\gamma\cdot\int_{p_H-\lambda}^{a}\frac{1}{a}(v_H-p_H+2\lambda)dv_H$ (note that the part of the stated objective function that is associated with consumer L has sunk at this stage), which yields the following optimal pricing $p_H^{\text{L buy}}=\frac{a(1-\gamma)+\lambda(1-2\gamma)}{2-\gamma}$. Inserting $p_H^{\text{L buy}}$ into the consumer's purchase probability yields $\alpha_H^{\text{L buy}}=\frac{a+(1+\gamma)\lambda}{a(2-\gamma)}$.

If consumer L does not make the purchase, then consumer H purchases only when $v_H\geq p_H$, which occurs with probability $\alpha_H^{\text{L not buy}}=1-\frac{1}{a}p_H$. The firm then chooses p_H to maximize $(1-\frac{1}{a}p_H)p_H+\gamma\cdot\int_{p_H}^{a}\frac{1}{a}(v_H-p_H)dv_H$, yielding $p_H^{\text{L not buy}}=\frac{a(1-\gamma)}{2-\gamma}$. Inserting $p_H^{\text{L not buy}}$ into the consumer's purchase probability yields $\alpha_H^{\text{L not buy}}=\frac{1}{2-\gamma}$.

Next we study consumer L's problem. The utility consumer L derives from the product purchase is $v_L+(1-\frac{1}{a}(p_H^{\text{L buy}}-\lambda))\lambda-p_L$. And she buys the product if and only if this utility is higher or equal to zero, which happens when $v_L\geq\bar{v}_L$, where

$$\bar{v}_L=p_L-\frac{\lambda(a+\lambda+\gamma\lambda)}{a(2-\gamma)}. \tag{A14}$$

Consumer L thus buys the product with probability $\alpha_L=1-\bar{v}_L$. Understanding consumer L's purchase strategy, the firm will choose p_L to maximize the stated objective function $\Pi=\pi+\gamma\cdot CS$, where

$$\pi=(1-\bar{v}_L)(p_L+(1-\frac{1}{a}(p_H^{\text{L buy}}-\lambda))p_H^{\text{L buy}})+\bar{v}_Lp_H^{\text{L not buy}}(1-\frac{1}{a}p_H^{\text{L not buy}}), \tag{A15}$$

and

$$\begin{aligned}CS=&\int_{\bar{v}_L}^{1}\int_{p_H^{\text{L buy}}-\lambda}^{a}(v_L-p_L+v_H-p_H^{\text{L buy}}+2\lambda)dv_Hdv_L\\&+\int_{\bar{v}_L}^{1}\int_{0}^{p_H^{\text{L buy}}-\lambda}(v_L-p_L)dv_Hdv_L+\int_{0}^{\bar{v}_L}\int_{p_H^{\text{L not buy}}}^{a}(v_H-p_H^{\text{L not buy}})dv_Hdv_L.\end{aligned}$$

Maximizing the stated objective function yields the optimal pricing p_L for consumer 1 as following:

$$p_L = \frac{\begin{pmatrix} -2a^2\gamma\left(\gamma^2-4\gamma+3\lambda+4\right)+2(\gamma+1)^2\lambda^2 \\ +a\left(\gamma^3\lambda^2+\gamma^2\left(-4\lambda^2+2\lambda+2\right)+\gamma\left(-5\lambda^2+2\lambda-8\right)+8\right) \end{pmatrix}}{2a(2-\gamma)^2(2-a\gamma)}. \tag{A16}$$

Next, inserting $p_H^{\text{L buy}}$, $p_H^{\text{L not buy}}$, and (A16) into (A15) we obtain the material payoff π as a function of γ. Taking the derivative of π with respect to γ and then setting $\gamma \to 0$ yields

$$\frac{d\pi}{d\gamma}\mid_{\gamma\to 0} = \frac{\lambda\left(9\lambda^3+4a^2(1+\lambda)+3a\lambda(4+5\lambda)\right)}{32a^2} > 0.$$

And the material payoff at $\gamma = 0$ is

$$\pi^{\text{Case 1}} = \frac{1}{64}\left(\frac{9\lambda^4}{a^2}+\frac{24(\lambda+1)\lambda^2}{a}+16a+16(\lambda+1)^2\right). \tag{A17}$$

Therefore, the optimal social responsibility must be positive, i.e., $\gamma^* > 0$.

We finally examine the consumers' purchase probability. When $\gamma \to 0$, consumer 2's purchase probabilities satisfy $\lim_{\gamma\to 0}\alpha_H^{\text{L buy}} = \frac{a+\lambda}{2a} < 1$ and $\lim_{\gamma\to 0}\alpha_H^{\text{L not buy}} = \frac{1}{2} < 1$. Consumer 1's purchase probability $\lim_{\gamma\to 0}\alpha_L = \frac{3\lambda^2}{8a} + \frac{1+\lambda}{2}$, which is strictly less than 1 when $a > \frac{3\lambda^2}{4(1-\lambda)}$, or equivalently, $\lambda < \frac{2(\sqrt{a(a+3)}-a)}{3}$.

Case 2: $\alpha_L = 1$**.** As discussed in Case 1, when consumer 1 buys the product, consumer 2 buys the product with probability $\alpha_H^{\text{L buy}}$ and the firm sets the price $p_H^{\text{L buy}}$ for consumer 2. Since $\alpha_L = 1$, the consumer 1 with the lowest willingness to pay wants to buy the product as well, suggesting that $p_L \leq \lambda\alpha_H^{\text{L buy}}$. The firm thus chooses p_L to maximize $\Pi = \pi + \gamma \cdot CS$, where $\pi = p_L + \alpha_H^{\text{L buy}} p_H^{\text{L buy}}$ and

$$CS = \int_0^1\int_{p_H^{\text{L buy}}-\lambda}^{a}(v_L - p_L + v_H - p_H^{\text{L buy}} + 2\lambda)dv_H dv_L + \int_0^1\int_0^{p_H^{\text{L buy}}-\lambda}(v_L - p_L)dv_H dv_L.$$

Taking the derivative of Π with respect to p_L yields $\frac{\partial \Pi}{\partial p_L} = 1 - a\gamma$. When $\gamma < \frac{1}{a}$, we know that $\lim_{\gamma \to 0} \frac{\partial \Pi}{\partial p_L} = 1 - a\gamma > 0$ regardless of the value of a.[7] So the firm will charge the highest possible price $p_L^* = \lambda \alpha_H^{\text{L buy}} = \lambda \frac{a+(1+\gamma)\lambda}{a(2-\gamma)}$. Inserting p_L^* into the material payoff yields

$$\pi = \frac{(1-\gamma)(a+3\lambda)(a+\gamma\lambda+\lambda)}{a(2-\gamma)^2}.$$

Taking the derivative of π with respect to γ and setting $\gamma \to 0$ yields $\frac{\partial \pi}{\partial p_L} = \frac{\lambda(a+3\lambda)}{4a} > 0$. The material payoff at $\gamma = 0$ is

$$\pi^{\text{Case 2}} = \frac{(a+\lambda)(a+3\lambda)}{4a}. \tag{A18}$$

Therefore, the optimal social responsibility $\gamma^* > 0$. A comparison of the material payoff (A17) and (A18) across the two cases yields that $\pi^{\text{Case 1}} > \pi^{\text{Case 2}}$. Therefore, when $\gamma \to 0$, the firm would like to induce $\alpha_L = 1$ only when $a < \frac{3\lambda^2}{4(1-\lambda)}$.

Proof of Proposition 4

Denote the consumer of type t that is indifferent between buying or not as $\bar{v}_t$. Then the purchase probability of the type t consumers is

$$\alpha_t = 1 - \frac{\bar{v}_t}{1+t}. \tag{A19}$$

Therefore, in equilibrium the total size of consumers N^* can be computed as follows:

$$N^* = \int_0^a \alpha_t df(t). \tag{A20}$$

For the type t consumer with the basic willingness to pay $\bar{v}_t$, the indifference condition sug-

[7]When $\gamma > \frac{1}{a}$, $\lim_{\gamma \to 0} \frac{\partial \Pi}{\partial p_L} < 0$ and thus the firm freely provides the good to consumer 1, i.e., $p_L^* = 0$. This case must be suboptimal to the firm's material payoff because the firm loses the profit from consumer 1.

gests that

$$\bar{v}_t = p_t - \lambda N^*. \tag{A21}$$

Thus, given γ, the firm chooses prices to maximize the stated objective function

$$\Pi = \int_0^a \alpha_t p_t df(t) + \gamma \cdot \lambda \int_0^a \left(\int_{\bar{v}_t}^{1+t} (v - p_t) dv \right) df(t) + \gamma \cdot \lambda (N^*)^2. \tag{A22}$$

Inserting (A19) and (A21) into (A22), maximizing (A22) with respect to p_t, and setting it to zero yields the the optimal price p_t as following (note that p_t affects N^* because $N^* = \int_{t' \neq t} \alpha_{t'} df(t') + \alpha_t$):

$$p_t = \frac{\gamma\lambda + (2\gamma - 1)\lambda N^* + \gamma\lambda t^2 + t(2\gamma\lambda - 1) - 1}{\gamma\lambda(t+1) - 2}. \tag{A23}$$

replacing p_t in equation (A19) yields the equilibrium purchase probability for the type-t consumers:

$$\alpha_t = \frac{\lambda N^*(\gamma(\lambda - 2) - 1) + t\left(\gamma\lambda^2 N^* - 1\right) - 1}{(t+1)(\gamma\lambda(t+1) - 2)}. \tag{A24}$$

Finally, the firm chooses γ to maximize the material payoff $\pi = \int_0^a p_t \alpha_t dt$, where p_t and α_t are given by (A23) and (A24) respectively. Taking the derivative of π with respect to γ and setting $\gamma \to 0$ yields

$$\frac{d\pi}{d\gamma} \mid_{\gamma \to 0} = \int_0^a \frac{(1 + t + N^*\lambda)^2}{4(1+t)} dt > 0.$$

Thus, the firm's optimal choice of social responsibility must be positive, i.e., $\gamma^* > 0$.

Proof of Proposition 5

Given γ, the subgame is the same as in the baseline model. we thus only need to derive the value of the true objective function $Z(\gamma; w)$ for a given w, which is as following:

(1) If $\bar{\gamma}_L < \gamma < \bar{\gamma}_H$,

$$Z(\gamma; w) = \frac{a \begin{pmatrix} a^2(\gamma-2)^2(-2\gamma+w+2) \\ +2a \begin{pmatrix} \gamma^3(\lambda^2+2\lambda-1)+\gamma^2(5-3\lambda^2) \\ -\gamma(3\lambda^2+12\lambda+8)+\lambda^2+8\lambda+4 \end{pmatrix} \\ +aw \begin{pmatrix} \gamma^2(\lambda^2-4\lambda+1)+2\gamma(\lambda^2+2\lambda-2) \\ +\lambda^2+8\lambda+4 \end{pmatrix} \\ +(\gamma+1)\lambda^2 \begin{pmatrix} \gamma^2(2-4\lambda)-4\gamma(\lambda+2) \\ +(\gamma+1)w+2 \end{pmatrix} \end{pmatrix}}{2\left(a(\gamma-2)^2-(\gamma+1)^2\lambda^2\right)^2}. \tag{A25}$$

Taking the derivative of $Z(\gamma, w)$ in (A25) with respect to γ and setting $\gamma = w$ yields

$$\frac{\partial Z(\gamma, w)}{\partial \gamma} \mid_{\gamma=w} = \frac{a\lambda(1-w)\Omega}{\left(a(w-2)^2-\lambda^2(w+1)^2\right)^3} > 0, \tag{A26}$$

where

$$\Omega = 2(\lambda-1)w^3\left(a^2-a(\lambda+1)\lambda+\lambda^3\right)-3w^2\left(a^2(\lambda-4)+a\left(-\lambda^3+6\lambda^2+\lambda\right)-(2\lambda+1)\lambda^3\right)$$
$$+4a^2(5\lambda+4)+6(\lambda+2)w\left(-2a^2+a(2\lambda-1)\lambda+\lambda^3\right)+a\left(7\lambda^2+36\lambda+20\right)\lambda+(2\lambda+7)\lambda^3,$$

and the inequality follows because Ω is decreasing in w and $\Omega(1) > 0$.

(2) If $\bar{\gamma}_L < \gamma < \bar{\gamma}_H$,

$$Z(\gamma;w) = \frac{\begin{pmatrix} a^2(-2\gamma+w+2) - 2a\left(\gamma^2+3\gamma-4\right)\lambda \\ +aw\left(\gamma^2+2\gamma(\lambda-2)+2(\lambda+2)\right) \\ +(\gamma+1)\lambda^2(-6\gamma+\gamma w+w+6) \end{pmatrix}}{2a(\gamma-2)^2}. \tag{A27}$$

Taking the derivative of $Z(\gamma, w)$ in (A27) with respect to γ and setting $\gamma = w$ yields

$$\frac{\partial Z(\gamma, w)}{\partial \gamma} \mid_{\gamma=w} = \frac{\lambda(1-w)(a+3\lambda)}{a(w-2)^2} > 0, \tag{A28}$$

where the inequality follows because $\gamma < \bar{\gamma}_H < 1$.

(3) If $\bar{\gamma}_H < \gamma < 1$, $Z(\gamma;w) = \frac{1}{2}(aw+4\lambda+w)$, and if $\gamma > 1$, $Z(\gamma;w) = \frac{1}{2}w(a+4\lambda+1)$. Clearly, when $\gamma > \bar{\gamma}_H$, $Z(\gamma;w)$ is independent of γ. So, when w induces the optimal γ to be higher than $\bar{\gamma}_H$, any $\gamma > \bar{\gamma}_H$ can be sustained in equilibrium.

As discussed above, if the optimal $\gamma^* < \bar{\gamma}_H$, we must have $\gamma^* > w$, whereas if $\gamma^* > \bar{\gamma}_H$, the optimal γ^* is not affected by w. We next determine the value of w that induces the optimal $\gamma^* = \bar{\gamma}_H$. Taking the derivative of $Z(\gamma;w)$ in (A27) with respect to γ and setting it to zero yields $\gamma(w) = \frac{aw+(2+w)\lambda}{a+(4-w)\lambda}$. Solving the equation $\gamma(w) = \bar{\gamma}_H$ yields $w = \bar{w} \equiv 1 - \frac{2\lambda}{a}$. As will be proved soon that $\gamma^*(w)$ is a non-decreasing function of w, we know that when $w < \bar{w}$, $\gamma^* < \bar{\gamma}_H$, whereas when $w > \bar{w}$, $\gamma^* > \bar{\gamma}_H$.

We finally complete the proof by proving that $\gamma^*(w)$ is a non-decreasing function of w. We can prove this when $\lambda \to 0$. Taking the derivative of $Z(\gamma, w)$ in (A25) with respect to γ and setting it to zero determines the optimal $\gamma^*(w)$ as a function of w. Applying the implicit function theorem to $\gamma^*(w)$ and setting $\lambda \to 0$ yields

$$\frac{\partial \gamma^*}{\partial w} \mid_{\lambda\to 0} = \frac{\gamma-2}{3w-2(\gamma+1)} > 0.$$

Similarly, we can show that when $\bar{\gamma}_L < \gamma < \bar{\gamma}_H$, $\frac{\partial \gamma^*}{\partial w} \mid_{\lambda \to 0} > 0$. And when $\gamma > \bar{\gamma}_H$, $\frac{\partial \gamma^*}{\partial w} = 0$. Thus, γ^* is non-decreasing in w.