Foreign Reserves and Capital Controls: Role of Financial Development

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Motivation

Key research questions:

- What is the optimal combination of capital controls and reserve policy?
- What explains the cross-country variation in these two policies?

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2. Developing countries used capital controls and accumulated reserves.
3. Policy coordination is important.

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Our objectives

- Provide a theory to justify the joint use of capital controls and reserves.
- Rationalize data pattern.

Our story

A liquidity role of international reserves. Evidence

- A liquidity shock hits which requires an early payment for private debts.
- Reserve holdings avoid costly liquidation of projects.

However, it is never optimal to fully self-insure against the liquidity shock b/c

- holding reserves is costly;
- benefits of holding reserves depend on the liquidation.

What is the role of policies?

- **Pecuniary externality** from liquidation.
- Price of projects is endogenous.

Optimal policies address the inefficiency by

- imposing capital controls;
- subsidizing reserve accumulation.

What we do in this paper

Motivating empirical facts about cross-country variation

- Foreign reserve-to-GDP ratio is non-monotonic in financial development: Countries with intermediate development have a high reserve-to-GDP ratio.
- Capital control index monotonically decreases in financial development index.

Small-open-economy model:

- Liquidity shock requires θ fraction of debt to be repaid before new borrowing.
- Domestic agents repay by reserves and costly asset liquidation.

Key points

- Our story can justify the joint use of capital controls and reserves.
- Our story highlights the role of financial development.

Key mechanism of our paper

- Domestic agents determine reserve holdings based on two factors:

  1. Cost of holding reserves.

  2. Relative advantage of reserve holdings over debt in liquidity management: liquidity risk measured by $-\theta b_t - s_t$.

     - One-unit reduction in debt $-b_t$ lowers liquidity risk by $\theta \leq 1$ units.

     - One-unit increase in reserves s_t lowers liquidity risk by one unit.

- High financial development (low θ) $\rightarrow$ Liquidity risk is low.

- Low financial development (high θ) $\rightarrow$ Relative advantage of reserves is low.

- Intermediate financial development $\rightarrow$ Reserve holdings become large.

- Fire-sale externality requires joint use of reserve policy and capital controls.

Relation to the literature

- the literature on reserve accumulation.
  - Jeanne and Ranciere 2011, Cespedes and Chang 2020, Matsumoto 2022, Hur and Kondo 2016, Cavallino 2019, Jeanne and Sandri 2020, etc.
- the literature on capital controls.
  - Bianchi 2011, Benigno et al. 2013, 2016, Bianchi and Mendoza 2018, Jeanne and Korinek 2020, Ma 2020, etc.
- Contributions of our work:
  - Fire-sale externality to justify joint use of capital controls and reserve policy.
  - Explain observed cross-country patterns in policy and financial development.

Motivating facts

Long-run relationship

- Data for 88 countries (economies) in $1980 - 2019$.
  - Financial development: IMF Financial Development Index
  - Foreign reserves and external liability: Lane and Milesi-Ferretti (2007)
  - Capital control index: Chinn and Ito (2006)

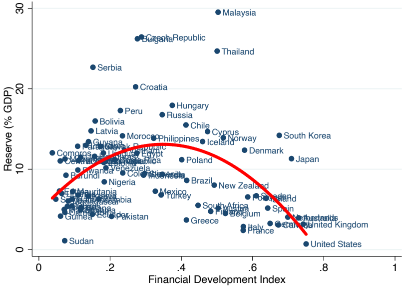
Business cycles relationship

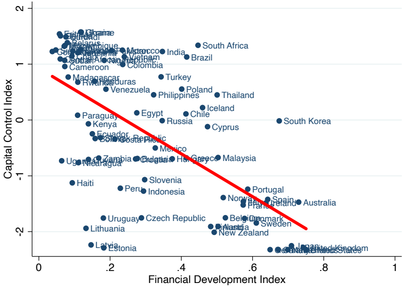
- Data for 47 emerging market economies in $1987 - 2019$.
  - Capital and reserve flows: Alfaro, Kalemli-Ozcan and Volosovych (2014)
  - EMBI spread: World Bank's Global Economic Monitor

Empirical patterns

The Financial Development Index has

- a non-monotonic relationship with reserves-to-GDP ratio;
- a negative correlation with capital control index.

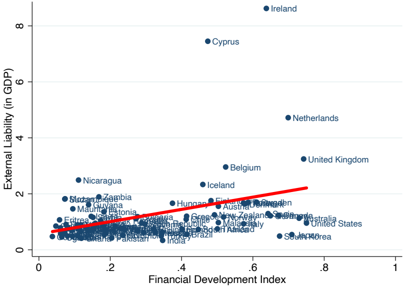


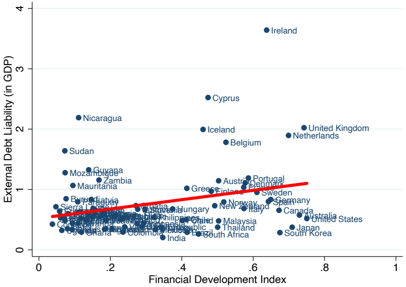


Empirical patterns (Cont.)

The Financial Development Index has

- a positive correlation with external liability.





Empirical pattern: long-run relationship

	Reserve/GDP		Capital Control Index		External Liability/GDP		External Debt Liability/GDP	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Financial Development	0.48***	0.43***	-3.83***	-3.78***	2.19***	2.19***	0.77***	0.77***
	(0.12)	(0.12)	(0.52)	(0.44)	(0.61)	(0.52)	(0.28)	(0.24)
Financial Development^2	-0.69***	-0.62***						
	(0.15)	(0.16)						
Pop (log)		-0.00		0.09		-0.34***		-0.21***
		(0.01)		(0.09)		(0.10)		(0.05)
GDP per capita (log)		-0.01		-0.57***		-0.33**		-0.14**
		(0.01)		(0.13)		(0.15)		(0.07)
Private credit		0.03		0.39		0.25		-0.11
		(0.02)		(0.40)		(0.48)		(0.22)
Trade		0.05***		-0.01		1.11***		0.32*
		(0.02)		(0.32)		(0.38)		(0.18)
Constant	0.05***	0.05***	0.93***	0.90***	0.57**	0.57***	0.52***	0.52***
	(0.02)	(0.02)	(0.19)	(0.16)	(0.23)	(0.19)	(0.10)	(0.09)
Observations	85	85	83	83	85	85	85	85
Adjusted R^2	0.186	0.282	0.396	0.570	0.124	0.368	0.074	0.333

Empirical pattern: business cycle frequency

Dep. Variables	Capital flows (% GDP)				Reserve flow (% GDP)	
	(1)	(2)	(3)	(4)	(5)	(6)
Reserve flows (% GDP)	0.57***	0.56**				
	(0.19)	(0.21)				
EMBI spread			-0.30***	-0.20***	-0.05**	-0.06**
			(0.09)	(0.07)	(0.02)	(0.03)
Population		13.76*		46.29**		2.10
		(7.01)		(19.22)		(4.94)
GDP per capita		7.33**		15.05***		-0.06
		(3.00)		(4.97)		(0.93)
Trade		-0.41		8.75*		5.97**
		(2.99)		(4.97)		(2.42)
Private credit		4.23		-7.83		-7.76***
		(3.23)		(7.92)		(2.68)
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	1269	961	663	574	664	575
Adjusted R^2	0.143	0.183	0.202	0.250	0.112	0.150

Model overview

- Small open economy with representative households:
  - Produce and consume tradable goods.
  - Borrow from abroad and hold reserves.
  - Invest to accumulate productive assets (semi-endogenous growth).
- Liquidity shock with an exogenous probability:
  - Need to repay a part of debt before new borrowing and production.
  - Repay by reserve holdings and liquidating productive assets.

Households

- Preference:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$$

- Budget constraint:

$$c_t+\frac{b_t}{R_t}+\frac{s_t}{R^s}+z_t=\underbrace{y_t}_{y_t=a_tL}+b_{t-1}+s_{t-1}+\underbrace{q_t a_t^{\ell}}_{\text{Liquidation value}}$$

  - b_t: foreign bond (negative is debt).
  - s_t: reserve holdings.
  - z_t: investment to accumulate productive assets.
  - a_t: productive assets and output, $y_t = a_t L$ with $L = 1$.
  - $q_t a_t^{\ell}$: proceeds from asset liquidation (later).

Asset accumulation and interest rate

- Law of motion for productive assets (growth):

$$a_t = a_{t-1} + \eta(z_{t-1})^{\gamma} \left[(1-\kappa)a_{t-1} + \kappa a^*_{t-1}\right]^{1-\gamma} - a^{\ell}_t$$

  - Households internalize a_t promotes future growth $\rightarrow$ No growth externality.
  - Spillover from foreign productivity a^*_t $\rightarrow$ Semi-endogenous growth.
  - a^{ℓ}_t: liquidated assets upon a liquidity shock (next slide).

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- Debt-elastic interest rate to control volatility:

$$R_t = R^b \exp(\varepsilon^R_t) + \psi^b \left[\exp\left(-\frac{b_t}{a_t} - \bar{b} \right) - 1 \right]$$

  - Households internalizes how b_t and a_t affect R_t $\rightarrow$ No externality.

Liquidity shock

- At the beginning of each period, liquidity shock may hit the economy.
  - Need to repay θ fraction of debt b_{t-1} before new borrowing and production.
  - θ: size of roll-over risk. Interpret as the measure of financial development.

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  - Reserve holdings s_{t-1}.
  - Liquidating a_t^ℓ units of productive asset to obtain liquidity $q_t a_t^\ell$.

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  - Reserve holdings s_{t-1}.
  - Liquidating a_t^ℓ units of productive asset to obtain liquidity $q_t a_t^\ell$.
- Liquidation a_t^ℓ needs to cover the liquidity shortage $-\theta b_{t-1} - s_{t-1}$:

$$q_t a_t^\ell \geq -\theta b_{t-1} - s_{t-1}$$

- Non-negative constraint on a_t^ℓ:

$$q_t a_t^\ell \geq 0$$

Fire-sale price

- Foreign buyers produce tradable goods using a_t^ℓ and a_t^*:

$$\pi_t^* = \max_{a_t^\ell} (a_t^*)^\zeta (a_t^\ell)^{1-\zeta} - F a_t^* - q_t a_t^\ell$$

  - a_t^* grows at a fixed rate $1 + \bar{g}$.

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$$\pi_t^* = \max_{a_t^\ell} (a_t^*)^\zeta (a_t^\ell)^{1-\zeta} - Fa_t^* - q_t a_t^\ell$$

  - a_t^* grows at a fixed rate $1+\bar{g}$.

- FOC determines the asset price q_t:

$$q_t = (1-\zeta)\left(\frac{a_t^*}{a_t^\ell}\right)^\zeta$$

  - Liquidation price q_t declines as aggregate liquidation a_t^ℓ increases.
    But individual households take q_t as given $\rightarrow$ Fire-sale externality.
  - Fa_t^*: Entry cost to enter the domestic asset market.
    Introduced to eliminate asset sales in normal times.

Decentralized equilibrium

- Households' maximization problem, taking q_t as given:

$$
\begin{aligned}
&V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) \\
&= \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) \\
&- \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q_t a^\ell_t \right] \\
&- \xi_t \left[a_t - a_{t-1} - \eta z^\gamma_{t-1} \left[(1-\kappa) a_{t-1} + \kappa a^*_{t-1} \right]^{1-\gamma} + a^\ell_t \right] \\
&+ \psi_t \left[q_t a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] \\
&+ \varphi_t q_t a^\ell_t \\
&+ \nu_t \frac{s_t}{R^s}
\end{aligned}
$$

  - $\theta_t = \{0, \theta\}$ is a liquidity shock.
  - Non-negativity constraint on liquidation a^ℓ_t and reserves s_t.

Optimality condition for liquidation a_t^ℓ

FOCs

$$a_t^\ell : \psi_t + \varphi_t = \frac{\xi_t}{q_t} - u'(c_t)$$

- When $a_t^\ell > 0$, liquidity constraint binds. $\psi_t > 0$ and $\varphi_t = 0$.
- When $a_t^\ell = 0$, non-negativity constraint binds. $\psi_t = 0$ and $\varphi_t > 0$.

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- When $a_t^\ell = 0$, non-negativity constraint binds. $\psi_t = 0$ and $\varphi_t > 0$.
- ψ_t: *private* value of one unit of liquidity when the liquidity constraint binds:
  - It reduces liquidation a_t^ℓ by $1/q_t$ units, whose value is ξ_t/q_t.
  - It also reduces available resources by one unit, whose value is $-u'(c_t)$.

Optimality conditions for debt and reserves

$$b_t : u'(c_t) = \beta \tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1} \right]$$

$$s_t : u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1} \right] + \nu_t$$

- ψ_{t+1} appears in the right-hand sides of the Euler equations.
- By giving up one unit of c_t, HHs can reduce liquidity risk $-\theta b_t - s_t$.
  - $\tilde{R}_t$-unit reduction in debt $\rightarrow$ Reduce liquidity risk by $\tilde{R}_t \theta$ units.
  - R^s-unit increase in reserve $\rightarrow$ Reduce liquidity risk by R^s units.

Trade-off of holding reserves

- Combining the two Euler equations,

$$\beta(\tilde{R}_t - R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[(R^s - \tilde{R}_t\theta_{t+1})\psi_{t+1}] + \nu_t$$

- LHS: the opportunity cost of holding reserves due to interest gap $\tilde{R}_t > R^s$.
- RHS: relative advantage of reserves over debt in liquidity management:
  - If $R^s > \tilde{R}_t\theta$, accumulating reserves is more efficient than reducing debt in lowering liquidity risk $-\theta b_t - s_t$.
- Households choose b_t and s_t to equalize cost and benefit of holding reserves.
  - If cost is too high, households choose $s_t = 0$ and $\nu_t > 0$.

Propositions 1 and 2

$$\beta(\tilde{R}_t - R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[(R^s - \tilde{R}_t\theta_{t+1})\psi_{t+1}] + \nu_t$$

Proposition

If $\theta = 0$, households do not hold reserves, $s_t = 0$.

- $\theta = 0$ implies $\psi_{t+1} = 0$ for any states. No liquidity risk in the first place.

Proposition

If $\theta \geq R^s/\tilde{R}_t$, households do not hold reserves, $s_t = 0$.

- Relative advantage of reserves in liquidity management vanishes.

Proposition 3

$$\beta(\tilde{R}_t - R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[(R^s - \tilde{R}_t\theta_{t+1})\psi_{t+1}] + \nu_t$$

Proposition

Households do not hold enough reserves to cover the entire early repayment $-\theta b_t$ and eliminate liquidation. $-\theta b_t > s_t$ always holds.

- $-\theta b_t \leq s_t$ implies $a^{\ell}_{t+1} = 0 \rightarrow \psi_{t+1} = 0$.
- Because of the opportunity cost of holding reserves, households do not fully insure against liquidity shock.

Relationship between reserves and financial development θ

$$\beta(\tilde{R}_t - R^s)\mathbb{E}_t[u'(c_{t+1})] = \beta\mathbb{E}_t[(R^s - \tilde{R}_t\theta_{t+1})\psi_{t+1}] + \nu_t$$

- If θ is close to 0,
  - Early repayment $-\theta b_t$ is close to 0. $-\theta b_t > s_t$ implies s_t is close to 0.
  - Low liquidity risk implies low need for reserve holdings.
- If θ is high,
  - $-\theta b_t$ can be large, but relative advantage of reserves $(R^s - \tilde{R}_t\theta)$ is low.
  - Debt is so risky that households just reduce debt rather than holding reserves.
- Intermediate value of $\theta \to$ Both liquidity risk and relative advantage exist.

Social planner's solution

detail

- Social planner internalizes that q_t is decreasing in a_t^ℓ.
- First-order condition regarding a_t^ℓ:

$$\psi_t^{SP} + \varphi_t^{SP} = \frac{\xi_t}{q_t - \zeta q_t} - u'(c_t)$$

  - $(\partial q_t / \partial a_t^\ell) a_t^\ell = -\zeta q_t < 0$ is the fire-sale externality.
  - *Social* value of one unit of liquidity ψ_t^{SP} is greater than ψ_t.
  - Reducing liquidation a_t^ℓ increases its price q_t and reduces a_t^ℓ even more.

Social planner's Euler equations

- Planner's Euler equations:

$$u'(c_t) = \beta \tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi^{SP}_{t+1} \theta_{t+1} \right]$$
$$u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi^{SP}_{t+1} \right] + \nu_t$$

- $\psi^{SP}_{t+1} > \psi_{t+1}$ implies households overborrow and hold too little reserves.
- Planner's allocation can be achieved by tax on debt and either of:
  - subsidy on reserves, or
  - public reserve holdings with no private reserves.
- Propositions 1, 2, 3 hold under the planner's solution.
- $\partial \tau^b_t / \partial \theta > 0$ can be analytically shown in a simplified two-period model. tax

Calibration: externally determined parameter values

- One period is one year. Utility function is $u(c_t) = \ln(c_t)$.

	Parameter	Value	Source
β	Discount factor	0.91	Bianchi (2011)
R^b	Gross interest rate on debt	1.06	Standard
R^s	Gross interest rate on reserves	1	Standard
γ	Investment curvature	0.8	Comin and Gertler (2006)
$\bar{g}$	Foreign growth rate	0.0261	Data
ε^R	Interest rate shock	0.0196	Mendoza (2010)

- Three-state Markov process for shocks:
  - $(\varepsilon_t^R, \theta_t) = \{(\varepsilon^R, 0), (-\varepsilon^R, 0), (\varepsilon^R, \theta)\}$
  - In normal times, same shock with 54%, and liquidity shock with 10%.
  - In liquidity crisis, $(\varepsilon^R, 0)$ with 90%, and (ε^R, θ) with 10%.

Calibrated parameter values

	Parameter	Value	Target		Model
η	Investment efficiency	0.1085	Mean CA-to-GDP	-0.017	-0.017
κ	Productivity spillover	0.25	Fire-sale price/normal price	0.37	0.36
ζ	Share of foreign assets	0.46	Elasticity of fire-sale price	1.74	1.87
ψ_b	Debt-elasticity of spread	0.01	S.D. of CA-to-GDP	0.063	0.064
$\bar{b}$	Baseline debt-to-GDP	0.8	Mean debt-to-GDP	0.53	0.53
θ	Size of liquidity shock	0.45	Mean reserve-to-GDP	0.17	0.17

- Fire-sale price and elasticity are based on Aguiar and Gopinath (2005).
- Other moments are average of 47 emerging economies in 1987-2019.
- Baseline parameter is $\theta = 0.45$. Later study how the value of θ affects policy.

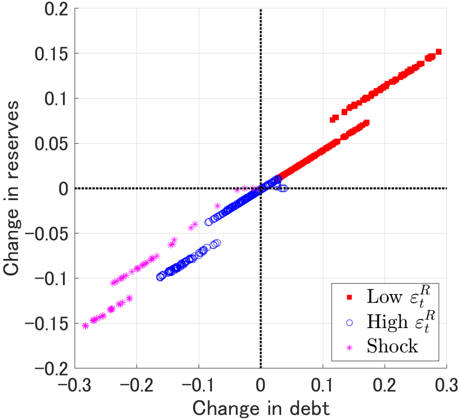
Simulation moments

- Moments are computed from 100,000-period stochastic simulations.

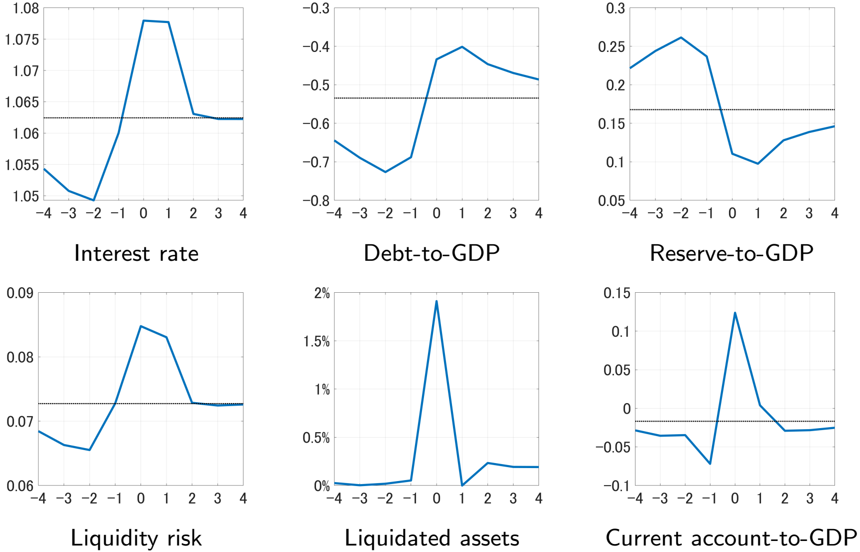
	Decentralized economy		Social planner	
	Mean	S.D.	Mean	S.D.
Consumption	0.807	0.035	0.811	0.036
Investment	0.181	0.164	0.171	0.172
Debt	−0.535	0.370	−0.530	0.342
Reserve	0.168	0.579	0.209	0.406
Current account	−0.017	0.065	−0.008	0.056
Mean tax on debt	...		4.78%	
Mean subsidy on reserve	...		10.49%	
Crisis probability	3.57%		0.27%	

Joint dynamics of debt and reserves

- Positive correlations between debt and reserve flows.
- Both debt and reserves increase when the interest rate is low, and vice versa.
  - Low interest rate → High debt and low opportunity cost → High reserves.

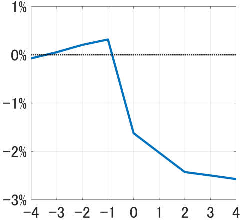


Crisis dynamics in decentralized economy

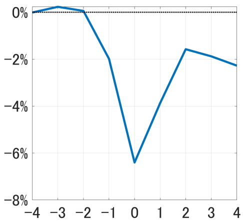


Crisis dynamics in decentralized economy

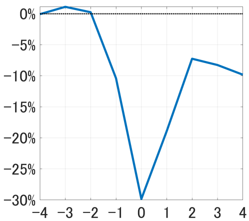
- Percentage deviations from pre-crisis 10-period log-linear trend.
- Persistent impacts are consistent with the empirical regularities of crises.



Output

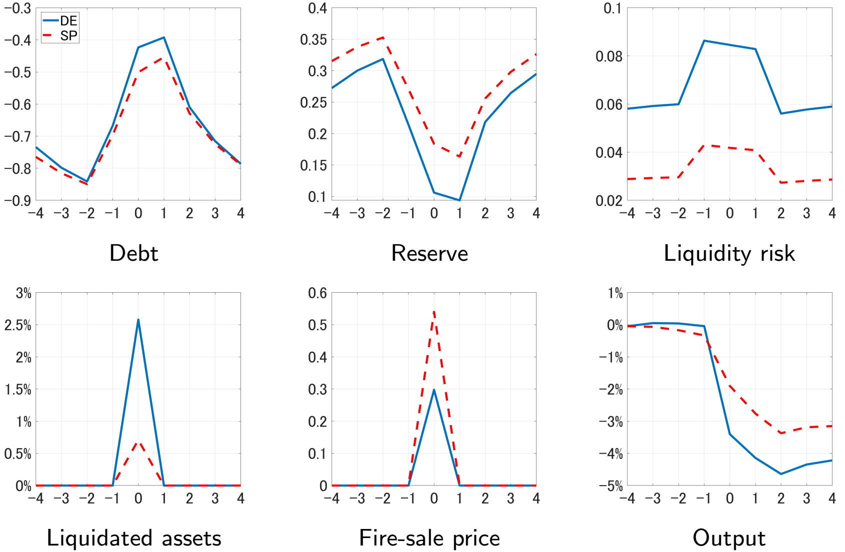


Consumption



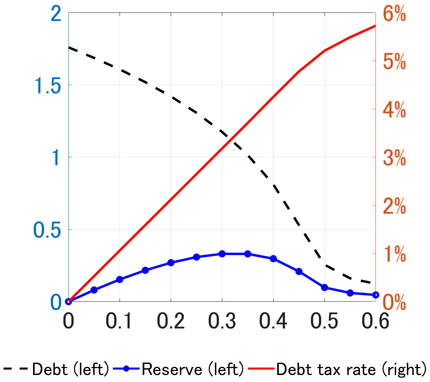
Investment

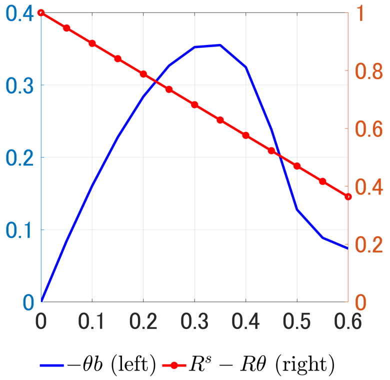
Crisis dynamics under DE and SP



Optimal policies and financial development θ

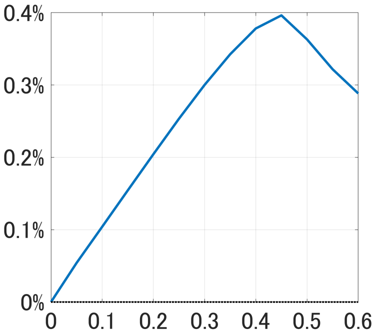
- Capital controls monotonically increase in θ.
- Reserve-to-GDP is non-monotonic and peaks at 33% when $\theta = 0.30$.
- Both liquidity risk and relative advantage are high for intermediate θ.

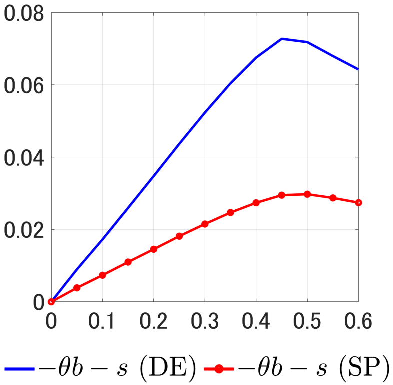




Welfare gains across different θ

- Expected welfare gain is the largest at 0.4% when $\theta = 0.45$.
- Excessive risk-taking in decentralized economy peaks when $\theta = 0.45$.





Conclusion and future work

What we do in this paper.

- We provide a liquidity theory for reserves and capital controls.
- The model can match several data patterns.
- We emphasize the importance of financial development for policy design.

To-do list

- Provide a Micro-foundation for financial development (θ).

Foreign reserves

- Foreign reserves: External assets that are readily available to and controlled by monetary authorities for
  - meeting balance of payments financing needs,
  - intervention in exchange markets to affect the currency exchange rate,
  - other related purposes (confidence in the currency and the economy, etc.).
- 60% of global reserves are in USD. US T-bills are typical reserve assets.
- IMF (2013) conducts a survey on the motivation for holding reserves.
  - 75% of countries: precautionary liquidity buffer.
  - 40% of countries: exchange rate level or volatility management.

Back

Optimality conditions by households

back

$$u'(c_t) = \beta \underbrace{\frac{R_t}{1 + \psi_b \exp\left(-\frac{b_t}{a_t} - \bar{b}\right) \frac{b_t/a_t}{R_t}}}_{\tilde{R}_t} \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1}\right]$$

$$u'(c_t) = \beta R^s \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\right] + \nu_t$$

$$u'(c_t) = \beta \mathbb{E}_t \left[\xi_{t+1} \eta \gamma \left(\frac{z_t}{(1-\kappa)a_t + \kappa a_t^*}\right)^{\gamma - 1}\right]$$

$$\xi_t = u'(c_t) \left[1 + \left(\frac{b_t/a_t}{R_t}\right)^2 \psi_b \exp\left(-\frac{b_t}{a_t} - \bar{b}\right)\right] + \beta \mathbb{E}_t \left[\xi_{t+1} \left\{\phi + \eta(1-\gamma)(1-\kappa)\left(\frac{z_t}{(1-\kappa)a_t + \kappa a_t^*}\right)^{\gamma}\right\}\right]$$

$$\psi_t + \varphi_t = \frac{\xi_t}{q_t} - u'(c_t)$$

Social planner's problem

back

$$
\begin{aligned}
&V(b_{t-1}, s_{t-1}, z_{t-1}, a_{t-1}; \Theta_t, a^*_{t-1}) \\
&= \max_{c_t, b_t, s_t, z_t, a^\ell_t, a_t} u(c_t) + \beta \mathbb{E}_t V(b_t, s_t, z_t, a_t; \Theta_{t+1}, a^*_t) \\
&- \lambda_t \left[c_t + \frac{b_t}{R_t} + \frac{s_t}{R^s} + z_t - a_t - b_{t-1} - s_{t-1} - q(a^\ell_t; a^*_t) a^\ell_t \right] \\
&- \xi_t \left[a_t - a_{t-1} - \eta (z_{t-1})^\gamma (a_{t-1} + \kappa(a^*_{t-1} - a_{t-1}))^{1-\gamma} + a^\ell_t \right] \\
&+ \psi^{SP}_t \left[q(a^\ell_t; a^*_t) a^\ell_t + \theta_t b_{t-1} + s_{t-1} \right] \\
&+ \varphi^{SP}_t q(a^\ell_t; a^*_t) a^\ell_t \\
&+ \nu_t \frac{s_t}{R^s}
\end{aligned}
$$

Tax on debt

back

- Tax on debt:

$$u'(c_t) = \beta(1+\tau_t^b)\tilde{R}_t \mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1} \right]$$

with

$$1+\tau_t^b = \frac{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}^{SP}\theta_{t+1} \right]}{\mathbb{E}_t \left[u'(c_{t+1}) + \psi_{t+1}\theta_{t+1} \right]}$$

- $\partial\tau_t^b/\partial\theta > 0$ can be shown in a simplified two-period model.
    - As θ becomes higher, the size of liquidation a_t^ℓ becomes larger and q_t lowers.
    - Lower q_t increases the value of liquidity ψ_t and ψ_t^{SP}.
    - Internalizing effect of a_t^ℓ on q_t, ψ_t^{SP} increases proportionally more than ψ_t.